Evaluating System Readiness Level Reversal Characteristics Using Incidence Matrices

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DEDICATION

This dissertation is dedicated to the late Dr. Gordon H. Clark. His collective corpus of writings in theology, logic, and philosophy guided me to think clearly, to live a life of purpose, and to work to the best of my God given ability. I never met Dr. Clark in my earthly life, but I look forward to discussing philosophy over a game of chess when we meet one day in the Celestial City.

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ABSTRACT

Evaluating System Readiness Level Reversal Characteristics Using Incidence Matrices

Contemporary system maturity assessment approaches have failed to provide robust quantitative system evaluations resulting in increased program costs and developmental risks. Standard assessment metrics, such as Technology Readiness Levels (TRL), do not sufficiently evaluate increasingly complex systems. The System Readiness Level (SRL) is a newly developed system development metric that is a mathematical function of TRL and Integration Readiness Level (IRL) values for the components and connections of a particular system. SRL acceptance has been hindered because of concerns over SRL mathematical operations that may lead to inaccurate system readiness assessments. These inaccurate system readiness assessments are called readiness reversals. A new SRL calculation method using incidence matrices, the Incidence Matrix System Readiness Level (IMSRL), was proposed to alleviate these mathematical concerns. The presence of SRL readiness reversal was modeled for four SRL calculation methods across several system configurations. Logistic regression analysis demonstrated that the IMSRL has a decreased presence of readiness reversal than other approaches suggested in the literature. The IMSRL was also analytically evaluated for conformance to five standard SRL mathematical characteristics and a sixth newly proposed SRL property. The improved SRL mathematical characteristics discussed in this research will directly support quantitative analysis of system technological readiness measurements.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

Symbol	Description
$A_{i,j} = \|a_{i,j}\|$	Adjacency matrix
$B_{i,j} = \|b_{i,j}\|$	Incidence matrix
$C_{n,m}$	Graph complexity measure as function of size (*m*) and order (n)
δ	Vertex degree
ε	Graph edge
$E\{\cdot\}$	Edge set
$G\{V,E\}$	Graph definition as set of Vertices and Edges
h	Head (termination) of a directed edge
$IRL_{a - b}$	Undirected IRL edge connecting vertex *a* and vertex *b*
$IRL_{a -> b}$	Unidirectional IRL edge starting at vertex *a* and ending at vertex *b*
$IRL_{a <-> b}$	Bidirectional IRL edge connecting vertex *a* and vertex *b*
m	Size of a graph
n	Order of a graph
t	Tail (origin) of a directed edge
TRL_x	TRL value of graph vertex *x*
$V\{\cdot\}$	Vertex set
X_x	Independent variables for logistic regression

LIST OF ACRONYMS

Acronym	Description
AD2	Advanced Degree of Difficulty
AT&L	Acquisition Technology and Logistics
CTE	Critical Technology Element
DOD	Department of Defense
DOT&E	Director of Operational Test and Evaluation
DT	Developmental Testing
EMD	Engineering, Manufacturing, and Development
GAO	Government Accountability Office
GT	Graph Theory
GTSRL	Graph Theory System Readiness Level
IMSRL	Incidence Matrix System Readiness Level
IOT&E	Initial Operational Test and Evaluation
IRL	Integration Readiness Levels
MC	Monte-Carlo
MDA	Missile Defense Agency
MRL	Manufacturing Readiness Level
NASA	National Aeronautics and Space Administration
RD3	Research and Development Degree of Difficulty
SAR	Selected Acquisition Reports
SoS	System of Systems

SRL	System Readiness Levels
SSRL	SysDML System Readiness Level
SysDML	Systems Development and Maturity Laboratory
T&E	Test and Evaluation
TA	Tropical Algebra
TASRL	Tropical Algebra System Readiness Level
TMP	Technology Maturation Plan
TPM	Technology Performance Measure
TPRI	Technology Performance Risk Index
TRA	Technology Readiness Assessment
TRI	Technology Risk Index
TRL	Technical Readiness Levels
TTRL	Technology Transfer Readiness Level
V&V	Validation and Verification
U.S.	United States

GLOSSARY OF TERMS

Adjacency Matrix: a symmetric *n x n* matrix, $A_{i,j} = |a_{i,j}|$, in which all non-zero elements define the sets of vertex connections of a graph

Adjacent Edges: any two edges that share a common vertex

Adjacent Vertices: any two vertices of a graph that are joined by a common edge

Complete Graph: a Simple graph in which each vertex is connected by one undirected edge with every other vertex in the graph

Degree: the number of proper edges of a graph incident on a common vertex, plus two times the number of self-loops of that vertex

Directed Edge: an edge with path flow from a designated originating vertex (tail) to a designated terminating vertex (head)

Edge: the connection link between adjacent vertices

General Graph: a graph that may have any number or combination of multiple edges (directed or undirected) and vertex loops

Graph: a mathematical construct that consists of a vertex set and an edge set; graphs are usually presented in pictorial form

Head: the designated terminating vertex of a directed edge

Incidence Matrix: an n x m matrix, $B_{i,j} = |b_{i,j}|$, in which all non-zero elements define the sets of edge connections of a general graph; incidence matrices do not necessarily have to be square symmetric

Integration Readiness Level: a nine level ordinal data scale that describes the capability or readiness of a given system connection between critical technology elements *maxDeg*: the maximum vertex degree present in a given graph

med(xRL): the median value of the set of TRL and IRL values present in a graph

min(xRL): the minimum value of the set of TRL and IRL values present in a graph

Mixed Graph: a graph that has both undirected and directed edges, but no vertex loops

Multi-Edge (or multiedge): a set of two or more edges with the same vertex connections

Multi-Graph: a graph that has both multi-edges and vertex loops

Neighbor: the set of vertices that are connected by edges to a common vertex.

Order: the number of distinct vertices in a graph (*n*)

Proper Edge: a graph edge that connects two distinct graph vertices

Regular Graph: a graph in which all vertices have the same degree.

Vertex-Loop: a graph edge that originates and terminates on the same vertex

Simple Graph: a graph with no directed edges, vertex loops, or multi-edges

Size: the number of distinct edges in a graph (*m*)

System Readiness Level: a [0,1] measurement scale that describes the relative advancement of sub-system and whole-system progress through development phases

Tail: the designated originating vertex of a directed edge

Technology Readiness Level: a nine level measurement scale that describes the relative technological advancement of critical technology elements

Tropical Algebra: a subset of mathematical geometry in which linear objects are used to fulfill the function of classical algebraic quantities

Vertex/Vertices: the connection endpoints of a given graph edge

CHAPTER 1 - INTRODUCTION

System maturity assessment approaches frequently fail to provide robust quantitative system evaluations. The 2013 US Government Accountability Office (GAO) Assessment of Selected Weapons Programs asserts that only 59% of evaluated systems achieved appropriate technological maturity upon entering formal technological development (GAO 2013). The promotion of immature systems can lead to increased program costs and developmental risks. DeNezza and Casey (2014) stress the need for system maturation metrics that support space system development. Additional GAO studies on acquisition best practices (GAO 1999 and 2006) emphasize a need for proper maturity of systems before integrating new technologies or progressing to subsequent acquisition phases. The GAO (2013) report makes a perceptive comment on the issue of system maturity metrics by affirming, "Demonstrating technology maturity is a prerequisite for moving forward into system development, during which the focus should be on design and integration." (GAO 2013, 22). Meier (2008) notes that unexpected technical issues may occur from immature technology insertion into systems leading to increased cost and schedule risks.

While there is a growing acceptance of the need to demonstrate system technical maturity, the suite of available measurement tools is of only recent origin. The Technology Readiness Level (TRL) was one of the first widely adopted technological assessment scales but is limited to assessing individual system Critical Technology Elements (CTE) or specific systems (Mankins 2002). This TRL limitation spawned the subsequent development of numerous system maturity metrics including the Integration

Readiness Level (IRL) (Sauser et al. 2006 and 2008) and the System Readiness Level (SRL) (Sauser et al. 2011). The newer metrics sought to address TRL limitations by evaluating whole-system and System-of-Systems (SoS) developmental maturity (Sauser et al. 2008).

1.1 Introduction

SRL measurements assess system maturity by calculating mathematical functions of a particular system's TRL and IRL values. Opponents of SRL criticized early SRL calculation methods (Sauser et al. 2006) for employing invalid mathematical operations on ordinal data values (Kujawski 2010 and 2013). The multiplication of ordinal data can produce decision rank reversals (Bowles 2008; Stevens 1946; Townsend and Ashby 1984; Velleman and Wilkinson 1993). SRL rank reversal--or to select a more appropriate term of "Readiness Reversal"—occurs when a particular combination of system TRL and IRL values produce an inaccurate system readiness value. Improper system readiness values may occur when: (1) various combinations of TRL and IRL system values produce the same SRL value (Kujawski 2013; McConkie et al. 2012; Sauser et al. 2011), or (2) the calculated SRL value exceeds the range supported by the system's constituent TRL or IRL values (Engle et al. 2009). SRL readiness reversal calculates a higher SRL value than what a system's constituent TRL and IRL values can support, thereby prematurely promoting a system to a subsequent acquisition phase before the system has attained the necessary characteristics and demonstrated the required performance.

This research evaluated SRL limitations, especially the extent of SRL readiness reversal for four SRL calculation methods. A new SRL calculation method, the

Incidence-Matrix System Readiness Level (IMSRL) approach, is presented as a solution to alleviate SRL readiness reversal and address other SRL calculation method shortfalls.

1.2 Research Problem Statement

Contemporary system maturity assessment approaches have failed to provide robust quantitative system evaluations resulting in erroneous decisions yielding increased program costs and developmental risks. Standard assessment metrics, such as the TRL, do not sufficiently evaluate increasingly complex systems. The SRL is a newly developed system development measure that mathematically combines the TRL and IRL values of individual system elements. Potentially erroneous system readiness assessments, however, have hindered SRL acceptance.

1.3 Purpose of Research

The primary purpose of this research is to improve SRL calculation validity. To accomplish this purpose, a new SRL calculation methodology, the IMSRL, was developed to provide superior calculation characteristics compared to legacy approaches. The secondary purpose of this research is to establish a quantitative relationship between system SRL models and the presence of SRL readiness reversal characteristics. A third purpose is to demonstrate the analytical relationship of the proposed IMSRL model and five desired mathematical SRL properties espoused by McConkie (2013) and McConkie et al. (2012). The readiness reversal analysis leverages logistic regression analysis methods to satisfy the first purpose. The proposed implementation of SRL models with reversal data analysis is demonstrated using a simple Monte-Carlo simulation analysis approach and logistic regression analysis. The third purpose of this study evaluates more complex graphical models than can presently be explored using contemporary methods.

The research described in this work, combined with findings from related literature, will support future research of the mathematical integration of system maturity metrics into performance growth assessments.

1.4 Research Scope and Limitations

Traditional measurement theory suggests two types of measurement properties, namely mathematical and utility properties (Bedford and Cooke 2001). This research focuses on the mathematical properties of SRL models. A quantitative analysis of the mathematical properties of SRL readiness reversal is presented that examines the relationship between SRL models and system structure parameters such as system order and system size. The assessment of SRL readiness reversal ensures that decision makers who utilize SRL will have confidence in their respective application and results.

Beside the specific research scope outlined above there are research scope limitations that bear mention. The research accepted the implementation of SRL values within a probabilistic structure as demonstrated by Tan (2011) but did not seek to demonstrate the validity of probabilistic SRL distributions for system evaluations. In addition this research did not propose improvements to legacy SRL approaches, but accepted the approaches employed by Sauser and Ramirez-Marquez (2009), Sauser (2011), Garrett et al. (2011), and McConkie (2013) as they were established in the literature.

1.5 Significance and Interest

This research will interest three primary stakeholders: systems engineers, program managers, and test and evaluation (T&E) engineers. The significance of this research to Systems Engineering is the expansion of the mathematical properties of SRL and empirical evaluation of SRL readiness reversal characteristics. Such validated analysis will support further implementation of SRL as a viable tool within the systems engineering community. Future applications of SRL models may enhance systems development monitoring and support capabilities for connecting system readiness to system performance monitoring. The proposed IMSRL framework should be implemented in systems engineering or program management bodies of knowledge thereby supplementing existing system evaluation tools such as Technical Performance Measure (TPM) monitoring and Technology Maturation Plan (TMP) development. Technical attributes of the proposed IMSRL model, in particular the synthesis of incidence matrix graph properties and mathematical set theory operations, may prove useful in other disciplines such as social network analysis, network implementation models, and organizational management. The T&E community will additionally benefit from validated SRL models by extending their respective application for complex systems within the DOD-5000.02 acquisition process (DODINST 2013) and other requirements tracking and analysis methods.

1.6 Rationale and Relevance

This research will support SRL implementation and provide a novel addition to the family of SRL calculation methods. Longer term application of this research is the extension of SRL methods to system configurations beyond those presently supported by existing methods. Such an extension of SRL capabilities would support complex System-of-Systems (SoS) analysis and mission threat capability analysis (Volkert et al. 2011, 2012 and 2013). The IMSRL model structure may also support improved technical performance metric tracking methods.

1.7 Research Methodology

A background literature review of system maturity metrics, SRL development, and recent SRL applications was performed. Limitations of SRL readiness reversal, system modeling methods using graph theory mathematics, and other SRL calculation methods are reviewed. Using this background material, a new incidence matrix based SRL model is developed using contributions from graph theory incidence matrices and mathematical set theory operations. The remainder of the research work focuses on the comparison and extension of the new IMSRL model with other competing SRL model approaches. The effects of SRL model type, system size, and system order on SRL readiness reversal are then modeled using a Monte-Carlo simulation approach and evaluated using logistic regression. The IMSRL model is analytically demonstrated for conformance with published SRL mathematical characteristic requirements. The research concludes with assessment of the results and planning for results feedback through the research methodology via future research efforts.

1.8 Organization of Dissertation

The research development, background, and results are presented in the following chapters. In chapter 2 a literature review is provided that discusses the development and application of SRL methods. Chapter 2 also provides working definitions of several key terms that will be used throughout the remainder of the work. Chapter 3 discusses specific contributions to the systems engineering body of knowledge, specifically SRL readiness reversal assessment and the development and validation of the new IMSRL model. Additional discussion in Chapter 3 expands on the research contributions to the larger body of knowledge including quantitative assessment of SRL readiness reversal.

The full research methodology is presented in Chapter 4 as a systems engineering oriented development process. The research method emphasizes the IMSRL model development and the pursuant SRL readiness reversal properties as well as SRL analytic evaluation validation of the new IMSRL model. Research results are provided in Chapter 5 with mathematical details of the IMSRL structure, discussions of the SRL readiness reversal results, and IMSRL analytical mathematical evaluation. Detailed research results on a Monte-Carlo readiness reversal simulation and logistic regression analysis results are also provided. Research conclusions and future research suggestions conclude the dissertation.

CHAPTER 2 – LITERATURE REVIEW

This chapter provides a substantive review of the pertinent literature related to SRL development and mathematical characteristics. The history and development of general system maturity metrics is presented along with background of TRL, IRL, and SRL formulation. Detailed discussion of legacy SRL calculation methodologies including matrix algebra, graph theory, and tropical algebra approaches are mentioned. Definitions of key terms establish a standard terminology for the remainder of the technical discussion.

2.1 Background

The 2006 Government Accountability Office (GAO) Assessment of Selected Weapons Programs considered the unintended costs of carrying immature technologies into subsequent phases of system development (GAO 2006). The report indicated that average unit procurement costs increased 1% for systems employing mature technologies but increased 27% for those systems using immature technologies (GAO 2006). The equivalent developmental cost increase for systems using immature technologies versus mature technologies increased 4.8% to 34.9% respectively (GAO 2006).

Subsequent GAO reports further assessed the concerns and performance risks of immature systems during acquisition development. A 2011 GAO report (GAO 2011) noted four significant program characteristics that engender increases in cost and schedule. One significant factor is the technology maturity integrated into the system. This report also noted that programs implementing previously demonstrated technologies had 33% lower cost and schedule growth than developmental programs employing technologies with unsuitably low TRL values (GAO 2011). The cost potential and inherent performance risks of advancing immature technologies into acquisition systems spurred the development of various system technology assessments (Mahafza et al. 2004), some of which are discussed below.

2.2 Definitions of Key Terms

The 20th century American philosopher Gordon H. Clark often stated that in the interest of clarity, one should carefully define one's terms at the beginning of a discussion. His injunction that, "…unless one knows the definition, he does not know what he is talking about," (Clark 2002, pg. 138), applies to the esoteric sphere of philosophical discourse as well as to the pragmatic world of systems engineering. To heed his poignant suggestion for this discussion requires a brief segue on the distinctions between the use of the terms "readiness" and "maturity" as applied to systems assessments, and "measurement" and "metric" for use with specific object parameters.

2.2.1 Readiness and Maturity

In systems technological development literature the terms system "readiness" and system "maturity" are not employed in a consistent manner. Mandelbaum (2005) suggested that the "readiness" of a technology implied a certain level of developmental maturity for a given application in that a particular technology may be "ready" for use in a particular environment or for a specific mission, but may be unsuited (or "immature") for a different use. Smith (2005) further elucidated a distinction between readiness and maturity by noting that a system considered mature in one context may not possess sufficient readiness for operation in a different environment. Bilbro (2007), however, used "maturity" as part of the definition of "readiness" and thereby implied a relationship between the two terms.   SoS literature also used the terms interchangeably (Azizian et al. 2011; Valerdi and Kohl 2004). Tetlay and John (2009 and 2010) noted that definitions of system readiness inherently included maturity as a component term. For this research, the author accepted that the terms “readiness” and “maturity” are neither denotatively interchangeable nor mutually exclusive. Yet due to the widespread use of “readiness” of a system capability to complete a given function, the author adopted the term “readiness” rather than “maturity” throughout this paper.

2.2.2 Metric and Measurement

In addition to clarifying a distinction between readiness and maturity within a systems development context, the terms “metric” and “measurement” also require additional elucidation. According to *The Merriam-Webster Dictionary*, denotative definitions of “measurement” include: (1) the act or process of measuring, or (2) a figure, extent, or amount obtained by measuring. Measurement can mean both a process of acquiring a given quantitative value of a given parameter, and the actual parameter itself. The term “metric” however contains the concept of measurement within itself. A quick consultation of *The Merriam-Webster Dictionary* reveals that “metric” may be defined as, “a standard of measurement.”   This research employed the first definition of “measurement” above as the process of measuring a particular readiness level of a given technology, integration, or system element.   “Metric” was correspondingly used to refer to a scale of reference against which such measurements were compared.

2.3 General Readiness Measures

The systems engineering requirement for processes and tools to assess the technological development of systems sowed the seeds for a fertile growth of qualitative and quantitative methods. While the TRL, discussed in section 2.4 below, attained preeminence among technological assessment tools, other methods offered competing approaches.

Qualitative measures such as the Technology Readiness Transfer Level (TTRL) (Holt 2007), the Missile Defense Agency (MDA) Checklist (Mahafza 2005), and the Technical Risk Index (TRI) (Garvey and Cho 2005) all employed alternate scales to assess system development risk other than the TRL. These tools have found only limited applications, however the Manufacturing Readiness Level (MRL) has found broad acceptance within the DOD for assessing system readiness for manufacturing and production (DOD[TRA] 2009).

Quantitative tools included the Advanced Degree of Difficulty (AD2) (Bilbro 2002), the Research and Development Degree of Difficulty (RD3) (Mankins 1998), and the Integrated Technology Analysis Methodology (ITAM) (Mankins 2002). Mahafza et al. (2004) proposed a Technology Performance Risk Index (TPRI) that tracked technology readiness throughout a system lifecycle. The TPRI leveraged the system performance requirements, the Bilbro (2002) AD2 measurement, and the set of unmet requirements via a feedback process to calculate a given system’s performance risk.

The details of these alternate methods will not be discussed here but the interested reader should consult two excellent papers by Azizian (2009 and 2011) and Chapter 2 of McConkie (2013) that provide an excellent review of the recent literature on system technology assessment methods.

2.4 Technology Readiness Levels

The system performance methods discussed in section 2.3 exhibit a range of capabilities yet have not achieved widespread adoption, with the exception of the MRL. A focus of the system measurements that have achieved a level of acceptance and application across system archetypes is therefore a fruitful exercise. The TRL is a foundational element of contemporary system technological assessment and deserving of particular attention.

TRL measurements assess the technological progression of Critical Technology Elements (CTE) of a given system or subsystem (DOD[TRA] 2009). Qualitative and quantitative TRL measurements were founded in the 1970s for spaceflight systems (Banke 2011; Dacus 2012; Mankins 2002 and 2005). TRL use rapidly expanded beyond the National Aeronautics and Space Administration (NASA) to the US Department of Defense (DOD) for use in Technology Readiness Assessments (DOD[TRA] 2009; GAO 2013; Mankins 2002; Sauser et al. 2010). As TRLs proliferated throughout the defense and commercial sectors, however, limitations of the TRL began to be openly discussed (Mankins 2002). TRL suitability was questioned and alternative solutions were sought to provide additional information for program decision makers (Azizian et al. 2009 and 2011; Garrett et al. 2011). TRLs only evaluated one critical system or technology component at a time and cannot evaluate inter-component connections or integrations (Azizian et al. 2011). These TRL limitations implied that TRL could not be easily extended to complex systems with integration connections between CTEs. Such considerations spawned the gestation of new system technology assessments approaches including the IRL and the SRL in the mid-2000s (Sauser et al. 2008). A representative 9-

level TRL scale definitions are provided in Table 2-1 following discussion of the IRL and SRL.

2.5 Integration Readiness Levels

The rapid development of SoS architectures supported the development of IRL to link disparate system technology elements within a comprehensive system model (Long 2011; Sauser et al. 2008). Since the TRL cannot evaluate complex systems inter-connections (Azizian et al. 2009 and 2011; Mankins 2002; Sauser et al. 2008, 2009, and 2011), IRLs were developed to address this limitation. IRL were proposed by Sauser et al. (2006) to assess the capabilities of system technology element inter-connections. Early IRL were 7-level integer-valued scales (Sauser et al. 2006) and were subsequently expanded to 9-level scales to better align with the TRL scale. Like the TRL before them, IRL initially focused on hardware applications but have expanded to software system applications (Long 2011). As TRL were limited to considering discrete technology elements, the IRL is also inherently limited to only considering the readiness of the connecting technology links between discrete components. The Sauser et al. (2008) IRL is currently the most commonly used IRL hardware metric presently considered in the literature. References to "IRL" throughout the rest of this dissertation will only consider the particular IRL configuration of Sauser et al. (2010). Definitions of the 9-level Sauser et al. (2010) IRL scale are listed in Table 2-1.

Although the IRL provided a valuable addition to the TRL for assessing technological system components, a holistic approach was required to evaluate complex systems rather than only individual components. In order to evaluate the technological state of a complex system comprised of both technology elements and integrated connecting technological links, a new assessment framework was required. This new framework became the SRL.

2.6 System Readiness Levels

The SRL was developed to evaluate whole-system development risk and to support program acquisition decisions (Sauser et al. 2006, 2008, and 2011). SRLs mathematically combined component TRL values with system integration IRL and created a separate measure of system technical progress (Sauser et al. 2011). SRLs were initially developed at the Systems Development Maturity Laboratory (SysDML) and were expanded in numerous subsequent works (Ramirez-Marquez et al. 2009; Sauser et al. 2012). The underlying motivation of SRL was that complex systems and SoS require a more robust readiness assessment than TRL component evaluations alone could provide. SRL values are calculated for individual components (both technology components and integration elements) and these subsystem component values produced a comprehensive system level SRL value. Early SRL development used pairwise matrix calculations to represent system structure. Recent SRL research has considered probabilistic distributions of SRL values (Tan et al. 2011 and 2013) and even tropical algebra computational approaches (McConkie et al. 2012).

Shortly after SRL were proposed, the systems engineering community rapidly explored SRL applications. SRL were subsequently leveraged to evaluate aircraft mechanical systems (Kober and Sauser 2009), shipboard mission systems (Forbes et al. 2008), mission thread analysis (Garrett et al. 2011) and multi-capability systems (Baron et al. 2011; Tan et al. 2013; Volkert et al. 2011, 2012, and 2013). Malone and Wolfarth

(2012) implemented SRL to support programmatic cost and schedule decisions by applying a modified version of the McCabe (1976) cyclomatic complexity.

Garrett et al. (2011) analyzed several contemporary methods for program maturity measurements and suggested a mathematical approach using graph theory to analyze System of Systems (SoS) architectural frameworks. The Garrett et al. (2011) paper, while not SRL-centric per-se, provided a useful discussion of the strengths and relative weaknesses of SRL implementation via a graph theory paradigm. A constructive paper by McConkie et al. (2012) and subsequent PhD dissertation (McConkie 2013) represented a significant and mathematically robust advancement in SRL calculations. The McConkie tropical algebra approach addressed certain mathematical limitations of the ordinal matrix methods and reduces the likelihood of illogical or unreliable calculation results.

Table 2-1 lists common definitions for TRL, IRL, and SRL scales, ~~and~~ representative acquisition phase value ranges, and source. The scales in Table 2-1 suggest that comparisons between individual TRL and IRL values and the equivalent SRL values can be made. The forthcoming discussion of SRL readiness reversals leverages this comparative structure between computed SRL values and system element TRL and IRL values.

Table 2-1: List of TRL, IRL, SRL values, acquisition phases, and source.

Scale Level	TRL Hardware Scale (DOD[TRA} 2009)	IRL Scale (Sauser et al. 2010)	SRL Definition (Sauser et al. 2010 and 2011)	Acquisition Phase (DOD(TRA) 2009)
1	Basic principles observed and reported	Interface defined to characterize component relationship.	Refine initial concept. Develop system/technology development strategy.	Material Solution Analysis (MSA)
2	Technology concept and/or application is formulated.	Characterize technology interface interactions.		
3	Analytical and experimental critical function and/or proof of concept demonstrated.	Compatibility demonstrated between technology elements.		
4	Component validated in a laboratory environment.	Quality assurance of integration quality.	Reduce technology risks and determine appropriate set of technologies to integrate into a full system.	Technology Demonstration (TD)
5	Component validated in a relevant environment.	Sufficient control established between technologies to establish, manage, and terminate the integration.		
6	System/subsystem demonstrated in a relevant environment.	Integrated technologies can accept, translate, and structure information.	Develop a system or capability increment; reduce integration and manufacturing risk; ensure operational supportability; reduce logistics footprint; implement human systems integration; design for producibility; ensure affordability and protection of critical program information; and demonstrates system integration, interoperability, safety, and utility.	Late Technology Demonstration - or- Engineering & Manufacturing Development (EMD)
7	System prototype demonstrated in a relevant environment.	Integration requirements are validated and verified.		
8	Actual system completed and qualified through test and evaluation.	Integration completed and mission qualified via test and evaluation.	Achieve operational capability that satisfies mission needs.	Production & Deployment (P&D)
9	Actual system proven through successful mission operations.	Integration is mission proven through successful mission operations.	Execute a support program that meets operational support performance requirements and sustains the system in the most cost-effective manner over its lifecycle.	Operations & Support (O&S)

2.7 SRL Applications

Shortly after SRL were proposed, the systems engineering community rapidly explored SRL applications. Malone and Wolfarth (2012) implemented SRL measurements for programmatic cost and schedule decisions by applying a modified version of the McCabe (1976) cyclomatic complexity model to determine the additional effort needed to increase the system SRL. Magnaye et al. (2010) considered cost optimization applications of SRL with a constrained optimization model. The Magnaye model used expanded SRL beyond simple readiness assessments. Garrett et al. (2011) analyzed several methods for program maturity assessments and proposed an approach using graph theory to analyze SoS architectural frameworks. The Garrett paper, while not an SRL-centric paper provided a useful discussion of the strengths and relative weaknesses of SRL implementation.

Signifying a departure from system cost or schedule considerations, Tan et al. (2010 and 2011) proposed a probabilistic SRL-based measurement for component importance analysis. This paper extended SRL by leveraging distributions of subject matter expert inputs for readiness levels. The underlying SRL calculation approach, however, was fundamentally identical to that used by Sauser et al. (2010). Guo et al. (2012) modeled time-domain system readiness assessment changes using a Markov-Chain approach, but as with the Tan (2010 and 2011) papers, the underlying SRL calculation approach mirrored that of Sauser et al. (2010).

A constructive paper by McConkie et al. (2012) and subsequent PhD dissertation (McConkie 2013) represented the most recent significant and mathematically robust advancements in SRL calculations. McConkie's work advanced SRL development in two key areas: (1) Evaluated a tropical algebra matrix calculation approach, rather than the original SRL pairwise ordinal matrix calculation approaches, that addressed certain mathematical limitations of the ordinal matrix methods, and (2) Provided analytical evaluations of several required mathematical properties for improved SRL rigor. These two contributions reduced the likelihood of illogical or unreliable calculation results of the originally proposed methods by Sauser et al. (2006 and 2008).

Dacus (2011) proposed SRL as a component of a comprehensive system readiness measurement for analyzing program cost and schedule data from 70 Selected Acquisition Reports (SAR). The Dacus paper suggested a weak relationship between TRL shortfalls, and program cost and schedule overruns. Although SRL usage and applications have steadily advanced, little quantitative work has been performed to assess the underlying stability of SRL calculation methods. The primary SRL calculation methods will be demonstrated by example in the next section leading to a subsequent discussion of specific SRL calculation limitations.

2.8 Representation of Systems as Graphs

SRL evaluations of systems are strongly dependent on the nature of the system representation used. Graph theory representations are foundational to SRL evaluations. Graph theory representations of networks and systems have enjoyed significant interest from a variety of technical disciplines including biological systems (Mesquita et al. 2002; Koch et al. 2004), electrical power systems (De La Ree et al. 2010), and social networking (Bonacich, Holdren, and Johnston 2004). Early SRL development by Sauser (2006, 2008) leveraged a modified weighted adjacency matrix approach to model simple system CTE components and interconnections. Garrett et al. (2011) and McConkie (2012 and 2013) employed a more accurately defined weighted adjacency matrix approach. Models of complex systems rely heavily on adjacency matrices (Bonacich, Holdren, and Johnston 2004; Newman 2004; Mesquita, Salazar, and Canazio 2002; Singh and Sharma 2012) yet incidence matrices comprise a robust alternative for system representation, especially for multigraphs and hypergraphs (Balbuena 2008; Bonacich, Holdren and Johnston 2004; Fulkerson and Gross 1965).

SRL calculation methods represent a particular system by encoding CTEs, and their respective TRL values, as graph nodes (Garrett et al. 2011). Connection paths between graph nodes, and the corresponding IRL values, are depicted as graph edges. For a given graph, the number of vertices, n, equals the number of TRL components. The graph edges, m, equal the number of IRL connections between connected graph nodes. A graph may then be mathematically defined as the sets of vertices and edge connections (Diestel, 2010). In mathematical form we may represent a graph as $G=\{V,E\}$ where $V=\{v_1, v_2, \ldots, v_n\}$ is the set of graph vertices and $E=\{\varepsilon_{1,1}, \varepsilon_{1,2}, \ldots, \varepsilon_{n,n}\}$ is the set of graph edges (Chartrand 1985; Chartrand and Zhang 2012; Diestel 2010; Gross and Yellen 2006). The edge set element subscripts denote the starting node and ending node connection for a given edge. Edges may have binary or non-binary weights and may be directed or undirected (Newman 2004; Singh and Sharma 2012).

The use of graphs for system representation requires a consideration of graph structure and complexity. Early complexity measures included the McCabe cyclomatic number for use in computer program control graphs (McCabe, 1976). Later graph complexity measures considered different graph topologies and measures of linear complexity (Costa et al. 2007; Jukna 2004; Neel and Orrison 2006). For simple graphs, in which only one non-directional edge between a given vertex pair is permitted, the set of permitted edges ranges from $m=\{(n-1), n, .., n(n-1)/2\}$ (Kim and Wilhelm 2008). This dissertation is not focused on the intricacies of graph complexity measures and therefore a simple graph complexity measure, $C_{n,m}=n+m$ was adopted as a graph complexity measurement. Other graph complexity measurements may be considered for future SRL research, but lie outside the scope of this dissertation.

To demonstrate the relationship between graph theory and SRL calculations consider the system graph in Figure 2-1(b). This graph contains three nodes *(TRL_1, TRL_2, TRL_3)* and three edges *($IRL_{1,2}$, $IRL_{2,3}$, $IRL_{1,3}$)*. The four graphs in Figure 2-1 are "complete" graphs in which each individual vertex, v_i, is connected to every other vertex in the total vertex set *V* using a using a non-directional edge (Chartrand and Zhang 2012, 19). Sample system configurations of order *n*=2, 3, 4, 5 and size *m*=1, 2, 6, 10 respectively are depicted in Figure 2-1.

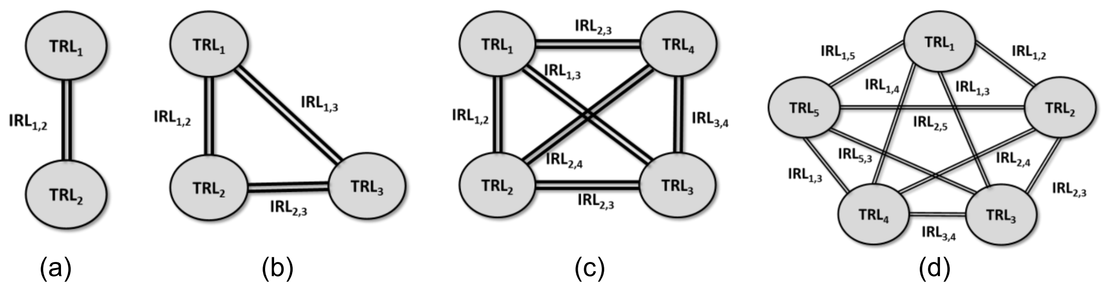


Figure 2-1: Complete graph systems various order and size.

This figure contains four system examples: (*a*) System of order *n*=2 and size *m*=1; (*b*) System of order *n*=3 and size *m*=3; (*c*) System of order *n*=4 and size *m*=6, and (d) System of order *n*=5 and size *m*=10.

The systems of Figure 2-1 are notional and do not necessarily correspond to any real system. A direct relationship exists between the order and size of a complete graph (Chartrand and Zhang 2012, pg. 19; Diestel 2010). Complete graphs will form the foundation of the later SRL readiness reversal analysis because they possess two key qualities: (1) ease of quantitative representation in adjacency and incidence matrices, and (2) ability to be represented and evaluated by all four SRL methods considered in this dissertation. The first quality permits easy simulation and consistency of analysis for system modeling. The second quality facilitates direct comparison of SRL methods that can evaluate the same system architectures.

The systems in Figure 2-1 may be mathematically represented using either adjacency or incidence matrices (Gross and Yellen 2006). The general adjacency matrix form $A=(a_{i,j})_{nxn}$ of a graph G, is a symmetric binary valued matrix in which each element is nonzero if and only if a particular edge $\varepsilon_{1i,j}$, belongs to the edge set E (Chartrand and Zhang 2012). In mathematical form this relationship is represented in Equation 2.1 (Diestel 2010).

$$a_{i,j} = \begin{matrix} 1 & v_i, v_j \subseteq E \\ 0 & otherwise \end{matrix} \tag{2.1}$$

In contrast to an adjacency matrix that encodes graph vertex-vertex connections, the incidence matrix $B=(b_{i,j})_{nxm}$ depicts a system graph by describing the relationship of vertices incident with a given edge. In mathematical form the binary-valued incidence matrix may be described in Equation 2.2 (Diestel 2010).

$$b_{i,j} = \begin{matrix} 1 & v_i \subseteq \varepsilon_j \\ 0 & otherwise \end{matrix} \tag{2.2}$$

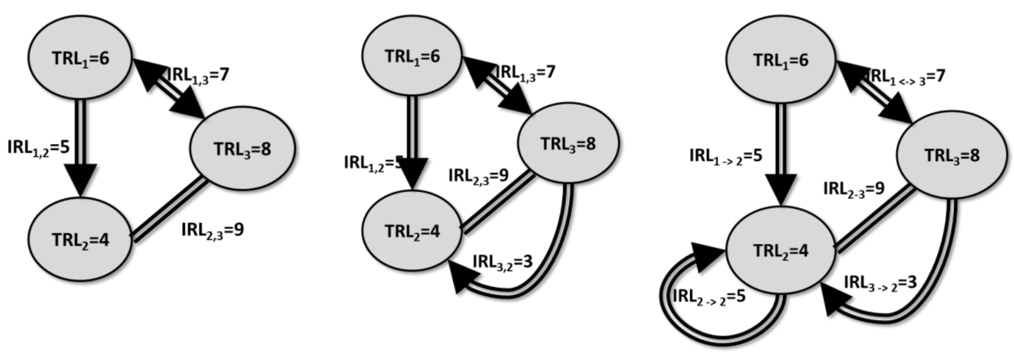
The corresponding adjacency and incidence matrices for Figure 2-1(b) are provided in Equation 2.3 and Equation 2.4 respectively.

$$a_{i,j} = \begin{matrix} & v_1 & v_2 & v_3 \\ v_1 & 0 & 1 & 1 \\ v_2 & 1 & 0 & 1 \\ v_3 & 1 & 1 & 0 \end{matrix} \tag{2.3}$$

$$b_{i,j} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array} \qquad (2.4)$$

The left hand adjacency matrix in Equation 2.3 depicts the mathematical relationships between connected vertex elements. Vertices v_1 and v_2 are connected by non-directional edges and the adjacency matrix shows a corresponding non-zero value at indices $a_{1,2}$ and $a_{2,1}$. The same relationship holds for the vertex sets $\{v_2, v_3\}$ and $\{v_1, v_3\}$.

In contrast to the adjacency matrix, the right hand incidence matrix in Equation 2.4 shows the edges $\varepsilon_i|_{i=1,m}$ that are incident to a given vertex $v_i|_{i=1,n}$. Edge e_i connects the adjacent vertices v_1 and v_2. This connection is depicted by the non-zero elements in the first column of the incidence matrix b_{ij} in Equation 2.3. The adjacency and incidence matrix example is sufficient for simple complete graphs in Figure 2-1, but more complex architectures are often encountered. Figure 2-2 provides examples of several more complex graph architectures.



(a) (b) (c)

Figure 2-2: Mixed graph, Multigraph, and General graph examples.

This figure provides examples of three system graph types: (a) Mixed graph with directed and undirected edges, (b) Multigraph with multiple edges but no vertex loops, and (c) General graph with both multi-edges and self-vertex loops.

Figure 2-2(a) is a mixed graph that has both undirected and directed edges. The multigraph of Figure 2-2(b) has both directional edges a multiple edge set between TRL_2 and TRL_3. The addition of a self-vertex loop on TRL_2 distinguishes the General graph of Figure 2-2(c) from Figure 2-2(b).

Both adjacency and incidence matrices can support simple graphs and the mixed graph of Figure 2-2(a), which contains both undirected and directed edges (Diestel 2010; Gross and Yellen 2006). Both matrix types can also support system configurations of Figure 2-2(b) that have vertex loops, in which a graph edge emanates and terminates on the same vertex. General graphs like Figure 2-2(c), which may contain multiple vertex loops or edges (either directional or nondirectional) between a given vertex pair, can be effectively represented using incidence matrices (Chartrand and Zhang 2012; Gross and Yellen 2006). Adjacency matrices, at least for the conventional graph theory considered in this dissertation, cannot be readily extended to the General graph case. Some authors have considered adjacency matrix extensions for complex biological systems using petri nets (Chaouiya 2007; Koch, Junker, and Heiner 2004) or system readiness functions as (Marchette 2010). These novel approaches are worthy of future consideration but have not been vetted in the literature and will therefore not be evaluated in this dissertation. The next section will briefly discuss SRL calculation methodologies.

2.9 SRL Calculation Methodologies

Three primary mathematical approaches for calculating SRL values are found in the literature: the Systems Dynamic Maturity Laboratory (SysDML) model (Sauser et al. 2006, 2008, 2010, and 2011), a formal graph theory approach (Garrett et al. 2011) and a tropical algebra method (McConkie 2013). In order to clearly distinguish among the SRL models under consideration, this research adopted a modified terminology of McConkie (2013) and will refer to the SysDML model as the SSRL (Sauser 2008 and 2011). The Garrett et al. (2011) graph theory method will be referred to as the GTSRL approach and the McConkie (2013) tropical algebra approach as the TASRL. Each calculation approach is described in more detail in Appendix A.

2.9.1 SSRL Method

The SSRL method employs a pairwise matrix-vector multiplication of a weighted IRL adjacency matrix and a TRL column vector (Sauser et al. 2008). The TRL and IRL values are converted from a [1,9] integer scale to a [0,1] continuous scale by dividing the TRL and IRL values by 9. The normalized quantities are matrix multiplied together to produce a new vector. A normalization factor is applied and the SSRL is calculated as the resulting vector mean. The IRL_{SSRL} square nxn matrix and the k_{SSRL} normalization factor are unique to the SSRL method. The SSRL method is unable to address configurations that have self-referencing vertex loops or multiple directed edges between adjacent vertices. An example of a self-referencing vertex loop may be seen in the IRL edge $IRL_{2\text{-}2}$ in Figure 2(c). A multiple directed edge may be seen in the pair of edges between TRL_3 and TRL_2 in Figure 2(b).

2.9.2 GTSRL Method

The SSRL matrix-vector method represents the earliest SRL calculation method, but research from Garrett et al. (2011) suggests that employing the full capability of adjacency matrices provides a more robust approach. The GTSRL approach generates the TRL vector and IRL matrix in an almost identical manner to the SSRL manner but differs from the SSRL in three specific ways. The GTSRL method employs a slightly different IRL matrix and normalization factor than the SSRL method as noted by the different k_{GTSRL} and IRL_{GTSRL} terms. The GTSRL method, unlike the SSRL, can represent systems with both directional and non-directional IRL edge connections and vertex loops. The GTSRL, like the SSRL, is also unable to effectively handle multiple edge connection paths between vertices. Consult Garrett et al. (2011) or McConkie (2013) for additional details on the GTSRL calculation approach. The primary weakness of the SSRL and GTSRL is that matrix multiplication of ordinal TRL and IRL data elements produces inaccurate computations of the resulting system readiness. This weakness was addressed by the TASRL considered next.

2.9.3 TASRL Method

In an attempt to address the weakness of the SSRL and GTSRL approaches McConkie et al. (2012) and McConkie (2013) offer a tropical algebra based TASRL calculation method. The TASRL method configures the TRL vector and IRL adjacency matrix in the identical fashion as the GTSRL method, but does not employ matrix multiplication to calculate the matrix-vector product. The TASRL instead uses tropical algebra operations of tropical algebraic sum and minimum value. A mathematical representation of the TASRL calculation approach is given below. The tropical algebraic sum operation, $\otimes$, is the sum of a given set of numbers. The minimal value operation, $\oplus$, denotes the minimum value of a set of numbers. Using these mathematical operations, the TASRL is calculated in the manner described by McConkie et al. (2012) and Sauser et al. (2011). The TASRL method has been demonstrated to exhibit superior mathematical properties than the matrix multiplication methods used by the SSRL and GTSRL methods (McConkie et al. 2012). Despite possessing superior properties than the

SSRL and GTSRL, the TASRL method is still fundamentally limited to evaluating system graph configurations that are structured using standard adjacency matrix formulations. A summary table of the three standard SRL calculation forms is listed in Table 2-2.

Table 2-2: SRL model calculation methodology and standard equation forms.

SRL Model	Calculation Methodology	Standard Mathematical Form
SSRL	Pairwise Matrix Multiplication	$SSRL = \frac{1}{n}\sum_{i=1}^{n}\left[k_{SSRL}\cdot\left[\left(\frac{1}{9}\cdot IRL_{SSRL}^{nxn}\right)*\left(\frac{1}{9}\cdot TRL_{n,1}\right)\right]\right]$
GTSRL	Pairwise Matrix Multiplication	$GTSRL = \frac{1}{n}\sum_{i=1}^{n}\left[k_{GTSRL}\cdot\left[\left(\frac{1}{9}\cdot IRL_{GTSRL}^{nxn}\right)*\left(\frac{1}{9}\cdot TRL_{n,1}\right)\right]\right]$
TASRL	Min-Plus Tropical Algebra	$TASRL = \oplus_{i=1}^{n}\left[\left(IRL_{i,1}\otimes TRL_{1}\right)\oplus\cdots\oplus\left(IRL_{i,n}\otimes TRL_{n}\right)\right]$

The SRL model calculation equation forms in Table 2-2 are fully explained in Appendix A. The next section discusses specific concerns of SRL calculation methods.

2.10 Mathematical Concerns of Readiness Levels

The growing use and expansion of SRL is not without skeptics. Kujawski (2010 and 2013) strongly warns against the SRL use, especially the calculation methods originally suggested by Sauser (2008 and 2010). Kujawsk (2013) suggested several notable mathematical limitations of SRL including the following:

1) *Invalid matrix operations on ordinal data*: Standard addition, multiplication, and division are not valid arithmetic operations for ordinal TRL and IRL data values (Agresti 1990; Velleman and Wilkinson 1993). Bowles (2004) and Cox (2005 and 2008) likewise discussed limitations of ordinal data manipulation. Ordinal data manipulation is a primary concern for SRL mathematics and can lead to readiness "reversal" discussed below.

2) Insufficient evidence for SRL benefits: This second objection suggested that SRL analyses are beneficial to programmatic decision–making processes. This objection may be addressed by expanding the set of SRL program case studies. Dacus (2011) provides an excellent methodology of assessing program SARs using an SRL–based measurement.

The three SRL methods considered in Section 2.8 used different calculation methods to determine the system readiness. The SSRL and GTSRL methods use pairwise matrix multiplication on IRL and TRL ordinal data values. The TASRL uses tropical algebra sum and min-plus operations. System development literature suggests that the SRL of a given system should not be higher than the lowest value of a given system's TRL or IRL values (Engle et al. 2009; Kujawski 2013; McConkie et al. 2012). This assertion is supported by Sauser et al. (2011) in which the authors suggest that a system should not be "more ready" than the "less ready" of its sub-systems. Readiness reversal produces inaccurate system development assessments and may promote a system to a developmental phase before the system has demonstrated the required characteristics. SRL readiness reversal considerations may be compared to the literature debates over invalid program risk matrix calculation considerations (Cox 2005 and 2008; Lansdowne 1999; Smith 2005) and the Analytic Hierarchy Process decision methodology (Forman and Gass 2001; Dyer et al. 1990; Barzilai and Golany 1994; Schenkerman 1994; Stam & Silva 1997; Saaty 2004). Given the SRL concerns noted above, several significant literature gaps were identified and leveraged to support the proposed research.

2.11 Gaps in the Literature

The literature review identified several areas of unexplored potential research. As noted by McConkie (2013) there is a concern of readiness reversal with SRL methods that may produce erroneous system readiness assessments. Developing objective assessments of SRL readiness reversal will support confidence in future use and application of SRL methods. In addition, SRL methods have not been extended beyond simple graph system architectures. Applying SRL assessments to more complex system structure including multigraphs and general graphs will expand SRL implementation options. The IMSRL method discussed in Chapter 5 addresses this limitation of contemporary SRL methods.

2.12 Research Questions

This research considered mathematical characteristics of SRL models. Evaluating SRL readiness reversal will support future applications of SRL to improve quantitative system evaluations. Given the concerns addressed in the literature with SRL calculation methods, three primary research questions are considered in this dissertation:

1. What is the relationship between SRL calculation methods and the likelihood of SRL readiness reversal?
2. What is the relationship of system structure parameters like size, order, degree, and complexity, to the likelihood of SRL readiness reversal?
3. What is the unitary TRL or IRL level input change sensitivity of the proposed IMSRL model as compared with existing SRL calculation methods?

The first question addresses the question of the impact of SRL calculation method on the occurrence of readiness reversal. The second question considers the impact of system complexity on the presence of SRL readiness reversal. The final research question addresses the sensitivity of the proposed IMSRL method. These research questions will be used to evaluate the supporting research hypotheses.

2.13 Research Hypotheses

The research hypotheses consider three aspects of SRL calculation model characteristics.

1) H_1: SRL calculation methods have no impact on readiness reversal.
2) H_2: Model structure parameters (e.g. system size, order, degree, TRL or IRL values) have no impact on SRL readiness reversal.
3) H_3: IMSRL has no unit input sensitivity difference from other methods.

The first hypothesis addresses the comparative impact each SRL calculation method has on the likelihood of readiness reversal. The second hypothesis is concerned with the effect of the system graphical structural components on readiness reversal. The third hypothesis considers a comparison of SRL model sensitivity to unit input changes. was that SRL calculation methods have no impact on SRL readiness reversal likelihood. The effects of SRL model type, size, order, and degree on SRL readiness reversal were considered in the second hypothesis. Finally, the third hypothesis compared the sensitivity analysis stability among four SRL models.

2.14 Research Goals and Objectives

Evaluating SRL readiness reversal and demonstration of the IMSRL method supports future SRL applications and other quantitative evaluation metrics. The proposed IMSRL model supports extension of SRL program assessments to system structures beyond those considered using legacy approaches. The overall research objective was to engender valid program assessments of system progress by providing valuable information to system engineering and program management personnel.

2.15 Summary

The literature review provided a review of the SRL development and pertinent mathematical characteristics. General maturity metrics were discussed and supported the formulation of SRLs. SRL calculation details and weaknesses of SRL calculation methods, including concerns of SRL readiness reversal, stimulated IMSRL development. Graph representations of system architectures provide a framework for IMSRL analysis in Chapter 5. Finally, a new SRL study was provided that emphasized the mathematical characterization of SRL readiness reversal using an incidence matrix approach.

CHAPTER 3 – SYSTEMS ENGINEERING ADVANCES

This chapter presents the primary and secondary contributions of this research to the systems engineering body of knowledge. These contributions were developed from observations of limitations of SRL models present in the literature. This research advances systems engineering by evaluating selected mathematical properties of SRL. As SRL expand in usage and applications (Ramirez-Marquez et al., 2010) the mathematical validity and rigor of SRL must rest on a solid foundation. SRL mathematical limitations need to be quantitatively described and developed in a straightforward manner (Azizian et al. 2011; McConkie et al. 2012; Sauser et al. 2010). Valid SRL mathematics will engender confidence in their continued and expanded applications for various program management needs (McConkie et al. 2012). This research contributes to the systems engineering literature in the following two fundamental areas:

1. <u>*IMSRL Model*</u>: The IMSRL model solves some of the mathematical limitations of SRL models, including readiness reversal. This new model leverages incidence matrix graph theory principles and mathematical set operations to provide a quantitative system readiness measurement. The IMSRL operation is described in Chapter 5 and further expanded in the following areas:

    a) *Complex system architectures*: The IMSRL is demonstrated for use with multigraphs and general graphs that possess self-referencing vertex loops and multiple parallel edges between vertex pairs. Such complex system formulations cannot be successfully evaluated using current SRL approaches.

b) *Analytical Demonstration*: This contribution analytically demonstrates the IMSRL model with respect to five primary SRL characteristics discussed in McConkie et al. (2012) and McConkie (2013). An additional sixth SRL property is also considered to extend SRL mathematical rigor.

2. Readiness Reversal Characterization: This contribution provides a Monte-Carlo simulation and logistic regression analysis approach to evaluate the SRL readiness reversal characteristics of SRL models. The evaluation of SRL readiness reversal properties directly contributes to the acceptance and propagation of SRL within the systems engineering community. Each contribution is discussed separately in the following sections.

3.1 IMSRL Model

The primary research contribution is the proposed incidence-matrix SRL model (IMSRL) as a newly developed method of calculating the SRL of a given system (London et al. 2014). The IMSRL model leverages graph theory mathematics, the characteristics of incidence matrices, and mathematical set theory to provide an improved SRL model that has a reduced likelihood of readiness reversal. The IMSRL model also supports system readiness evaluations of more complex system models than may be evaluated using existing approaches.

3.1.1 IMSRL Model Formulation

The IMSRL model is presented using a worked example of a general graph system. This example exemplifies a key advantage of the IMSRL model, namely the ability to assess systems with multi-edges and vertex loops. Such complex system formulations cannot be successfully evaluated using current SRL approaches. The GTSRL and TASRL can effectively model a single undirected or bi-directional edge between a given vertex pair, but cannot evaluate the system readiness when more than one edge structure is present between the same vertex pair. Systems that possess multiple redundant functional pathways between nodes cannot presently be evaluated using the SSRL, GTSRL, or TASRL models.

3.1.2 IMSRL Analytic Demonstration

This research contribution provides an analytical demonstration of the IMSRL satisfaction of five primary SRL properties developed by McConkie et al. (2012) and McConkie (2013). A sixth SRL mathematical property is proposed to further support IMSRL model rigor. The first five properties are adapted from McConkie (2013, pgs. 37-39).

An analytical demonstration of the IMSRL for each of the six considered SRL properties is provided in Chapter 5.

3.2 SRL Readiness Reversal Characterization

The second area of research contribution evaluates the readiness reversal characteristics of the various SRL models. This research develops a Monte-Carlo based SRL simulation model and a quantitative logistic-regression based model of SRL readiness reversal. Quantitative readiness reversal evaluation will lend credibility for future SRL application verification. Extensive details of the readiness reversal evaluation process are provided in Chapter 4 with data results discussed in Chapter 5.

CHAPTER 4 – RESEARCH METHOD

The research methodology progressed from fundamental literature reviews of general methods of system technological development through assessments of mathematical underpinnings of SRL models. The specific weaknesses of existing SRL methods were assessed and informed the new IMSRL model that addressed these limitations. IMSRL model validation was performed using notional system architectures and simulated data. The approach outlined below differs from prior quantitative research on SRL. McConkie (2013) employed a four-step linear research process to develop the TASRL calculation approach and generate a set of desired SRL mathematical characteristics. Magnaye (2012) used a constrained optimization model approach to evaluate SRL assessments of system development cost and schedule. The research method employed in this work was a modified systems engineering V-model approach as depicted in Figure 4-1.

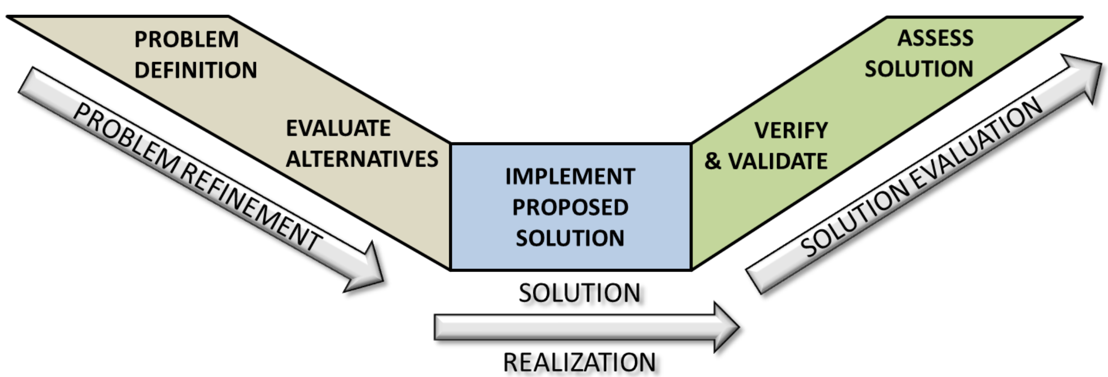


Figure 4-1: Research methodology system V-model approach.

In Figure 4-1 the proposed research methodology approach is divided into three sequential phases comprising five cumulative stages. Each research method phase is described separately in the following sections with results provided in Chapter 5.

4.1 Problem Refinement

The problem refinement phase included two steps of problem definition and solution evaluation.

4.1.1 Problem Definition

The problem definition phase generated a literature review for system development assessment measurements in general, and for SRL in particular. The SRL literature discussion included recent developments in probabilistic SRL calculation methods (Tan 2011), system maturity metric usage for analyzing system component importance (Tan 2010), and SRL use for system develop cost minimization models (Malone 2012; Magnaye 2010). The state of SRL development was assessed and preliminary solutions developed to address SRL readiness reversal.

4.1.2 Evaluation of Alternate Solutions

Two different SRL focus areas were considered.. The first area—subsequently discarded—pursued the expansion of SRL applications to system reliability growth applications. This initial work, presented in London et al. (2013a and 2013b), correlated system readiness progress with reliability growth predictions using simulated data sets. As this research progressed, the limitations and concerns of SRL calculation methods became evident and focus shifted to assessing the underlying SRL mathematics before SRL should be further extended to new application areas.

The research focus adjustment to SRL mathematical assessments provoked a decision point of whether to simply adapt an existing model or develop an entirely new approach. The limitations of the existing SRL models described in Chapter 2, however, compelled the development of a new SRL calculation approach. The new incidence matrix SRL formulation became the IMSRL model.

4.2 Solution Realization

Incidence matrix structures provide provide greater flexibility in modeling complex system structures like digraphs, multigraphs, and general graphs (Gross and Yellen 2006). The adoption of an incidence matrix formulation, coupled with the the use of the Cartesian Product set operation, and a min-min calculation approach completed the IMSRL formulation structure.

Two primary SRL readiness reversal definitions are present in the literature. The first readiness reversal definition (Sauser et al. (2010); McConkie (2013)) suggests evaluating all known TRL and IRL combinations for a given system architecture that generate a particular SRL value. This definition is impractical due to the large set of TRL and IRL permutations theoretically available for anything other than a very simple system. For example, a simple system like Figure 2-2(b) comprised of 3-TRL nodes and 3-IRL edges has a theoretical maximum number of $9^{(3+3)}$=531,441 component value permutations. Such large permutation combinations proved intractably difficult to support distinct reversal assessment measures.

Since the first readiness reversal definition option was eliminated, a second readiness reversal evaluation option was considered. The second readiness reversal definition in the literature asserts that a readiness “reversal” occurs when a calculated SRL value pushes a readiness assessment higher than what can be adequately supported from the constituent TRL and IRL values (Engel et al. 2010). By adopting this definition scheme for use within an acquisition development lifecycle framework, a much more useful and quantifiable assessment measurement tool was devised.

4.3 Solution Evaluation

Once the IMSRL was formulated, it was evaluated for both mathematical tractability and for readiness reversal considerations. Analytical evaluation of the IMSRL was performed to determine satisfaction of five primary SRL mathematical properties formulated by McConkie et al. (2012) and McConkie (2013) as well as an additional sixth property.

The readiness reversal characteristics of four SRL models (SSRL, GTSRL, TASRL, and IMSRL) were evaluated using a Monte-Carlo simulation modeland were analyzed using a logistic regression approach that produced a regression model of the output model variable of the presence or absence of readiness reversal.

4.3.1 Solution Verification and Validation

Proper model Verification and Validation (V&V) processes are fundamental in the solution evaluation process. These processes encompass a wide range of definitions across different system domains but definition consistency is emerging (Gardner III, 2014). One definition of verification notes that, "…verification tells us if we build the system right, that is, that the specifications were satisfied." (Gardner III 2014, 242). Verification is therefore focused on evaluating a given system to determine whether basic fundamental requirements have been satisfied. Validation, however, requires focused stakeholder input and answers the question of whether the system satisfies the user needs (Gardner III 2014). For the purpose of this research, V&V processes were fundamentally focused on the IMSRL method demonstration and the Monte-Carlo readiness reversal simulation validation.

This research phase considered the IMSRL and readiness reversal model verification and validation. The focus was to demonstrate the attractiveness of the IMSRL model against the alternate models with respect to both readiness reversal characteristics and the scope of system architectures that may be adequately supported. Readiness reversal performance of the SRL frameworks was demonstrated using the Monte-Carlo simulation and logistic regression analysis approach discussed in Chapter 5.

The IMSRL model was verified using a Symbolic Evaluation static verification approach that assessed the model output by exercising the IMSRL operations using symbolic entries (Balci 1998). Assertions Checking dynamic verification techniques were used to confirm both IMSRL and readiness reversal model outputs at discrete stages of model operation—both at the submodule and global level (Balci 1998). Extensive model structural analysis and Boundary Value Input Testing were also performed to ascertain the IMSRL adherence to valid mathematical operations at extreme values of model input values. The readiness reversal Monte-Carlo model code was generated using Matlab (2011) and was subjected to extensive module interface checks, execution testing, and code functional checks (Balci 2014).

As noted by Birta and Ozmizrak (1996, 77) one can never fully validate a given model, rather the validation emphasis is to gain a "reasonable level" of confidence in the model results. Consequently the IMSRL and readiness reversal model validation focused on demonstrating the relative strengths of the IMSRL model and the distinctive readiness reversal characteristics that the IMSRL possesses. Pearson and Hosmer-Lemeshow p-value goodness of fit tests supported the research assertions of model performance and accuracy (Hosmer et al. 1997; Bedford and Cooke 2001). The data results produced during this phase supported the final research method phase in which the results were thoroughly assessed and evaluated for further improvement.

4.3.2 Assessment of Proposed Solution

This research phase evaluated the collective analysis and data results from the IMSRL and readiness reversal models. Readiness reversal characteristics of the IMSRL were assessed and compared with the other three models. IMSRL extension to complex system architectures like multigraphs and general graphs was considered. Limitations of the IMSRL model framework were considered and supported discussion of future research extensions of the present work.

CHAPTER 5 – DATA RESULTS

This chapter describes the data results generated using the methodology in Chapter 4. The IMSRL is demonstrated using a General graph example. An IMSRL analytical demonstration with six SRL properties is presented followed by a sensitivity analysis. SRL readiness reversal model results are presented along with a multivariate correlation analysis. Hypothesis test results and data results discussion completes the chapter.

5.1 IMSRL Development

This section presents the IMSRL model development and assessment. Subsections explore IMSRL formulation and analytically evaluate the IMSRL model with respect to several desired SRL mathematical properties described in McConkie et al. (2012) and McConkie (2013).

5.1.1 IMSRL Model Formulation

The IMSRL is a novel method for calculating sub-system and whole-system readiness levels using an incidence matrix minimum-value approach.

Consider the general graph of Figure 5-1 with three TRL component values {TRL_1=6, TRL_2=4, TRL_3=8} and five IRL edges {$IRL_{1->2}$=5, IRL_{2-3}=9, $IRL_{1<->3}$=7, $IRL_{3->2}$=9, $IRL_{2->2}$=5}. The IRL edge set contains both undirected {IRL_{2-3}=9} and directed edges {$IRL_{1->2}$=5, $IRL_{1<->3}$=7, $IRL_{3->2}$=9}, as well as a vertex loop at TRL_2 {$IRL_{2->2}$=5}. Edge labels ε_1 through ε_5 denote arbitrarily defined edge numbering that does not impact IMSRL calculations.

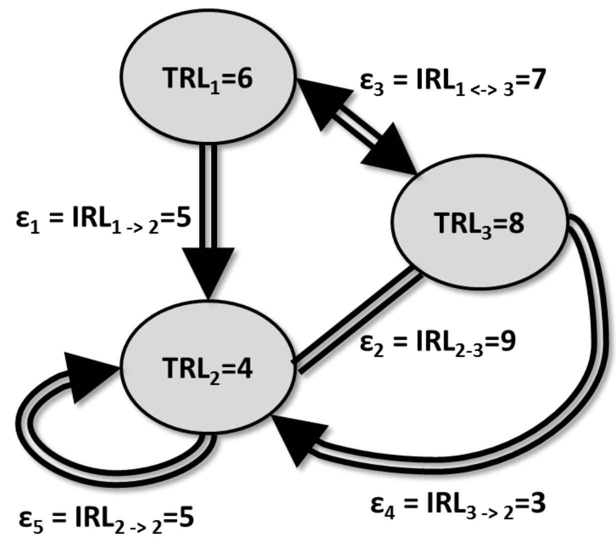


Figure 5-1: Notional general graph system model for IMSRL demonstration.

The IMSRL first establishes a TRL column vector in Equation 5.1 in the same manner as the SSRL, GTSRL, and TASRL models.

$$TRL_{IMSRL} = \begin{bmatrix} 6 \\ 4 \\ 8 \end{bmatrix} \tag{5.1}$$

The IMSRL model uses incidence matrices to represent the IRL connection values. Each IRL matrix column in the IMSRL model corresponds to a numbered edge in the system graph with the IRL value of that particular edge placed in the incidence matrix column vector. The column row in which each IRL value is placed equates to the vertex (v_i) of the start and end points of a particular edge. Each column therefore represents the vertices to which a given edge is incident.

$$IRL_{IMSRL} = \begin{matrix} \\ TRL_1 \\ TRL_2 \\ TRL_3 \end{matrix} \begin{matrix} \begin{matrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 & \varepsilon_5 \end{matrix} \\ \begin{bmatrix} 5^t & 0 & 7 & 0 & 0 \\ 5^h & 9 & 0 & 3^h & 5^{t,h} \\ 0 & 9 & 7 & 3^t & 0 \end{bmatrix} \end{matrix} \tag{5.2}$$

The zero values in the IRL_{IMSRL} matrix columns represent vertices to which the particular edge, ε_x, represented in that column is not connected. Comparison of Equation 5.2 with Figure 5-1 reveals directed edges ε_1, ε_4 and a vertex loop for ε_5. The "h" and "t" superscripts in Equation 5.2 denote the tail (origin) and head (termination) vertex for each directed edge. This directed edge origin and termination notation was adapted from Gross and Yellen (2006) but other graph theory texts may suggest alternate notation schemes (see Chartrand and Zhang 2002; Diestel 2010). As shown below, the head and tail designations for a directed edge are not explicitly used in the IMSRL computation process, but are preserved at this step to demonstrate the flexibility of the IMSRL to assess both directed and undirected edges between vertex pairs.

The values in the TRL_{IMSRL} vector and IRL matrix are normalized from a [1-9] integer scale to a [0-1] continuous scale by multiplying each by 1/9. The normalized TRL and IRL values are listed below.

$$TRL_{IMSRL} = \frac{1}{9} \cdot \begin{bmatrix} 6 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 0.67 \\ 0.44 \\ 0.89 \end{bmatrix} \tag{5.3}$$

The corresponding IRL_{IMSRL} matrix configuration is expressed in Equation 5.4.

$$IRL_{IMSRL} = \frac{1}{9} \cdot \begin{matrix} \\ TRL_1 \\ TRL_2 \\ TRL_3 \end{matrix} \begin{matrix} \begin{matrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 & \varepsilon_5 \end{matrix} \\ \begin{bmatrix} 5^t & 0 & 7 & 0 & 0 \\ 5^h & 9 & 0 & 3^h & 5^{t,h} \\ 0 & 9 & 7 & 3^t & 0 \end{bmatrix} \end{matrix} = \begin{bmatrix} 0.56 & 0 & 0.78 & 0 & 0 \\ 0.56 & 1 & 0 & 0.33 & 0.56 \\ 0 & 1 & 0.78 & 0.33 & 0 \end{bmatrix} \tag{5.4}$$

Once the normalized TRL column vector and IRL matrix are created, the matrix transpose of each is calculated. This calculation produces a TRL row vector and a transposed IRL matrix in Equations 5.5 and 5.6 respectively. The transpose operation simplifies the Cartesian Product calculation later in the process.

$$TRL^{T}_{IMSRL} = \begin{bmatrix} 0.67 & 0.44 & 0.89 \end{bmatrix} \tag{5.5}$$

$$IRL^{T}_{IMSRL} = \begin{bmatrix} 0.56 & 0.56 & 0 \\ 0 & 1 & 1 \\ 0.78 & 0 & 0.78 \\ 0 & 0.33 & 0.33 \\ 0 & 0.56 & 0 \end{bmatrix} \tag{5.6}$$

Once the IRL^T and TRL^T matrices are generated, a Cartesian Product operation for each IRL^T row and the row vector TRL^T is performed (Gross & Yellen, 2006). The Cartesian Product is a mathematical set operation that produces ordered pair combinations of set elements. For example, the Cartesian Product C=A**X**B of the notional sets A={1,2} and B={5, 6} produces the ordered pair set C={(1,5), (1,6), (2,5), (2,6)}. The IMSRL model calculates a Cartesian Product ordered pair set, called the $ITRL_{IMSRL}$, of the positive IRL^T row values in each i^{th} row, and TRL^T vector elements. For the example of Figure 5-1 this calculation in symbolic form is in Equation 5.7.

$$ITRL^{j}_{i=1} = \left[\left\{ IRL^{T}_{i,j} > 0 \right\}^{j}_{i=1} X \left\{ TRL^{T} \right\} \right] \tag{5.7}$$

Inserting the IRL^T and TRL^T values produces the result of Equation (5.8).

$$ITRL_{IMSRL} = \left\{ \begin{bmatrix} 0.56 & 0.56 & 0 \\ 0 & 1 & 1 \\ 0.78 & 0 & 0.78 \\ 0 & 0.33 & 0.33 \\ 0 & 0.56 & 0 \end{bmatrix} > 0 \right\} X \begin{bmatrix} 0.67 & 0.44 & 0.89 \end{bmatrix} \tag{5.8}$$

Reducing the Cartesian Product calculation results in the large matrix set of Equation 5.9.

$$ITRL_{IMSRL} = \begin{bmatrix} \{(0.56,0.67),(0.56,0.44),(0.56,0.89)\} & \{(0.56,0.67),(0.56,0.44),(0.56,0.89)\} & \{(0,0.67),(0,0.44),(0,0.89)\} \\ \{(0,0.67),(0,0.44),(0,0.89)\} & \{(1,0.67),(1,0.44),(1,0.89)\} & \{(1,0.67),(1,0.44),(1,0.89)\} \\ \{(0.78,0.67),(0.78,0.44),(0.78,0.89)\} & \{(0,0.67),(0,0.44),(0,0.89)\} & \{(0.78,0.67),(0.78,0.44),(0.78,0.89)\} \\ \{(0,0.67),(0,0.44),(0,0.89)\} & \{(0.33,0.67),(0.33,0.44),(0.33,0.89)\} & \{(0.33,0.67),(0.33,0.44),(0.33,0.89)\} \\ \{(0,0.67),(0,0.44),(0,0.89)\} & \{(0.56,0.67),(0.56,0.44),(0.56,0.89)\} & \{(0,0.67),(0,0.44),(0,0.89)\} \end{bmatrix} \quad (5.9)$$

Zero values in the IRL matrix convey the absence of integrations between adjacent vertices. Stated simply, zero IRL values are placeholders for connections that do not exist in a given system graph incarnation. The SSRL and GTSRL models effectively ignore zero IRL values through their use of matrix multiplication and so the authors believe that the IMSRL model selection of nonzero elements in the Cartesian Product set operation is justified.

Proceeding with the IMSRL, minimum values for each $ITRL_{IMSRL}$ Cartesian Product row element combination is calculated. Taking the minimum value of that reduced set produces a column vector in which each $ITRL_{Min}$ row element is the minimum value of the Cartesian Product set of combination pairs. Equation 5.10 expresses the symbolic form of the final ITRL column vector reduction.

$$ITRL_{\min} = \left\lfloor \min\left(\min\left(\left\{IRL_{i,j}^{T} > 0\right\}_{i=1}^{m} \times \left\{TRL^{T}\right\}\right)\right)\right\rfloor \quad (5.10)$$

Applying the numerical inputs from Equation 5.9 operating on only the non-zero values produces a minimum value for each $ITRL_{IMSRL}$ row set shown in Equation 5.11.

$$ITRL_{Min} = \begin{bmatrix} \min(0.44,0.44,0.44) \\ \min(0.44,0.44,0.44) \\ \min(0.44,0.44,0.44) \\ \min(0.44,0.33,0.33) \\ \min(0.44,0.44,0.44) \end{bmatrix} = \begin{Bmatrix} 0.44 \\ 0.44 \\ 0.44 \\ 0.33 \\ 0.44 \end{Bmatrix} \quad (5.11)$$

The $ITRL_{Min}$ column vector represents the minimum SRL value for each sub graph comprised of two connected vertices and the set of edge connections between the two vertices. This subsystem readiness level set equates to the ITRL calculation stage of the SSRL, GTSRL, and TASRL methods (Garrett et al. 2011; McConkie et al. 2012; Sauser et al. 2008).

The final IMSRL method step calculates the overall system SRL value by taking the minimum value of the $ITRL_{Min}$ column vector. This operation therefore reduces the IMSRL value to the minimum subsystem ITRL value as demonstrated in Equation 5.12.

$$IMSRL = \left[\min\left(ITRL_{Min}\right)\right] = \min\begin{bmatrix} 0.44 \\ 0.44 \\ 0.44 \\ 0.33 \\ 0.44 \end{bmatrix} = 0.33 \tag{5.12}$$

To calculate the final SRL value, the IMSRL model implements a min-min calculation approach for non-zero TRL and IRL connection pairs. The IMSRL min-min approach ensures that the calculated readiness level value cannot exceed the range supported by the lowest TRL or IRL value. By forcing the readiness level calculation to a minimum value set, the IMSRL method greatly reduces the likelihood of the system being placed into a more advanced acquisition phase for which the system is not yet technologically ready. The IMSRL method was evaluated for sensitivity analysis and readiness reversal characteristics in the following sections.

5.1.2 Sensitivity Analysis

A sensitivity analysis of the four SRL models ascertained the comparative stability of the four SRL models for identical input conditions. All TRL and IRL values were set to identical values [1,2,…,8] for the 2-TRL system of Figure 2-1(a). Each TRL and IRL component value was separately increased by 1 and the percentage change in SRL value was recorded. The mean percentage change of SRL value versus the TRL and IRL value change for each baseline TRL and IRL value is provided in Figure 5-2.

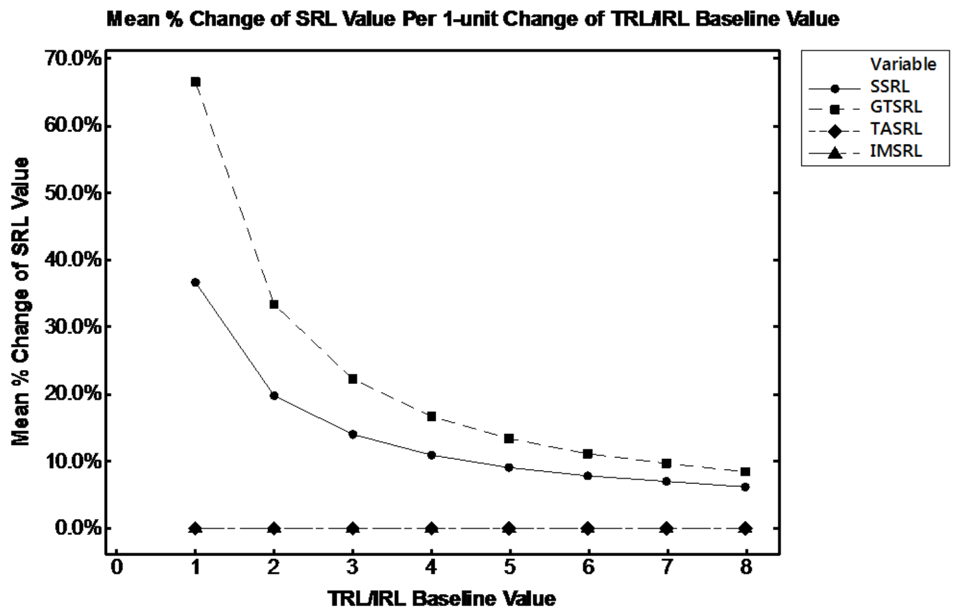


Figure 5-2: Mean SRL percentage change per unit TRL or IRL value change.

Figure 5-2 denotes the average SRL value percentage change per a one-unit TRL or IRL value increase from the baseline value. The "TRL/IRL Baseline Value" term on the x-axis of Figure 5-2 refers to the case in which all TRL and IRL values in Figure 2-1(a) are set to the same value of the set {1,2,…,9}. The sensitivity analysis demonstrates that the TASRL and IMSRL models exhibited no SRL value changes (0% change) from unitary changes of TRL or IRL component values. The SSRL and GTSRL models have significant sensitivity to unit changes in TRL or IRL values. The GTSRL sensitivity exceeds the SSRL model for low TRL/IRL values but the sensitivity nearly converges at higher TRL or IRL values. This simple example provides critical insight into the underlying mechanics of the four SRL models. The SSRL and GTSRL models possessed a larger set of calculable SRL values in the [0-1] range than the TASRL or IMSRL models. Minimum value calculations mapped the set of allowable TASRL and IMSRL values to the set of (k/9) where k={1,2,…,9}. This SRL set value limitation provided a very stable SRL evaluation model but prevented the TASRL or IMSRL models from increasing their calculated system readiness value until each individual TRL and IRL value in the system achieved the next integer value on the set {1,2,..,9}. The data suggested that a marked difference in readiness reversal existed between matrix multiplication SRL methods (SSRL and GTSRL) and min-plus or min-min approaches (TASRL and IMSRL). Lower IMSRL and TASRL sensitivity rates provided greater assurance that these models provide stable readiness assessments through the range of allowable TRL and IRL value ranges.

5.2 IMSRL Analytical Demonstration

In addition to IMSRL model demonstration and a basic sensitivity analysis comparison among four SRL models, the IMSRL model was also demonstrated to assess conformance with the five desired SRL properties developed by McConkie et al. (2012) and McConkie (2013). A new sixth SRL property was also considered. Each of these properties is evaluated below for the IMSRL. For a full analytic evaluation of the SSRL, GTSRL, and TASRL the interested reader is invited to consult the two primary McConkie sources noted above.

5.2.1 SRL Property #1

The first property considered was the Closure property that asserted ITRL and SRL of a system cannot exceed the maximum available TRL or IRL value. The intent of this property was to prevent calculated SRL values from exceeding the supportable range of the underlying TRL and IRL values.

Equation 5.13 restates the IMSRL symbolic form of Equation 5.10.

$$ITRL_{\min} = \left[\min\left(\min\left(\left\{IRL_{i,j}^{T} > 0\right\}_{i=1}^{m} \times \left\{TRL^{T}\right\}\right)\right)\right] \qquad (5.13)$$

Using Figure 5-1 we set all TRL node and IRL edge values equal to the lowest possible TRL and IRL value of 1 (normalized to 1/9 or 0.11). The IMSRL model process using Equations 5.1 through 5.12 produces the new ITRL' result of Equation 5.14.

$$ITRL'_{Min} = \left[\min\left(\min\left(\left\{IRL_{i,j}^{T} > 0\right\}_{i=1}^{m} X \left\{TRL^{T}\right\}\right)\right)\right] = \begin{bmatrix} 0.11 \\ 0.11 \\ 0.11 \\ 0.11 \\ 0.11 \end{bmatrix} \qquad (5.14)$$

The new calculated IMSRL' for this scenario becomes the value in Equation 5.15.

$$IMSRL' = \left[\min\left(ITRL_{Min}\right)\right] = 0.11 \qquad (5.15)$$

From this demonstration we see that both the $ITRL_{Min}$ and the IMSRL satisfies the closure property for this most restrictive set of TRL and IRL values.

5.2.2 SRL Property #2

The second SRL property posits that increasing a particular TRL component value without changing other TRL or IRL values will not decrease the calculated ITRL or SRL values. Again considering Figure 5-1 we set all TRL node and IRL edge values equal to 1 except for $TRL_{1\text{->}2}$ that is set to 2 (normalized to 2/9 or 0.22). Working through the IMSRL model with Equations 5.1 through 5.12 and using these values produces the following new ITRL' and IMSRL' values.

$$ITRL'_{Min} = \left[\min\left(\min\left(\left\{IRL^{T}_{i,j} > 0\right\}_{i=1}^{m} X \left\{TRL^{T}\right\}\right)\right)\right] = \begin{bmatrix} 0.11 \\ 0.11 \\ 0.11 \\ 0.11 \\ 0.11 \end{bmatrix} \quad (5.16)$$

The new calculated IMSRL' for this scenario becomes the value in Equation 5.17.

$$IMSRL' = \left[\min\left(ITRL_{\min}\right)\right] = 0.11 \quad (5.17)$$

This demonstration shows that increasing one particular TRL value decreases neither the new $ITRL_{min}$ nor the IMSRL value. This demonstration, along with the sensitivity analysis provided above, suggests that the IMSRL satisfies this second SRL property by use of a min-min calculation approach.

5.2.3 SRL Property #3

The third SRL mathematical property states that if a new system component is introduced, that has both a TRL and IRL connection greater than or equal to the existing SRL value, the resulting new SRL value (denoted SRL') will also be greater than or equal to the existing SRL value. In the case of the IMSRL model, the min-min approach forces the calculated ITRL' and IMSRL' values to collapse to those of the minimum TRL or IRL values. Thus, for the case of a new system component integrated into the existing system we would have the following new ITRL' and IMSRL' calculations of Equations 5.18 and 5.19 respectively.

$$ITRL'_{Min} = \left[\min\left(\min\left(\left\{IRL'^{T}_{i,j} > 0\right\}_{i=1}^{m} \times \left\{TRL'^{T}\right\}\right)\right)\right] \geq ITRL_{\min} \quad (5.18)$$

And

$$IMSRL' = \left[\min\left(ITRL'_{\min}\right)\right] \geq IMSRL \quad (5.19)$$

Consider Figure 5-1 but with a new component $TRL_4 = 9$ connected to TRL_1 via an undirected edge $IRL_{1\text{-}4}$=9 with TRL_4 and $IRL_{1\text{-}4}$ values normalized to (9/9=1.00) respectively. For this new system configuration, the $ITRL_{Min}$ value would equal Equation (5.12) but with an extra row value of 1.00. Following the IMSRL calculation process through completion produces a IMSRL value of 0.33 that is the same as the original system of Figure 5-1.

From this example we see that the new $ITRL'_{Min}$ and IMSRL' values, corresponding to the calculations using the new system addition, will necessarily be equal to or exceed the prior ITRL and IMSRL values. This demonstration suggests that the IMSRL satisfies this third SRL property by not decreasing the ITRL or IMSRL value with the introduction of new technologies.

5.2.4 SRL Property #4

The fourth McConkie (2013) SRL property asserts that the ITRL of a component or subsystem cannot exceed the maximum TRL or IRL of that same particular component or subsystem. This property effectively bounds the ITRL value range for a given system configuration by restricting the ITRL to values less than or equal to the maximum TRL or IRL values.

Again, consider Figure 5-1 but with all TRL node and IRL edge values equal to the maximum value of 9 (normalized to 1.0). For this system configuration in which all TRL and IRL components are set to their maximum value, the $ITRL_{Min}$ value must be less than or equal to 1.0. The IMSRL populated with these values produces the following new $ITRL'_{Min}$ and IMSRL' values in Equations 5.20 and 5.21.

$$ITRL'_{Min} = \left[\min\left(\min\left(\left\{IRL^{T}_{i,j} > 0\right\}_{i=1}^{m} \times \left\{TRL^{T}\right\}\right)\right)\right] = \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{bmatrix} \tag{5.20}$$

And

$$IMSRL' = \left[\min\left(ITRL'_{Min}\right)\right] = 1.0 \tag{5.21}$$

This example demonstration suggests that neither the $ITRL'_{min}$ nor the IMSRL' exceeds the maximum TRL or IRL value even in this restrictive case. The IMSRL model therefore satisfies this fourth SRL property.

5.2.5 SRL Property #5

The fifth and final SRL property discussed by McConkie (2013) affirms that if all TRL and IRL values equal a given constant, the calculated SRL should equal the same constant. If all TRL and IRL values of Figure 5-1 are set equal to five (normalized to 5/9 or 0.56), this property asserts that the IMSRL model should also equal 0.56. Evaluating the IMSRL for the system configuration in Figure 5-1, but with all TRL and IRL values equal to five, produces the new $ITRL'_{Min}$ and IMSRL' values in Equations 5.22 and 5.23.

$$ITRL'_{Min} = \left[\min\left(\min\left(\left\{IRL^{T}_{i,j} > 0\right\}_{i=1}^{m} \times \left\{TRL^{T}\right\}\right)\right)\right] = \begin{bmatrix} 0.56 \\ 0.56 \\ 0.56 \\ 0.56 \\ 0.56 \end{bmatrix} \tag{5.22}$$

And

$$IMSRL' = \left[\min\left(ITRL'_{Min}\right)\right] = 0.56 \tag{5.23}$$

From this demonstration, both the $ITRL_{min}$ IMSRL values equal the same constant normalized TRL and IRL values. This demonstration can be readily repeated for all TRL and IRL scale values {1,2,…,9} thereby affirming that the IMSRL satisfies this fifth and final SRL property by use of a min-min calculation approach.

5.2.6 New SRL Property #6

In addition to the five desired SRL properties espoused above, this research also considered a sixth and newly developed SRL mathematical property. This new SRL property states that adding a new edge connection between existing TRL nodes, but with an IRL value greater than or equal to the present minimum TRL or IRL value, will not increase the new calculated SRL value. This property accentuates the flexibility of the IMSRL model that can support multigraph and general graph configurations. The SSRL, GTSRL, and TASRL models can support, at most, two delineated edge structures between the same TRL node pair. The IMSRL model, however, can theoretically manage an unlimited number of edge connections between the same node pair.

For this property consider a new Figure 5-3, which is a modified version of Figure 5-1 but with one additional edge between TRL_1 and TRL_2. Let the new IRL edge, denoted $\varepsilon_6 = IRL_{1 \to 2} = 9$ assume a maximum value of 9 that is greater than the minimum TRL or IRL value of 4.

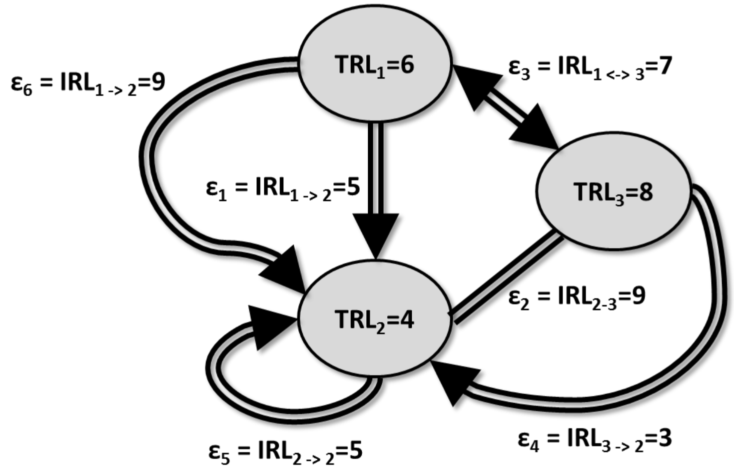


Figure 5-3: Modified version of Figure 5-1 with added edge ε6=IRL1 -> 2=9.

Evaluating the IMSRL model for the new model values in Figure 5-3 produces the following ITRL'$_{Min}$ and IMSRL' results in Equation 5.24 and 5.25..

$$ITRL'_{Min} = \left[\min\left(\min\left(\left\{IRL_{i,j}^{T} > 0\right\}_{i=1}^{m} \times \left\{TRL^{T}\right\}\right)\right)\right] = \begin{bmatrix} 0.44 \\ 0.44 \\ 0.44 \\ 0.33 \\ 0.44 \end{bmatrix} \qquad (5.24)$$

And

$$IMSRL' = \left[\min\left(ITRL'_{Min}\right)\right] = 0.44 \qquad (5.25)$$

Equations 5.24 and 5.25 demonstrate that the new ITRL'$_{Min}$ and IMSRL' values equal the same constant normalized to the minimum system TRL and IRL value. The IMSRL results are unaffected by the introduction of an additional IRL edge with values greater than the existing minimum TRL and IRL values. This result also confirms the stability of the IMSRL model even in the presence of complex multi-edge system configurations.

The IMSRL analysis in this section demonstrates satisfaction of the six SRL properties above. This demonstration, while not a fully rigorous symbolic mathematical assessment, provides additional support for IMSRL validity. The IMSRL satisfaction of these six listed properties confirms that the IMSRL, like the TASRL, is a stable and mathematically robust model capable of assessing complex systems while preserving mathematical tractability. In the next section the IMSRL model and definitions of SRL readiness reversal in Chapter 2 are leveraged to evaluate SRL readiness reversal.

5.3 Readiness Reversal

The SRL readiness reversal definition provided in Chapter 2 suggested a method by which such reversals may be evaluated for a given system and SRL calculation approach. The proposed evaluation method evaluated a particular system's calculated SRL value and confirmed whether that value conforms to the acquisition phase concurrent with the minimum TRL or IRL technology element in the system. SRL readiness reversal characteristics for different SRL calculation approaches require an evaluation of SRL characteristics across different system architectures.

This research considered the acquisition phase for a given SRL value to be established according to the SRL mapping scale provided in Table 5-1. This research defined SRL "readiness reversal" as the calculation of a particular SRL value that is higher than what a particular TRL and IRL combination can support.

Table 5-1: TRL and IRL levels, SRL value ranges, and acquisition phases.

	TRL / IRL Scale Value				
	1 2 3 4	5 6	7	8	9
Acquisition Phases (DOD(TRA}2008)	Material Solution Analysis (MSA)	Technology Demonstration (TD)	Engineering & Manufacturing Development (EMD)	Production & Deployment (P&D)	Operations & Support (O&S)
SSRL Value Range	[0.00,0.19]	[0.20,0.49]	[0.50,0.79]	[0.80,0.89]	[0.90,1.00]
GTSRL Value Range	[0.00,0.19]	[0.20,0.49]	[0.50,0.79]	[0.80,0.89]	[0.90,1.00]
TASRL Value Range	[0.00,0.44]	[0.44,0.55]	[0.56,0.79]	[0.80,0.89]	[0.90,1.00]
IMSRL Value Range	[0.00,0.44]	[0.44,0.55]	[0.56,0.79]	[0.80,0.89]	[0.90,1.00]

The acquisition phase SRL value ranges for the SSRL and GTSRL methods were derived from Sauser et al. (2008, 2010, and 2011) and the DOD[TRA} (2009). For the TASRL and IMSRL methods, the acquisition phase SRL value ranges were calculated using the system of Figure 2-1(a) with all TRL and IRL system elements set equal to values {1,2,…,9}. The calculated TASRL and IMSRL values were mapped to the equivalent acquisition phases. The working definition of SRL readiness reversal provided above served as a foundation for SRL reversal simulation analysis described in the following analysis.

Consider a simple example of a 2-TRL system like Figure 2-1(a) with TRL_1=4, TRL_2=6, and IRL_{1-2}=7. The calculated TASRL and IMSRL values, however, are 0.44 for this system, which maintain the system within the MSA phase. The SSRL value, however, is 0.4935 and places the system in the late Technology Demonstration (TD) phase or early Engineering Manufacturing and Development (EMD) phase with equivalent TRL range of [6-7] according to Table 5-1. For this hypothetical system, however, the minimum TRL_1=4 suggests that the system should not yet proceed beyond the Material Solution Analysis (MSA) phase. This disunity between the equivalent SSRL acquisition phase and the acquisition phase equivalent to the minimum TRL value is one example of SRL readiness reversal for the SSRL.

5.3.1 Monte-Carlo Simulation Data Collection

This research employs a simulation experimental approach. The data instrument was a Monte-Carlo based simulation model using Matlab (Matlab, 2011). Monte-Carlo methods allow effective analysis of large parameter problems or problems for which analytical solutions cannot readily be obtained (Hougen et al. 2014). The calculation code used for this research is provided in Appendix B.

The Monte-Carlo method employed numerous simulation runs each of which had different randomly generated input parameters according to a defined probability distribution (Ross 2006). The generated data used the input variables and variable conditions in Table 5-2 and were analyzed using a logistic regression approach. The set of dependent and independent model variables and their respective value ranges and variable coding are provided in Table 5-2.

Table 5-2: Variable list used in SRL readiness reversal evaluation.

Variable Type	Name	Range and Value Type	Coding	Label
Independent	SRL Method	SSRL (Categorical)	(0=No, 1=Yes)	$X_{1,1}$
		GTSRL (Categorical)	(0=No, 1=Yes)	$X_{1,2}$
		TASRL (Categorical)	(0=No, 1=Yes)	$X_{1,3}$
		IMSRL (Categorical)	(0=No, 1=Yes)	$X_{1,4}$
	System Size (n)	n = [2,3,6] Interval	(2, 3, 6)	X_2
	System Order (m)	m = [1, 2,3,7] Interval	(1, 2, 3, 7))	X_3
	TRL_n / IRL_m values	TRL/IRL= Uniform ({1,2,…,9})	(1, 2, …, 9)	X_4
Intermediate	SRL Value	[0,1] Continuous	Continuous	X_5
	Min (TRL or IRL)	[1,2,…,9] Ordinal	(1, 2, …, 9)	X_6
	Max (TRL or IRL)	[1,2,…,9] Ordinal	(1, 2, …, 9)	X_7
	Med (TRL or IRL)	[1,2,…,9] Ordinal	(1, 2, …, 9)	X_8
	Range (TRL or IRL)	[0,1,,…,8] Ordinal	(0,1, …,8)	X_9
	Maximum Degree	maxDeg [1, 2, 5] Interval	(1, 2, 5)	X_{10}
Dependent	Readiness Reversal	[0, 1] Binary	(1 = Reversal Present) (0 = Reversal Not Present)	Y

The dichotomous dependent variable was the occurrence of readiness reversal coded (0=no reversal, 1=reversal present) for each model simulation run. The presence of readiness reversal is confirmed if the acquisition phase equivalent of a calculated SRL value is of a later development phase than the acquisition phase of the minimum TRL or IRL value as listed in Table 5-1. The independent variables were the specific SRL model selected for a specific model run ($X_{1,1\text{-}1,4}$), the system size and order (X_2 and X_3), and the specific set of TRL and IRL values for a specific system architecture (X_4). The independent variables were randomly generated from a pseudorandom uniform integer distribution to ensure independent samples. A given model iteration selected one of the four SRL models and then separately selected one of the four complete graph system architectures in Figure 2-1. The system size, order, and degree parameters were determined directly from the system architecture. Finally the individual TRL and IRL values were randomly generated using a pseudorandom uniform integer distribution to populate the system vertex and edge structure.

Once the independent variables were generated, the intermediate variables were calculated. The intermediate variables included the calculated SRL value for a given SRL model (X_5), as well as the minimum, maximum, median, and range of the set of TRL and IRL values (X_6-X_9). The maximum system degree (labeled “MaxDeg” or “δ”) is used as a system complexity parameter (X_{10}).

5.3.2 Data Analysis

The readiness reversal data analysis focused on logistic regression and multi-variate correlation analysis of the simulated Monte-Carlo readiness reversal data. Sample size was carefully considered for logistic regression analysis. Prior logistic regression simulation approaches suggested that a minimum of 20 samples per variable were required to reduce logistic regression coefficient errors (Peduzzi et al. 2006). Subsequent research efforts suggested that fewer than 10 samples per variable may be sufficient to reduce coefficient bias, but only for specific cases (Vittinghoff and McCulloch 2006). Applying a standard of 15-20 samples per variable suggested a sample size of about 3000 model iterations to address coefficient bias errors across the set of independent and intermediate variables and variable levels.

5.3.2.1 Logistic Regression

Logistic regression analysis evaluated the readiness reversal characteristics of SRL mathematical approaches. Logistic regression represents the relationship of independent variables to a dichotomous dependent variable (Hosmer and Lemeshow 2000). Logistic regression is similar to linear regression in that both methods posit a mathematical relationship between a set of input variables and one or more output variables. Linear regression posits a linear relationship between continuous input variables and continuous output variables. A logistic regression model, however, assumes that the relationship between a dichotomous dependent variable and the set of independent variables can be represented by a logistic function similar to that of Figure 5-4.

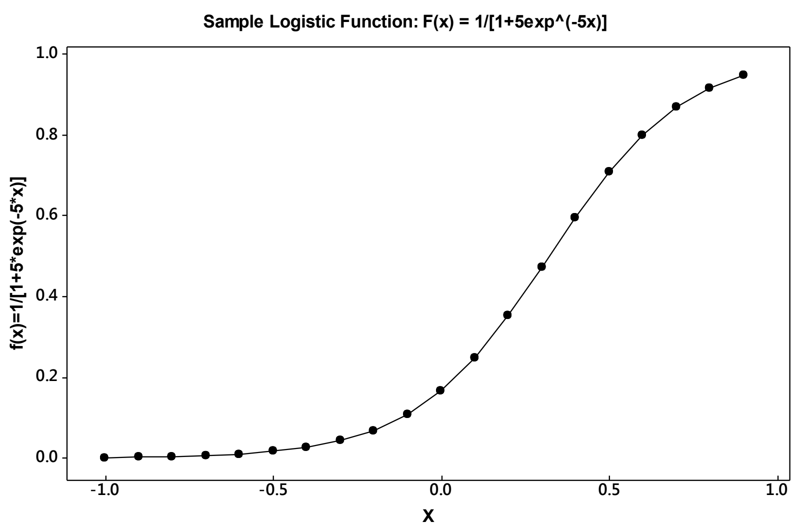


Figure 5-4: Sample logistic function plot.

The sample logistic function plot in Figure 5-4 depicts an increasing-valued logistic function but this is only one example of numerous logistic function shapes. Logistic regression generated the probability of a dichotomous event Y given the values of the input variables X_i according to the model structure in Equation 5.26 (Pampel 2000).

$$P(Y|X_i) = \frac{1}{1+e^{-(\alpha_o+\sum\beta_iX_i)}} = \frac{e^{(\alpha_o+\sum\beta_iX_i)}}{1+e^{(\alpha_o+\sum\beta_iX_i)}} \quad (5.26)$$

The model coefficient α_o was the y-axis intercept and the logistic regression coefficients β_i corresponded to the respective X_i independent variables (Peng et al. 2002). The logit of $P(Y|X_i)$, shown in Equation 5.27, was the natural logarithm of the odds ratio of $P(Y|X_i)$ and mapped the P(Y|Xi) from a (0-1) scale to a continuous parameter on a ($-\infty$,$+\infty$) scale.

$$logit[P(Y|X_i)] = ln\left(\frac{P(Y|X_i)}{1-P(Y|X_i)}\right) = \alpha_o + \sum\beta_iX_i) \quad (5.27)$$

Using the logistic regression relationships above, the generated model data were evaluated using Minitab 17 (Minitab 2014). A sample set of the generated data is provided in Table 5-3.

Table 5-3: Sample set of generated model data.

Run	n	m	Max Deg (δ)	Min (xRL)	Med (xRL)	Max (xRL)	Range (xRL)	xRL Phase	SRL Phase	SRL Model	SRL Value	Reversal
1	6	7	5	1	6	9	8	MSA	EMD	SSRL	0.5185	1
2	3	2	1	1	8	9	8	MSA	MSA	IMSRL	0.1111	0
3	6	7	5	1	6	8	7	MSA	TD	GTSRL	0.3077	1
4	6	7	5	1	5	8	7	MSA	MSA	TASRL	0.1111	0
:	:	:	:	:	:	:	:	:	:	:	:	:
3000	2	1	3	3	4	4	1	MSA	MSA	IMSRL	0.3333	0

The min(xRL), med(xRL), and range(xRL) variables were the minimum, median, and range respectively of the generated sets of TRL and IRL system values for a given model iteration. The xRL Phase variable was the acquisition phase of the minimum TRL or IRL values of Table 5-1. The SRL Phase was the acquisition phase equivalent of the calculated SRL value listed in Table 5-1. The SRL model lists the particular SRL model used for a given model iteration and the SRL value was the value calculated using the specific SRL model. The variable Reversal denoted the presence (1) or absence (0) of SRL readiness reversal. The model parameters listed above established the set of conditions on model input variables to the Monte-Carlo model results discussed in the next section.

5.3.2.2 Descriptive Data Results

The SRL readiness reversal data were evaluated using descriptive statistics and logistic regression analysis. Goodness of fit tests assessed the accuracy of the fitted model to the generated data. Hypothesis tests evaluated the significant difference in readiness reversal among the four SRL models. The summary descriptive statistics in Table 5-4 are accumulated from Minitab analysis.

Table 5-4: Descriptive statistics for SRL readiness reversal model evaluation.

SRL Model	Trials	Reversals	Non-Reversals	% Reversal
SSRL	794	727	67	91.56%
GTSRL	723	454	269	62.79%
TASRL	747	181	566	24.23%
IMSRL	736	62	574	8.42%
Total	3000	1424	1576	47.74%

The four SRL models were uniformly evaluated through the model iterations. The SSRL demonstrated the highest rate of readiness reversal (91.56%) and the IMSRL model had the lowest rate of reversal (8.42%). The GTSRL (62.49%) and TASRL (24.34%) models exhibited progressively lower rates of readiness reversal. The descriptive statistics above clearly demonstrated that the IMSRL had the lowest rate of readiness reversal among the four SRL models considered in this paper.

5.3.2.3 Logistic Regression Data Results

In addition to the basic set of summary statistics provided above a logistic regression analysis of the generated model results is also provided. Logistic regression model results with a 95% confidence interval (CI) are provided in Table 5-5.

Table 5-5:Logistic regression results for SRL readiness reversal data (α=0.05).

Term	β	SE	SE 95% CI	z	p	Wald (χ^2)	BIC	OR = e^β
Constant	-2.915	0.658	(-4.205, -1.625)	-4.43	0.000	19.625	11.618	0.054
SRL Value	23.71	1.27	(21.22, 26.21)	18.63	0.000	348.542	339.070	1.98e10
SRL Model								
SSRL	0.507	0.225	(0.065, 0.949)	2.25	0.025	5.077	-2.943	1.66e9
GTSRL	0.000	0.000	(0.000, 0.000)	0.000	0.000	0.000	0.000	0.000
TASRL	-4.424	0.262	(-4.938, -3.911)	-16.88	0.000	285.119	276.928	0.012
IMSRL	-4.004	0.258	(-4.510, -3.499)	-15.52	0.000	294.029	232.864	0.012
Range(xRL)	0.038	0.064	(-0.0869, 0.1643)	0.60	0.546	0.364	-7.646	1.039
Med(xRL)	0.114	0.067	(-0.0179, 0.2473)	1.69	0.090	2.870	-5.158	1.121
Min(xRL)	-1.157	0.350	(-1.410, -0.904)	-8.97	0.000	10.927	72.454	0.314
N	-0.531	0.350	(-1.217, 0.154)	-1.52	0.129	2.301	-5.695	0.588
M	0.732	0.391	(-0.034, 1.498)	1.87	0.061	3.504	-4.509	2.079
maxDeg (δ)	-0.582	0.322	(-1.213, 0.049)	-1.81	0.071	3.266	-4.730	0.558

The logistic regression results included coefficients for seven primary variables: the SRL value, the SRL models under consideration, the range and median of the modeled TRL or IRL values of a given model iteration, the system size (n), system order (m), and the graph degree (δ). The Wald statistic (squared ratio of the regression coefficient to the standard error) followed a Chi-square distribution and was used to test for coefficient significance (Pampel 2000). Analysis of the Wald statistic for each variable revealed that SRL Value, the SSRL, TASRL, and IMSRL models exceeded significance levels. The GTSRL model is the reference model for the logistic regression and had a regression coefficient of 0.000, but exhibited notable odds ratio characteristics discussed below. The min(xRL) was also significant but the range(xRL), median(xRL), as well as the system size, order, and maximum graph degree coefficients did not exceed normal significance levels. P-values for all variables were significant at the 95% confidence level (CL), except for range(xRL) (0.546), med(xRL) (0.090), system size (0.129), system order (0.061), and maxDeg (0.071).

Recent research by Raftery (1995) suggested that p-values do not provide useful significance tests for logistic regression and recommended the Bayesian Information Criteria (BIC) as a viable alternative. A BIC greater than zero confirms the coefficient is significant. BIC values in Table 5-5 suggested that the SRL Value, the TASRL and IMSRL coefficients, and min(xRL) were all significant contributors to SRL readiness reversal. The above significance analysis suggested that system size, order, and maximum degree parameters were not significant contributors to SRL readiness reversal regardless of SRL model type. This assertion reveals that the underlying SRL model mathematical constructions produce reversals, not the particular system configuration parameters.

5.3.2.4 Logit Odds Ratio Results

Logit odds ratios in Table 5-5 conveyed the effect on the dependent variable for unit increases of the independent variables. A one unit increase in range(xRL) or med(xRL) increased the logged odds of readiness reversal by 0.0387 and 0.1147 respectively. A one unit increase in min(xRL), however, decreased the logged odds of reversal by 1.157. Increased system size or maxDeg parameters produced lower logged odds of reversal while the system order raised the logged odds of reversal. Of the four SRL models considered, the SSRL model evidenced increased logged odds of reversal while the TASRL and IMSRL models demonstrated strongly decreased reversal rates. The strongest continuous variable was the SRL Value, which demonstrated a significant increase in logged odds of reversal. Using the regression model coefficients, β_x, in Table

5-5, the full logistic regression estimation equation was generated. Binary valued regression plots of each SRL model versus SRL value were generated and plotted in Figure 5-5.

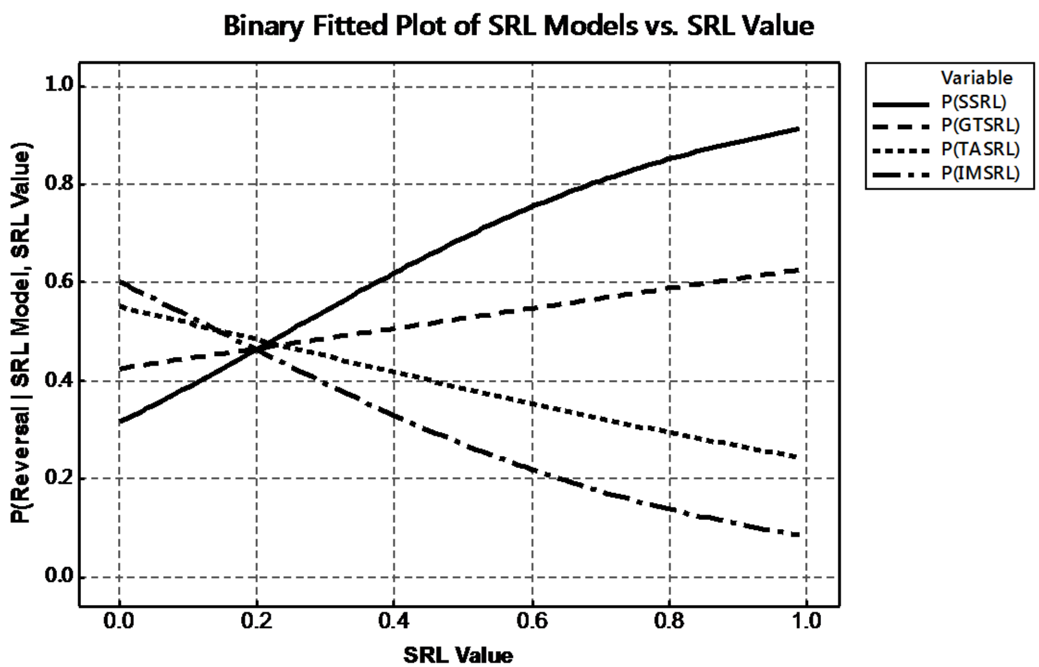


Figure 5-5: Fitted binary valued plots of P(Reversal | SRL Model, SRL Value).

This plot depicts fitted binary logistic regression plot of the probability of readiness reversal, P(RR), across a range of SRL values from [0,1] and for each of four SRL models. Each curve portrays a regression fit at a 95% confidence level.

The plot of Figure 5-5 depicts the binary regression model plots of readiness reversal for each SRL model. The regression plot does not consider the effects of system parameters or TRL and IRL values, yet still provides insight into qualitative differences among the four models. The SSRL and GTSRL models showed increased probability of readiness reversal as SRL values increase. The TASRL and IMSRL models demonstrated decreased rates of readiness reversal as SRL values increase. For calculated SRL values greater than about 0.2, the four SRL models demonstrated remarkable divergence. In early acquisition phases (e.g. MSA or TD) the SSRL and GTSRL phases had lower rates of readiness reversal than the TASRL or IMSRL models. As systems progress through subsequent acquisition phases, however, the TASRL and IMSRL models exhibited significantly lower reversal rates. Direct paired comparison of SRL model odds ratios were evaluated and listed in Table 5-6.

Table 5-6: Odds ratios of logistic regression model variables.

Model A	Model B	Odds Ratio	95% CI	Variable	Odds Ratio	95% CI
SSRL	GTSRL	1.66	(1.07, 2.58)	Range(xRL)	1.0395	(0.9168, 1.1786)
SSRL	TASRL	138.89	(232.56, 81.97)	Med(xRL)	1.1215	(0.9822, 1.2805)
SSRL	IMSRL	91.02	(51.11, 162.08)	m	2.0787	(0.9664, 4.4714)
GTSRL	TASRL	83.33	(138.89, 50.00)	n	0.5879	(0.2963, 1.1666)
GTSRL	IMSRL	54.95	(90.91, 33.11)	maxDeg	0.5589	(0.2972, 1.0504)
TASRL	IMSRL	1.52	(2.58, 089)	Min(xRL)	0.3144	(0.2442, 0.4048)

Larger logit odds ratios between two SRL models implied a greater impact on readiness reversal. Consider the first line of Table 5-6 that lists a pair-wise odds ratio of 1.66 for the comparison of the SSRL (Model A) against the GTSRL (Model B). The odds ratio of 1.66 in this comparison implies that readiness reversal using the SSRL model was 1.66 times more likely to occur than for the GTSRL method. Although not explicitly included in the logistic regression model coefficients above in Table 5-4, the GTSRL model was evaluated for odds ratio comparison. The SSRL model had 1.6 times higher odds of readiness reversal than the GTSRL but 139 times higher than the TASRL. The TASRL model has only marginally higher odds of readiness reversal than the IMSRL model (1.52).

In addition to the SRL model comparisons, odds ratios were calculated for individual model factors. Increased system order raised the odds of readiness reversal by 2.07 times. Range(xRL) and med(xRL) had odds ratios close to one, which implied little effect on readiness reversal. The system size, maxDeg, and min(xRL) all had odds ratios less than one that suggested these variables were not strong contributors to readiness reversal. System order was a contributing factor to SRL readiness reversal but the system size and degree had little or no substantive impact. The results of Table 5-6 demonstrate that the SSRL model had significantly higher odds of readiness reversal than the TASRL or IMSRL models. The GTSRL model also generated rates of reversal notably higher than the TASRL and IMSRL models. Comparing the TASRL and IMSRL models, however, yielded only marginally different rates of readiness reversal. Given the logit odds ratio results, the IMSRL model demonstrated the lowest effect of generating SRL readiness reversal. The TASRL model was a close second place while the SSRL and GTSRL models had substantially higher odds of readiness reversal than either the IMSRL or TASRL models.

In addition to the binary value regression plot in Figure 5-5, a series of supplemental contour surface model plots were also generated and listed in Appendix C. Each of the twelve contour plots in Appendix C depict a three dimensional contour surface of the probability of readiness reversal for all four SRL models as compared with the min(xRL), med(xRL), and range(xRL) variables respectively. The set of twelve plots reinforce the foregoing analysis and provide additional demonstration of the reduced likelihood of IMSRL readiness reversal versus the competing approaches.

5.3.2.5 Logistic Regression Error Analysis

Logistic regression model analysis fit using Pearson and Hosmer-Lemeshow tests produces p-values of 0.000 that suggests a poor fit between the logistic regression model and the raw simulation data. The Deviance test, however, has a p-value of 1.000 that supports a strong model fit. Somers' D, Goodman-Kruskal Gamma, and Kendall's Tau tests produce p-values of 0.95, 0.95, and 0.47 respectively that all support a strong fit between the fitted model and the generated data set. The strong model fit parameters reinforced the validity of the simulated SRL readiness reversal and logistic regression analysis approach.

5.3.2.6 One-Proportion and Two-Proportion Test Results

A series of one and two-proportion nonparametric Z-tests assessed the comparative mean readiness reversal rates for each SRL model. These tests are commonly used to support analysis of categorical data input factors with binary dependent variables (Freeman, Hutto, and Mackertich 2014). The one-proportion Z-test compares the cumulative readiness reversal rate for each individual SRL model against the cumulative readiness reversal rate of all four models. A Fisher Binomial hypothesis test compares the mean readiness reversal rate of each model against the cumulative reversal of all four models. Table 5-7 lists the results for the one-proportion Z-test and the Fisher Binomial p-value tests.

Table 5-7: One-proportion Z-test and Fisher binomial test results (α=0.05).

SRL Model	# Reversals	Sample Size	Reversal Rate	One-Proportion p-value	95% CI	Fisher Binomial p-value
SSRL	727	794	91.56%	0.9156	(0.8941, 0.9340)	0.000
GTSRL	454	723	62.7%	0.6279	(0.5916, 0.6633)	0.000
TASRL	181	747	24.23%	0.2423	(0.2120, 0.2747)	0.000
IMSRL	62	736	8.42%	0.0842	(0.2120, 0.2747)	0.000
Total	1424	3000	47.47%			

The one-proportion Z-test p-values were all higher than α=0.05, which suggested that the reversal rate of each of the four SRL models is not significantly different from the cumulative readiness reversal rate. If a confidence level α=0.10 is used, however, then the IMSRL has a significantly different mean reversal rate from the cumulative readiness reversal rates. In contrast with the one-proportion Z-test, the Fisher binomial test p-values suggests that each SRL model has a significantly different mean from the cumulative mean at α=0.05. A series of two-proportion Z-tests compares the mean reversal rate of each SRL model with the other SRL models. The two-proportions Z-values confirms that each SRL model comparison test was valid and the p-values (0.000) suggestd that a significant difference in readiness reversal rates exists among the SRL models at α=0.05.

Table 5-8: Two-proportion Z-test results.

SRL Model	Z value	p-value
SSRL vs. GTSRL	14.03	0.000
SSRL vs. TASRL	36.35	0.000
SSRL vs. IMSRL	58.48	0.000
GTSRL vs. TASRL	58.46	0.000
GTSRL vs. IMSRL	26.28	0.000
TASRL vs. IMSRL	8.44	0.000

The IMSRL model exhibits the lowest rate of readiness reversal (8.42%) among the four models evaluated while the SSRL model clearly possesses the highest rate of readiness reversal (91.56%) given the definition of readiness reversal proposed in this research. The system graph parameters of size, order, and maximum graph degree are not significant contributors to SRL readiness reversal rates at a 95% confidence level. If α=0.10 is selected, however, then the maximum graph degree and system order would be significant factors. Odds ratios suggest that system order produced higher odds of readiness reversal even with the moderately significant p-values. The range(xRL) and med(xRL) are not significant contributors to readiness reversal from either their respective p-values or BIC values. The min(xRL) is significant according to both its p-value (0.000) and BIC parameter (72.45) at α=0.05.

5.3.2.7 Multi-Variable Correlation

A multi-variate Pearson correlation analysis was also performed. This analysis calculates a Pearson correlation coefficient (ρ_{XY}) analysis for each paired combination of the full set of independent and independent variables. The correlation coefficient results are provided in Table 5-9.

Table 5-9: Multi-variate correlation matrix of SRL readiness reversal variables.

	Reversal	IMSRL	TASRL	GTSRL	SSRL	Range (xRL)	Max (xRL)	Med (xRL)	Min (xRL)	Max deg	Cnm
IMSRL	-0.446										
TASRL	-0.268	-0.328									
GTSRL	0.173	-0.321	-0.342								
SSRL	0.530	-0.342	-0.345	-0.338							
Range (xRL)	0.015	-0.017	-0.022	0.009	0.029						
Max (xRL)	0.198	0.003	-0.018	-0.011	0.025	0.701					
Med (xRL)	0.299	0.010	-0.014	-0.006	0.009	0.017	0.432				
Min (xRL)	0.183	0.027	0.012	-0.024	-0.015	-0.680	0.047	0.422			
Max (deg)	0.034	0.005	-0.033	-0.001	0.028	0.502	0.357	0.017	-0.337		
Cnm	0.026	0.001	-0.031	-0.001	0.030	0.541	0.384	0.019	-0.362	0.982	

The correlation analysis reveals that the IMSRL model has a moderately strong negative correlation with readiness reversal (ρ = -0.446) whereas the SSRL has a moderately strong positive correlation with readiness reversal (0.530). This contrast of correlation coefficients between the IMSRL and SSRL suggests that the SSRL model has a notably strong correlative relationship with readiness reversal than the IMSRL model. The TASRL (ρ = -0.268) and GTSRL (ρ = -0.446) respectively exhibited weakly negative and positive correlation with readiness reversal.

5.4 Data Conclusions

The foregoing discussion provides a significant breadth of data on the IMSRL model evaluation and on SRL readiness reversal characteristics. This section discusses the pertinent conclusions from this chapter.

5.4.1 IMSRL Model Formulation

The IMSRL was demonstrated using a general graph system comprised of directed edges, multi-edges, and vertex loops. The new model exhibitd a simple formulation and a methodology that leveraged Cartesian Product set theory operations and a min-min calculation approach. The IMSRL provides greater flexibility at handling complex systems than do the SSRL, GTSRL, or TASRL models. The IMSRL flexibility to model complex system architectures serves as a notable enhancing characteristic of the IMSRL compared to the SSRL, GTSRL, and TASRL methods. The IMSRL demonstration in section 5.1 coupled with the descriptions of the other SRL models in Chapter 2 and Appendix A support a comparison of the model structures supported by the various RL models. This SRL model comparison versus system structure is provided in Table 5-10.

Table 5-10: SRL model comparison for various system graph structures.

Graph Structure	SSRL	GTSRL	TASRL	IMSRL
Simple Graph (single undirected edges)	X	X	X	X
Digraph (single directed edges)		X	X	X
Mixed Graph (single directed and undirected edges, no loops)		X	X	X
Multi-Graph (multiple directed or undirected edges, no loops)				X
General Graph (any directed, undirected edges and/or vertex loops)				X

The comparison in Table 5-10 reveals a significant advantage of the IMSRL versus competing models. The IMSRL can adequately handle any of the listed system graph representations whereas each of the other SRL approaches have some level of limited applicability. The SSRL is restricted to assessing only simple graph system representations. The GTSRL and TASRL can both handle simple graphs and digraphs, and mixed graph implementations. For the more complex graph structures of multigraphs and general graphs, however, only the IMSRL can assess the readiness of such complex system structures. This ability to ascertain the readiness of such complex system structures is a fundamental IMSRL enhancement.

5.4.2 IMSRL Analytic Demonstration

The IMSRL displays excellent mathematical characteristics and satisfies the five desired SRL properties of McConkie (2013). This research also demonstrates a new desirable SRL mathematical property for which the IMSRL is also satisfactory. In addition, the IMSRL demonstrates superior sensitivity to unit input changes as compared to the SSRL and GTSRL models.

5.4.3 Readiness Reversal Conclusions

SRL readiness reversal data suggests that the four SRL models have divergent readiness reversal characteristics. The pairwise matrix multiplication based methods (i.e. SSRL and GTSRL) have significantly higher rates of reversal than the tropical algebra and incidence matrix based methods (i.e. TASRL and IMSRL respectively). In contradistinction to the SSRL and GTSRL models, the TASRL and IMSRL methods demonstrates notably lower readiness reversal rates (24% and 8% respectively). The TASRL model, however, has nearly a three times greater reversal rate than the IMSRL method, but yet still has far lower reversal rates than the SSRL and GTSRL models. Given the dramatically lower readiness reversal rates of the TASRL and IMSRL as compared to the other two models, which of these two models should be preferred? The answer depends on the system characteristics under consideration. For simple system architectures with no multi-edges or vertex loops, either method generates significantly lower risk of readiness reversal than the SSRL or GTSRL models. Systems engineers evaluating complex systems modeled as multigraphs or general graphs may opt for the IMSRL as this method can handle such structures in a more straightforward manner. The IMSRL, however, lacks the intuitive simplicity of the SSRL or GTSRL pairwise matrix multiplication approach (Garrett et al. 2011; Sauser et al. 2008). Furthermore, the IMSRL does not possess the mathematical elegance of the TASRL method (McConkie et al. 2012; McConkie 2013). Additional criticisms of the IMSRL model may arise from producing seemingly simplistic results that are concordant with a min-min approach. Given the research results discussed above, the IMSRL definitively has the lowest risk of readiness reversal. The low readiness reversal risk directly implies that the IMSRL will provide more accurate readiness assessments of complex system architectures than other SRL methods. Ongoing research on System Readiness Functions by Marchette (2013) suggest that advanced adjacency matrix configurations may be leveraged to support such complex system architectures but practical model formulations have not yet been demonstrated.

5.4.4 Hypothesis Test Results

This section discusses the three research hypotheses and supporting data resultsw.

1) H_1: SRL calculation methods have no impact on readiness reversal.

The accumulated logistic regression model results and individual hypothesis testing support a rejection of the first hypothesis that SRL model type had no significant effect on the presence of SRL readiness reversal at α=0.05. The IMSRL had significantly lower readiness reversal rates than the other methods and so this research accepts the alternate hypothesis that the SRL model selection is in fact a significant factor on the presence of SRL readiness reversal at a CL of α=0.05.

2) H_2: Model structure parameters (e.g. system size, order, degree, TRL or IRL values) have no impact on SRL readiness reversal.

The second hypothesis in this research was that changes of system size or system order did not affect SRL readiness reversal. Neither system size nor system order are significant contributors to readiness reversal at α=0.05 but system order would be a significant factor at α=0.10. This author therefore accepts the null hypothesis that system order and size have no significant effect on SRL readiness reversal at α=0.05. The cumulative results confirm that readiness reversal is impacted by the selection of SRL model but is not affected by system characteristics of order or size.

3) H_3: IMSRL has no unit input sensitivity difference from other methods.

Sensitivity analysis of the four models revealed that the IMSRL and TASRL models have zero sensitivity to unit input changes of baseline TRL or IRL component levels. Both the SSRL and GTSRL models, however, exhibit significant sensitivity to such input changes. These sensitivity analysis results suggest that this third null hypothesis should be rejected for a comparison of the IMSRL with either the SSRL or GTSRL models. The he alternate hypothesis that sensitivity analysis differences are indeed present among the four SRL models is accepted for IMSRL comparison with the SSRl and GTSRL models..

5.5 Summary

In this chapter the new IMSRL model was presented and evaluated for a complex general graph system configuration. Several demonstration examples of the IMSRL were provided as well as a sensitivity analysis comparison of the IMSRL with competing models. A summary table of the statistical tests and data results discussed in this chapter that were used to assess SRL readiness reversal are presented in Table 5-11.

Table 5-11: Summary table of data assessment results.

Data Assessment Method	Summary of Results
IMSRL Model	
- IMSRL Model Formulation	Demonstrated using General graph example
- Sensitivity Analysis	IMSRL and TASRL had no sensitivity change
- Mathematical Properties	IMSRL satisfies all six properties
Readiness Reversal Analysis	
- Descriptive Statistics	SRL reversal rates for SSRL (91.6%), GTSRL (62.5%), TASRL (24.3%), and IMSRL (8.4%)
- Regression Coefficient Analysis	SSRL, GTSRL, TASRL, IMSRL significant
- Wald Statistic	SRL value and all SRL model types significant
- Bayesian Information Criterion	SRL value and TASRL, IMSRL, all significant
- Logit Odds Ratios	SRL value, SSRL, and system Order all significant
- Logit Odds Ratio Comparisons	SRL comparisons, and system Order all significant
Error Analysis	
- Pearson	p (0.00) suggest poor model fit
- Hosmer-Lemeshow	p (0.00) suggest poor model fit
- Deviance	p (1.00) suggest strong model fit
- Somers-D	p (0.95 suggest strong model fit
- Goodman-Kruskal Gamma	p (0.95) suggest strong model fit
- Kendall Tau	p (0.47) suggest strong model fit
Comparison Tests	
- 1-Proportions Z-test	No SRL model reversal rates were significant
- Fisher Binomial test	All SRL model reversal rates were significant
- 2-Proportions Z-test	All SRL moderl comparisons were significant
Multivariate Correlation	IMSRL and TASRL had negative correlation GTSRL and, SSRL had positive correlation
Hypothesis Tests	H1: Reject – SRL model type has impact H2: Accept – graph structure has no impact H3: Reject – IMSRL has sensitivity difference

The IMSRL demonstrates decreased presence of readiness reversal compared to the SSRL, GTSRL, and TASRL models. Multiple correlation analysis suggests a moderately negative correlation between the presence of readiness reversal and the IMSRL model. The new IMSRL model also satisfies all six desired SRL mathematical properties.

CHAPTER 6 – CONCLUSIONS AND FUTURE WORK

This research evaluated the readiness reversal of four SRL methods by a simulation experimental approach and logistic regression data analysis methods. The IMSRL model exhibited notably decreased presence of SRL readiness reversal compared with competing models. In addition the IMSRL was extended to more complex system configurations than can be supported using current SRL methods. While not explicitly discussed in this dissertation, the IMSRL method aligns with the mathematical properties of System Readiness Functions of Marchette (2013) by employing a minimum value set operation. The IMSRL satisfies five desired SRL mathematical properties suggested by McConkie et al. (2012) and McConkie (2013), as well as a sixth new SRL property.

6.1 SRL Cautions

SRL models are still in their relative infancy. Quantitative system evaluation methods are still in development but the IMSRL represents a notable advance in the system readiness assessments. In concert with other authors (Kujawski 2010 and 2013; McConkie et al., 2012), caution is urged to not rely exclusively on SRL calculations for system developmental progress assessment. As George Box noted, "Essentially, all models are wrong, but some are useful." (Box and Draper 1987, pg. 424). This observation certainly applies to SRL methods. SRL are not a silver bullet that eliminates technological developmental concerns. The IMSRL significantly lower readiness reversal rate will support superior management decisions when employed as an element of a structured acquisition development process. Legacy SRL in general do not provide quantitative estimates of the cost or schedule expenditures required to attain the next acquisition phase. The IMSRL provides readiness assessments at the sub-system and whole-system architecture levels and can thereby provide useful information to program managers on the progression of system readiness at various levels of system architectural hierarchy. As McConkie et al. (2012, 5) astutely notes, "Increasing the validity of SRL's calculation will give program managers more assurance when making acquisition decisions on systems or SoSs."

6.2 Future Research

While this research presented a notable advance in the study of SRL models with a particular emphasis on readiness reversal characteristics, more research is needed in several key areas.

(1) *Evaluate Fundamental SRL Assumptions*: Further work on extensions of the present IMSRL model should address the fundamental SRL constraint of using values of "0" for non-integration paths. This feature was preserved in accordance with the contemporary SSRL, GTSRL, and TASRL models, yet such zero values effectively provide no added information. By employing a min-min calculation approach, the IMSRL provides an effective method of ignoring such non-informative values. Other approaches that afford equivalent calculation stability in more compact formats without using uninformative zero values should be considered. Incidence tables may be useful in this regard (Gross and Yellen, 2006).

(2) *Develop SRL Calculation Options Beyond Graph Theory*: Solutions to expand SRL model structures beyond graph theory should also be considered. Advanced system network structures such as Petri nets (Diestel 2010; Koch, Junker, and Heiner 2005) or

Bayesian belief networks may prove useful. The use of adjacency matrix structures for SRL calculations will particularly limit the applicability of the published models by restraining their effective implementation in hierarchical system models, nested models, or more complex multi-layer frameworks. The IMSRL incidence matrix structure provides a framework that can assess hierarchical system architectures, nested models, and complex system structures. The system readiness function mathematics of Marchette (2013) may provide insight in this particular area.

(3) *Broaden Readiness Reversal System Evaluations*: Quantitative evaluation of SRL readiness reversal is a unique new feature of this dissertation yet this evaluation was limited in two main areas. The first readiness reversal limitation was the confinement of the evaluation to the three primary published SRL methods heretofore mentioned and the new IMSRL presented in this research. In addition, the readiness reversal simulation model was limited to only those system architectures discussed in Chapter 5 and Figure 2-1. This limitation was enacted to make the assessments of the readiness reversal impact of system structure parameters (e.g. size, order, and degree) more tractable. Extension of the readiness reversal analysis can be easily extended to any single level system architecture of simple graphs, complete graphs, mixed graphs, or general graphs. The IMSRL framework offers a convenient approach for evaluating any class of single-layer or hierarchical graph systems including general graph architectures. Extending the Monte-Carlo simulation model calculation code listed in Appendix B to a wider set of graph configurations may generate a relationship of readiness reversal probability to any single-level graph model structure.

(4) *Expanding Readiness Reversal Definition Evaluations:* This research focused on one particular definition of readiness reversal, but other research (McConkie 2013 and Sauser et al. 2010) offers a competing readiness reversal definition. This other readiness reversal definition suggests that reversal may be evaluated by assessing the set of TRL and IRL value combinations for a given system that produce a specific SRL value. SRL readiness reversal properties should be evaluated for both readiness reversal concepts to demonstrate the reliability and comparability of readiness reversal characteristics.

(5) *SRL Applications to Diminished System Capacity:* SRL applications for system evaluation and program management considerations have steadily grown since SRL were introduced in the mid-2000s (see Chapter 2 for further discussion). All such applications, however, consider the TRL and IRL constituent system components within their conventionally defined [1-9] scale. Such constraints do not consider system operational use and eventual diminishing capacity due to aging components and of SRL use for systems operations and sustainment considerations. That is, systems may degrade in readiness as they age during their operational lifetime. This aspect of system lifecycle support is not presently considered within SRL analysis. System aging and diminished capacity considerations lie outside the scope of this research, but raise interesting opportunities for expanding SRL applications beyond system development and technical capability growth.

(6) *Disparate TRL and IRL Weighting*: Current SRL calculation methods employ TRL and IRL values that are equally weighted in the calculation methodology. This implicit assumption that TRL and IRL values are of equal weight in the calculation process is unverified. TRL are fairly well characterized and well implemented as part of

formal TRA processes whereas IRL frameworks are somewhat novel and not well validated at the present time. Disparate weighting scheme should be considered to account for the relative strength of TRL assessments versus the relative uncertainty of IRL assessments. Accounting for disparate TRL and IRL weighting may be more easily accommodated using the IMSRL approach than legacy approaches because the IMSRL does not need to consider unintended impacts of multiplicative weighting schemes int eh readiness calculations.

(7) *SRL Utility Assessment*: SRL development in this research was primarily concerned with quantitatively evaluating readiness reversal and proposing a new model to address readiness reversal concerns. Little formal work in the literature, however, has addressed the usefulness of SRL methods. Formal SRL usefulness analysis, especially as part of a comprehensive SRL risk assessment, would be a strong addition to the SRL literature by demonstrating the information content and decision influencing capability of SRL.

(8) *Consideration of Graph Complexity Measures.* Early complexity measures included the McCabe cyclomatic number for use in computer program control graphs (McCabe, 1976). Later graph complexity measures considered different graph topologies and measures of linear complexity (Costa et al. 2007; Jukna 2004; Neel and Orrison 2006). Other graph complexity measurements should be considered for future SRL readiness reversal research, but lie outside the scope of this dissertation. This area of future work would be of particular interest for expanding IMSRL use as the IMSRL

could serve as a foundational system representation methodology regardless of the system complexity measurement structure under consideration.

(9) *Expanding GTSRL and TASRL Models.* Both the GTSRL and TASRL could be easily modified to handle self-referencing vertex loops by using non-zero values in their IRL matrix diagonal elements, but neither model explored this potential capability. This dissertation did not explore GTSRL or TASRL properties beyond their respective formulations espoused in the literature. Expanded GTSRL and TASRL model architectures may alleviate concerns over the breadth of system structures that these models can support. Such expansions of the GTSRL or TASRL models, however, would merely expand their capabilities to those closer to the IMSRL.

6.3 Summary

This research evaluated variant implementations of the System Readiness Level and proposed the new IMSRL model. IMSRL mathematical confidence will provide legitimate system readiness assessment information to program decision makers. The viable IMSRL method will foster broader acceptance of readiness level assessments by systems engineering professionals, and will support program development risk reduction efforts. The IMSRL is a demonstrably better arrow in the systems engineering quiver than prior readiness level assessment methods. Expanded IMSRL implementation will cultivate system technical readiness assessments into a vibrant and flourishing part of the systems engineering garden.

REFERENCES

1. Agresti, Alan. 1990. *Categorical Data Analysis*. Vol. 218: Wiley-Interscience,
2. Azizian, Nazanin, Thomas Mazzuchi, Shahram Sarkani, and David F. Rico. 2011. "A Framework for Evaluating Technology Readiness, System Quality, and Program Performance of U.S. DoD Acquisitions." *Systems Engineering* 14, no. 4: 410-26.
3. Azizian, Nazanin, Shahram Sarkani, and Thomas Mazzuchi. 2009. "A Comprehensive Review and Analysis of Maturity Assessment Approaches for Improved Decision Support to Achieve Efficient Defense Acquisition." *Lecture Notes in Engineering and Computer Science* 2179, no. 1: 1150-57.
4. Balbuena, C. 2008. "Incidence Matrices of Projective Planes and of Some Regular Bipartite Graphs of Girth Six with Few Vertices." *SIAM Journal on Discrete Mathematics* 22, no. 4: 1351-63.
5. Baron, Neil T., O. Thomas Holland, David J. Marchette, and Sara E. Wallace. 2011. "Integrating through the Fire Control Loop." In *National Fire Control Symposium*, 19. Lake Buena Vista, FL.
6. Balci, Osman. 1998. "Verification, Validation, and Testing." Chap. 10 In *Handbook of Simulation: Principles, Methodology, Advances, Applications, and Practice*, edited by Jerry Banks, 335-93: John Wiley & Sons, Inc., co-published by Engineering & Management Press.
7. Barzilai, Jonathan, and Boaz Golany. 1994. "Ahp Rank Reversal, Normalization and Aggregation Rules." *Information Systems and Operational Research* 32, no. 2 (1994): 57-64.
8. Bedford, T., and Roger M. Cooke. *Probabilistic Risk Analysis: Foundations and Methods*. Cambridge, UK: Cambridge University Press, 2001.
9. Bilbro, James W. 2007. "A Suite of Tools for Technology Assessment." In *Air Force Research Laboratory Technology Maturity Conference*. Virginia Beach, VA.

10. Bonacich, Phillip, Annie Cody Holdren, and Michael Johnston. 2004. "Hyper-Edges and Multidimensional Centrality." *Social networks* 26, no. 3: 189-203.
11. Birta, Louis G., and F. Nur Ozmizrak. 1996. "A Knowledge-Based Approach for the Validation of Simulation Models: The Foundation." *ACM Trans. Model. Comput. Simul.* 6, no. 1: 76-98.
12. Box, G. E. P. and N. R. Draper. 1987. *Empirical Model Building and Response Surfaces*. John Wiley & Sons, New York, NY.
13. Bowles, John, B. 2004. "An Assessment of RPN Prioritization in a Failure Modes Effects and Criticality Analysis." *Journal of the IEST* 47: 51.
14. Chaouiya, Claudine.2007. "Petri Net Modelling of Biological Networks." *Briefings in Bioinformatics* 8, no. 4: 210-19.
15. Chartrand, Gary. 1985. *Introductory Graph Theory*. New York: Dover, DE.
16. Chartrand, Gary, and Ping Zhang. 2012. *A First Course in Graph Theory*. Mineola, N.Y: Dover Publications.
17. Clark, Gordon H. 2002. The Theologian's Besetting Sin. In *Against the Churches: The Trinity Review, 1989-1998*, ed. John R. Robbins, 127-139. The Trinity Foundation, Unicoi, TN.
18. Costa, L da F, Francisco A. Rodrigues, Gonzalo Travieso, and P. R. Villas Boas. 2007. "Characterization of Complex Networks: A Survey of Measurements." *Advances in Physics* 56, no. 1: 167-242.
19. Cox, Louis Anthony. 2008. "What's Wrong with Risk Matrices?" *Risk analysis* 28, no. 2: 497-512.
20. Cox, Louis Anthony. 2009. "What's Wrong with Hazard-Ranking Systems? An Expository Note." *Risk Analysis* 29, no. 7: 940-48.
21. Dacus, Chad L. 2012. "Improving Acquisition Outcomes through Simple System Technology Readiness Metrics." *Defense Acquisition Review Journal* 19, no. 4: 17.
22. De La Ree, J., V. Centeno, J. S. Thorp, and A. G. Phadke. 2010. "Synchronized Phasor Measurement Applications in Power Systems." *Smart Grid, IEEE Transactions on* 1, no. 1: 20-27.

23. DeNezza, Eugene J., and Aloysius G. Casey. 2014. "Future Space System Acquisitions: Is the Key "What" or "When"?". *Defense Acquisition, Technology and Logistics*, no. Defense Acquisition University, January-February 2014: 9-14.
24. Department of Defense. 2009. Technology Readiness Assessment (TRA) Deskbook. *United States Department of Defense*. Washington, DC.
25. Department of Defense. 2013. Operation of the DoD Acquisition System, Interim 5000 Revision, United States Department of Defense, November 26, 2013.
26. Diestel, Reinhard. 2010. *Graph Theory*. 4th ed. Vol. 173., Heidelberg: Springer.
27. Dyer, James S., Thomas L. Saaty, Patrick T. Harker, and Luis G. Vargas. 1990. "Remarks on the Analytic Hierarchy Process; an Exposition Of." *Management Science* 36, no. 3: 249.
28. Engle, M., S. Sarkani, and T. Mazzuchi. 2009. "Technical Maturity Evaluations for Sensor Fusion Technologies." Paper presented at the Applied Imagery Pattern Recognition Workshop (AIPRW), 2009 IEEE, 14-16 Oct.
29. Forbes, Eric, Richard Volkert, Peter Gentile, Ken Michaud, and Tom Sondi. 2009. "Implementation of a Methodology Supporting a Comprehensive System-of-Systems Maturity Analysis for Use by the Littoral Combat Ship Mission Module Program " INCOSE Chesapeake Chapter Dinner Meeting, 19 August.
30. Forman, Ernest H, and Saul I Gass. 2001. "The Analytic Hierarchy Process-an Exposition." *Operations research* 49, no. 4: 469-86.
31. Freeman, Laura, Gregory Hutto, and Neal Mackertich. 2014. "*Scientific Methods for Improving DoD Test and Evaluation: Statistical Test Optimization Synthesis Panel*." ITEA Journal, 35, 1: 31-39.
32. Fulkerson, Delbert R, and Oliver A Gross. 1965. "Incidence Matrices and Interval Graphs." *Pacific Journal of Mathematics 15*, no. 3: 835-55.

33. Gardner III, George H. 2014. "The Difference between Verification & Validation in Systems Engineering." *International Test and Evaluation Association Journal* 35, no. 4: 240-47.
34. Garvey, Paul R, and Chien-Ching Cho. 2005. "An Index to Measure and Monitor a System-of-Systems' Performance Risk." *MITRE Technical Paper*.
35. Government Accountability Office. 1999. Best Practices: Better Management of Technology Development Can Improve Weapon System Outcomes. *Government Accountability Office*, GAO/NSIAD-99-162, Washington, DC.
36. Government Accountability Office. 2006 Best Practices: Stronger Practices Needed to Improve DOD Technology Transition Processes. *Government Accountability Office*, GAO-06-883, Washington, DC.
37. Government Accountability Office. 2011. Defense Acquisitions: Assessments of Selected Weapon Programs. *Government Accountability Office*, GAO-11-233SP, Washington, DC.
38. Government Accountability Office. 2013. Defense Acquisition: Assessment of Selected Weapon Systems. *Government Accountability Office*, GAO-13-294SP, Washington, DC.
39. Garrett, Robert K., Steve Anderson, Neil T. Baron, and James D. Moreland. 2011. "Managing the Interstitials, a System of Systems Framework Suited for the Ballistic Missile Defense System." *Systems Engineering* 14, no. 1: 87-109.
40. Gross, Jonathan L., and Jay Yellen. 2006. *Graph Theory and Its Applications*. Boca Raton: Chapman & Hall/CRC.
41. Guo, Qisheng, Guo Li, Liang Li, and Jinghua Song. 2012. "Study on Markov Chain-Based System Readiness Assessment Method." In *Asiasim 2012*, 490-98: Springer.
42. Holt, Daniel T, Summer E Bartczak, Steven W Clark, and Martin R Trent. 2007. "The Development of an Instrument to Measure Readiness for Knowledge Management." *Knowledge Management Research & Practice* 5, no. 2: 75-92.

43. Hougen, Kevin J., Michael T. Frazier, Jason E. Wright, and Don D. Nguyen. 2014. "Flight Test Correlation Modeling and Autoland Monte Carlo Analysis for Aircraft Certification." *International Test and Evaluation Association Journal* 35, no. 3: 232-39.
44. Hosmer, David W, T Hosmer, Saskia Le Cessie, and Stanley Lemeshow. 1997. "A Comparison of Goodness-of-Fit Tests for the Logistic Regression Model." *Statistics in Medicine* 16, no. 9: 965-80.
45. Jukna, S. 2006. "On Graph Complexity." [In English]. *Combinatorics, Probability & Computing* 15, no. 6: 855-76.
46. Kim, Jongkwang, and Thomas Wilhelm. 2008. "What Is a Complex Graph?". *Physica A: Statistical Mechanics and its Applications* 387, no. 11: 2637-52.
47. Kober, Brian J., and Brian Sauser. 2008. "A Case Study in Implementing a System Maturity Metric." In *29th American Society of Engineering Management Conference*. West Point, NY.
48. Koch, Ina, Björn H Junker, and Monika Heiner. 2005. "Application of Petri Net Theory for Modelling and Validation of the Sucrose Breakdown Pathway in the Potato Tuber." *Bioinformatics* 21, no. 7: 1219-26.
49. Kujawski, Eduoard. 2010. "The Trouble with the System Readiness Level (SRL) Index for Managing the Acquisition of Defense Systems." In *13th Annual Systems Engineering Conference*, 24. San Diego, CA: National Defense Industrial Association.
50. Kujawski, Eduoward. 2013. "Analysis and Critique of the System Readiness Level." *Systems, Man, and Cybernetics: Systems, IEEE Transactions on* PP, no. 99: 1-9.
51. Lansdowne, Zachary F. 1999. "Risk Matrix: An Approach for Prioritizing Risks and Tracking Risk Mitigation Progress." *Proceedings of the 30th Annual Project Management Institute, Philadelphia, PA, October 10-16*.
52. London, Mark A., Thomas Holzer, Timothy Eveleigh, and Shahryar Sarkani. 2013. “Using System Readiness Levels to Improve Reliability Predictions and

T&E Investment Decisions." *30th Annual Symposium of the International Test and Evaluation Association*, November 11-14, 2013.

53. London, Mark A., Thomas Holzer, Timothy Eveleigh, and Shahryar Sarkani. 2013. "Using System Readiness Levels to Improve Reliability Predictions and T&E Investment Decisions." *National Defense Industrial Association 16th Annual Systems Engineering Conference*, October 28-31.
54. London, Mark A., Thomas Holzer, Timothy Eveleigh, and Shahryar Sarkani. 2014. "Incidence Matrix Approach for Calculating Readiness Levels." *Journal of Systems Science and Systems Engineering*. Publication date TBD.
55. Long, J. M. 2011. "Integration Readiness Levels." Paper presented at the Aerospace Conference, 2011 IEEE, 5-12 March.
56. Maclagan, Diane and Bernd Sturmfels. 2009. "*Introduction to Tropical Geometry*." Unpublished manuscript.
57. Magnaye, Romulo B., Brian J. Sauser, and Jose E. Ramirez-Marquez. 2010. "System Development Planning Using Readiness Levels in a Cost of Development Minimization Model." *Systems Engineering* 13, no. 4: 311-23.
58. Mahafza, Sherry, Paul Componation, and Donald Tippett. 2005. "A Performance-Based Technology Assessment Methodology to Support DoD Acquisition." *Defense Acquisition Review Journal* 11, no. 3 (December 2004-March 2005): 268-83.
59. Malone, Patrick, and Lawrence Wolfarth. 2012. "System-of-Systems: An Architectural Framework to Support Development Cost Strategies." Paper presented at the 2012 IEEE Aerospace Conference.
60. Mandelbaum, Jay. 2005. "Enabling Technology Readiness Assessments (TRAs) with Systems Engineering." *Paper presented at the NDIA 8th Annual Systems Engineering Conference*.
61. Mankins, John C. 1998. "Research & Development Degree of Difficulty (R&D3)." *White Paper.* March 10.
62. Mankins, John C. 2002. "Approaches to Strategic Research and Technology (R&T) Analysis and Road Mapping." *Acta Astronautica* 51, no. 1: 3-21.

63. Marchette, David J. 2013. "An Analysis of System Readiness Functions." *Paper presented at the Joint Mathematics Meetings*, San Diego, CA, Jan. 30.
64. Mathworks. 2011. *Matlab (Version 7.12.0.635, R2011a)*. Computer Software.
65. McCabe, T. J. 1976. "A Complexity Measure." *IEEE Transactions on Software Engineering* SE-2, no. 4: 308-20.
66. McConkie, Eileen B. 2013. *A Systems Engineering Approach to Mathematical Properties of System Readiness Levels.* PhD Diss. The George Washington University, Washington, DC.
67. McConkie, Eileen, Thomas A. Mazzuchi, Shahram Sarkani, and David Marchette. 2012. "Mathematical Properties of System Readiness Levels." *Systems Engineering* 14, no. 4: 391-400.
68. Meier, Steven R. 2008. "Best Project Management and Systems Engineering Practices in the Preacquisition Phase for Federal Intelligence and Defense Agencies." *Project Management Journal* 39, no. 1: 59-71.
69. Mesquita, A., F. A. Salazar, and P. P. Canazio. 2002. "Chromosome Representation through Adjacency Matrix in Evolutionary Circuits Synthesis." Paper presented at the Evolvable Hardware, 2002. Proceedings. NASA/DoD Conference.
70. Mikhalkin, Grigory. 2006. "Tropical Geometry and Its Applications." *Unpublished Manuscript*.
71. Michaud, K., B. Sauser, E. Forbes, and P. Gentile. 2008. "Evaluating Complex System Development Maturity: The Creation and Implementation of a System Readiness Level for Defense Acquisition Programs." In *NDIA Systems Engineering Conference*, 38. San Diego, CA.
72. Minitab. *Minitab (Ver. 17)*. Computer Software. Minitab, Inc.
73. Neel, David L, and Michael E Orrison. 2006. "The Linear Complexity of a Graph." *The Electronic Journal of Combinatorics* 13, no. 1:R9.
74. Newman, Mark E. J. 2004. "Analysis of Weighted Networks." *Physical Review E* 70, no. 5: 056131.

75. Pampel, Fred. C. 2000. *Logistic Regression: A Primer*. Quantitative Applications in the Social Sciences. Ed. Michael S. Lewis-Beck, Sage Publications, Inc.
76. Peduzzi, Peter, John Concato, Elizabeth Kemper, Theodore R. Holford, and Alvan R. Feinstein. 1996. "A Simulation Study of the Number of Events Per Variable in Logistic Regression Analysis." *Journal of Clinical Epidemiology* 49, no. 12: 1373-79.
77. Peng, Chao-Ying Joanne, Kuk Lida Lee, and Gary M. Ingersoll. 2002. "An Introduction to Logistic Regression Analysis and Reporting." *The Journal of Educational Research* 96, no. 1: 3-14.
78. Raftery, A. E. 1995. ”Bayesian Model Selection in Social Research.” *Sociological Methodology*, no. 25): 111-163.
79. Ramirez-Marquez, J. E., and B. J. Sauser. 2009. "System Development Planning Via System Maturity Optimization." *Engineering Management, IEEE Transactions on* 56, no. 3: 533-48.
80. Ross, Sheldon M. 2000. *Simulation*. 4th ed. Burlington, MA: Elsevier.
81. Saaty, Thomas L. 2004. "Decision Making - the Analytic Hierarchy and Network Processes (Ahp/Anp)." *Journal of Systems Science and Systems Engineering* 13, no. 1. March 2004: 1-35.
82. Sauser, B, Eric Forbes, Michael Long, and S McGrory. 2009. "Defining an Integration Readiness Level for Defense Acquisition." Paper presented at the *International Conference of the International Council on Systems Engineering*, Singapore.
83. Sauser, Brian, and Jose E Ramirez-Marquez. 2012. Multi-Objective Optimization of System Capability Satisficing in Defense Acquisition. *Defense Technical Information Cente*r (2012), Accession No. ADA563343.
84. Sauser, Brian J., Jose Emmanuel Ramirez-Marques, Devandham Henry, and Donald DiMarzio. 2008. "A System Maturity Index for the Systems Engineering Life Cycle." *International Journal of Industrial and Systems Engineering* 3, no. 6: 673-91.

85. Sauser, Brian J., Jose Emmanuel Ramirez-Marques, Romulo B. Magnaye, and Weiping Tan. 2008. "A Systems Approach to Expanding the Technology Readiness Level within Defense Acquisition." *International Journal of Defense Acquisition Management* 1: 39-58.
86. Sauser, Brian J., Jose E. Ramirez-Marquez, David Nowicki, Abhi Deshmukh, and Martin Sarfaraz. 2011. "Development of Systems Engineering Maturity Models and Management Tools." *Defense Technical Information Center Report*, 2011-TR-014, Systems Engineering Research Center, Hoboken, NJ : 65.
87. Sauser, Brian J., Jose Emmanuel Ramirez-Marques, Dinesh Verma, and Ryan Gove. 2006. Determining System Interoperability Using an Integration Readiness Level. *Stevens Institute of Technology*.
88. Sauser, Brian J., Dinesh Verma, Jose Emmanuel Ramirez-Marques, and Ryan Gove. 2006. "From TRL to SRL: The Concept of System Readiness Levels." In *Conference on Systems Engineering Research*, 1-10: Stevens Institute of Technology.
89. Schenkerman, Stan. 1994. "Avoiding Rank Reversal in AHP Decision-Support Models." *European Journal of Operational Research* 74, no. 3: 407-19.
90. Singh, Harmanjit, and Richa Sharma. 2012. "Role of Adjacency Matrix & Adjacency List in Graph Theory." *International Journal of Computers & Technology* 3, no. 1: 179-83.
91. Smith, James D. 2005. "An Alternative to Technology Readiness Levels for Non-Developmental Item (NDI) Software." Paper presented at the System Sciences, 2005. HICSS'05. *Proceedings of the 38th Annual Hawaii International Conference on System Sciences*.
92. Stam, Antonie, and A. Silva. 1997. "Stochastic Judgments in the AHP: The Measurement of Rank Reversal Probabilities." *Decision Sciences* 28, no. 3: 655-88.
93. Stevens, Stanley Smith. 1946. "On the Theory of Scales of Measurement." Science, no.103: 677-680.

94. Tan, Weiping, Jose Ramirez-Marquez, and Brian Sauser. 2011. "A Probabilistic Approach to System Maturity Assessment." *Systems Engineering* 14, no. 3: 279-93.
95. Tan, Weiping, Brian J. Sauser, and Jose Emmanuel Ramirez-Marques. 2011. "Analyzing Component Importance in Multifunction Multicapability Systems Development Maturity Assessment." *IEEE Transactions on Engineering Management* 58, no. 2: 275-94.
96. Tan, Weiping, B. J. Sauser, J. E. Ramirez-Marquez, and R. B. Magnaye. 2013. "Multiobjective Optimization in Multifunction Multicapability System Development Planning." *Systems, Man, and Cybernetics: Systems, IEEE Transactions on* 43, no. 4: 785-800.
97. Tetlay, Abideen, and John Phillip. 2009. “Determining the Lines of System Maturity, System Readiness and Capability Readiness in the System Development Lifecycle.” *7th Annual Conference on Systems Engineering Research*.
98. Tetlay, Abideen, and John Phillip. 2010. "Clarifying the Concepts of System Maturity, System Readiness and Capability Readiness through Case Studies." In *8th Annual Conference on Systems Engineering Research*. Hoboken, NJ.
99. Townsend, James T., and F. Gregory Ashby 1984.. "Measurement Scales and Statistics: The Misconception Misconceived." *Psychological Bulletin* 96, no. 2: 394-401.
100. Valerdi, Ricardo, and Ron J Kohl. 2004. "An Approach to Technology Risk Management." In *Engineering Systems Division Symposium*, 29-31.
101. Velleman, Paul F., and Leland Wilkinson. 1993. "Nominal, Ordinal, Interval, and Ratio Typologies Are Misleading." *The American Statistician* 47, no. 1: 65-72.
102. Vittinghoff, Eric, and Charles E. McCulloch. 2007. "Relaxing the Rule of Ten Events Per Variable in Logistic and Cox Regression." *American Journal of Epidemiology* 165, no. 6: 710-18.

103. Volkert, Richard, and Carly Jackson. 2011. "Proposed Methodology for Performance Prediction and Monitoring for an Acknowledged Systems of Systems." *Naval Postgraduate School*.

104. Volkert, Richard, Carly Jackson, Jerrell Stracener, and Junfang Yu. 2012. "Development and Extension of a Deterministic System of Systems Performance Prediction Methodology for an Acknowledged System of Systems." 47. *Space and Naval Warfare Systems Center*, San Diego, CA,.

105. Volkert, Richard, Carly Jackson, and Cecil Whitfield. 2010. "Development of Modular Mission Packages Providing Focused Warfighting Capability for the Littoral Combat Ship." *Naval Engineers Journal* 122, no. 4: 75-92.

106. Volkert, Richard, Jerrell T. Stracener, and Junfang Yu. 2012. "A Framework for Performance Prediction During Development of Systems of Systems." *International Journal of System of Systems Engineering* 3, no. 1: 76-95.

107. Volkert, Richard, Jerrell T. Stracener, and Junfang Yu. 2013. "Incorporating a Measure of Uncertainty into Systems of Systems Development Performance Measures." *Systems Engineering*: 1-17.

APPENDIX A: SRL METHOD CALCULATION DEMONSTRATION

This appendix provides detailed calculation examples for the SSRL, GTSRL, and TASRL methods. Each method is discussed separately below using notional system examples. Sauser et al. (2010), Garrett et al. (2011), and McConkie (2013) have detailed calculation examples for the SSRL, GTSRL, and TASRL methods respectively.

A.1 SSRL Calculation Example

Consider the simple graph in Figure A-1 with three TRLs and three IRLs.

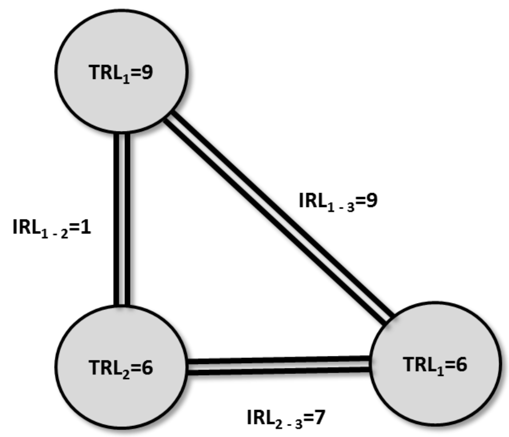


Figure A-1: SSRL calculation system example.

The system in Figure A-1 is a simple graph (i.e. one that has only undirected edges) with n=3 TRL components of values $\{TRL_1=9, TRL_2=6, TRL_3=6\}$ and m=3 IRL connections of values $\{IRL_{1\text{-}2}=1, IRL_{2\text{-}3}=7, IRL_{1\text{-}3}=9\}$. For this simple example the TRL_{SSRL} vector and IRL_{SSRL} matrix are listed in matrix notation and then populated with the component values in Figure A-1.

$$TRL_{SSRL} = [trl_j] = \begin{bmatrix} trl_1 \\ trl_2 \\ \dots \\ trl_m \end{bmatrix} \tag{A.1}$$

$$IRL_{SSRL} = \left[irl_{jk}\right] = \begin{bmatrix} irl_{11} & \dots & irl_{1n} \\ \dots & \dots & \dots \\ irl_{n1} & \dots & irl_{nn} \end{bmatrix} \tag{A.2}$$

$$TRL_{SSRL} = \begin{bmatrix} TRL_1 \\ TRL_2 \\ TRL_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 6 \end{bmatrix} \tag{A.3}$$

$$IRL_{SSRL} = \begin{bmatrix} IRL_{1,1} & IRL_{1,2} & IRL_{1,3} \\ IRL_{2,1} & IRL_{2,2} & IRL_{2,3} \\ IRL_{3,1} & IRL_{3,2} & IRL_{3,3} \end{bmatrix} = \begin{bmatrix} 9 & 1 & 9 \\ 1 & 9 & 7 \\ 9 & 7 & 9 \end{bmatrix} \tag{A.4}$$

In the above TRL_{SSRL} and IRL_{SSRL} matrices the TRL value of 9 represent a full technology maturity whereas the IRL values of 9 in the IRL diagonal components represent a perfect IRL self-integration—i.e. an ideal integration component connection with itself. In this instance a perfect integration is assumed (McConkie 2013). An IRL value of 0 represents no integration (e.g. in the above example there is no integration pathway between TRL_1 and TRL_3. The standard SSRL calculation approach of Sauser (2006, 2008) then normalizes the initial TRL and IRL matrix values by dividing by nine. This step converts the ordinal TRL and IRL 9-level scale values to a continuous scale (Sauser et al. 2010). This basic calculation produces normalized TRL_{SSRL} and IRL_{SSRL} matrices in Equations A.5 and A.6.

$$TRL_{SSRL} = \begin{bmatrix} TRL_1 \\ TRL_2 \\ TRL_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.67 \\ 0.67 \end{bmatrix} \tag{A.5}$$

$$IRL_{SSRL} = \begin{bmatrix} IRL_{1,1} & IRL_{1,2} & IRL_{1,3} \\ IRL_{2,1} & IRL_{2,2} & IRL_{2,3} \\ IRL_{3,1} & IRL_{3,2} & IRL_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 0.11 & 1 \\ 0.11 & 1 & 0.77 \\ 1 & 0.77 & 1 \end{bmatrix} \tag{A.6}$$

The Integration-Technology ($ITRL_{SSRL}$) vector represents the individual component readiness levels and is determined by matrix multiplying the IRL_{SSRL} matrix and TRL_{SSRL} column vector. The value for each of the i^{th} components of the n total components is represented by the i^{th} row of the ITRL calculation shown below. This SSRL calculation stage is depicted in Equations A.7 through A.9.

$$ITRL_{SSRL} = \left[itrl_{jk}\right] = \left[irl_{jk}\right] * \left[trl_{j}\right] \tag{A.7}$$

$$ITRL_{SSRL} = \left[itrl_{jk}\right] = \sum_{k=1}^{n} irl_{jk} \cdot trl_{k} \tag{A.8}$$

$$ITRL_{SSRL} = \begin{bmatrix} ITRL_1 \\ ITRL_2 \\ ITRL_3 \end{bmatrix} = \begin{bmatrix} 1 & 0.11 & 1 \\ 0.11 & 1 & 0.78 \\ 1 & 0.78 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0.67 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 1.74 \\ 1.30 \\ 2.19 \end{bmatrix} \tag{A.9}$$

The individual $ITRL_{SSRL}$ components above are still represented according to a [0, n] scale, where n is the system size and equals the number of TRL connections. To convert these values to a more useful scale, the $ITRL_{SSRL}$ values are converted to a [0,1] scale by multiplying each ITRL vector element by the number of edge connections adjacent to each edge plus one additional value for the assumed perfect self-vertex integration. The SSRL method includes self-integrations whereas the GTSRL method discussed below does not. Again considering Figure A-1, TRL_1, TRL_2, and TRL_3 each have two integration connections plus an assumed self-vertex connection. Placing these values into a column vector k_{SSRL} and performing a scalar multiplication of k_{SSRL} with the $ITRL_{SSRL}$ vector produces the result of Equation A.10.

$$ITRL_{SSRL} = k_{SSRL} * \begin{bmatrix} ITRL_1 \\ ITRL_2 \\ ITRL_3 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} * \begin{bmatrix} 1.74 \\ 1.30 \\ 2.19 \end{bmatrix} = \begin{bmatrix} 0.58 \\ 0.43 \\ 0.72 \end{bmatrix} \tag{A.10}$$

Equation A.10 effectively maps the $ITRL_{SSRL}$ column vector to a continuous [0,1] scale. The converted $ITRL_{SSRL}$ column vector mean is then calculated to produce the system SSRL value as shown in Equation A.11.

$$SSRL = \frac{1}{n}\sum_{i=1}^{n} ITRL = \frac{1}{3}\sum_{i=1}^{3} ITRL = \frac{1}{3}\left(0.58 + 0.43 + 0.72\right) = 0.57 \qquad (A.11)$$

The admittedly simply hypothetical example above is used to demonstrate the basic SSRL calculation method employed by Sauser (2006 and 2008) and does not represent any real system. The above SSRL calculation approach represents the original SSRL pairwise matrix-vector calculation method. The next section discusses the GTSRL method that differs according to certain graph theory matrix rules.

A.2 GTSRL Calculation Example

The GTSRL method is configured in a manner more consistent with graph theory principles than the SSRL. Although a graph is defined as the mathematical set of vertex elements and edges, conventional graph representation is usually done in pictorial form (Chartrand and Zhang 2012). Consider a modified form of Figure A-1 but updated with directional edges between $IRL_{1 \to 2}$, $IRL_{1 \to 3}$, and $IRL_{2 \leftrightarrow 3}$.

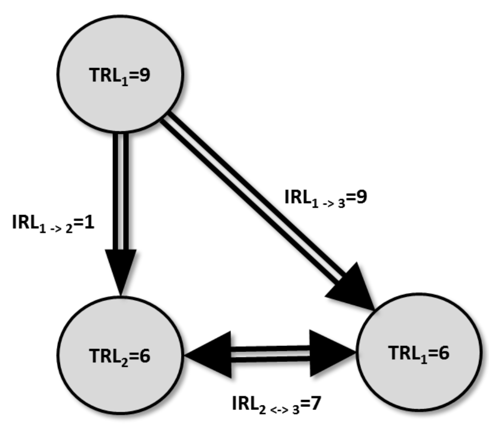


Figure A-2: GTSRL calculation system example.

The system in Figure A-2 contains three TRL components and IRL edges, but introduces directionality on the IRL paths edges. The introduction of directional edges necessarily changes the calculation technique because the SSRL used a symmetric adjacency matrix that cannot assess directional edges.. For the GTSRL we have the same TRL column vector as the SSRL (see Equation A.12), but a modified IRL matrix comprising a formal weighted adjacency matrix with the IRL values for each edge element (Equation A.13). As noted by Garrett et al. (2011) the adjacency matrix is one in which each row and column corresponds to a vertex if is an edge is present between adjacent vertices.

$$TRL_{GTSRL} = \begin{bmatrix} TRL_1 \\ TRL_2 \\ TRL_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 6 \end{bmatrix} \tag{A.12}$$

$$IRL_{GTSRL} = \begin{bmatrix} IRL_{1,1} & IRL_{1,2} & IRL_{1,3} \\ IRL_{2,1} & IRL_{2,2} & IRL_{2,3} \\ IRL_{3,1} & IRL_{3,2} & IRL_{3,3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 9 \\ 0 & 0 & 7 \\ 0 & 7 & 0 \end{bmatrix} \tag{A.13}$$

The IRL_{GTSRL} matrix presents two notable differences from the IRL_{SSRL}. The first difference is that the diagonal elements of the IRL matrix assume a value of 0. This is in contrast with the IRL_{SSRL} matrix in which diagonal values of 9 represent perfect vertex self-integration. This change of self-integration value is because graph theory adjacency matrix mathematics dictates that zero values in the matrix diagonal unless a self-integration loop is present (Chartrand and Zhang 2012; Diestel, 2010).

The second difference between the IRL_{GTSRL} and the IRL_{SSRL} is the introduction of directionality for the integration edges. The SSRL does not account for directionality of paths between technology components whereas the GTSRL permits the use of single and bi-directional integration edges between technology components. Thus, we can see that for the example of Figure A-2 the IRL_{GTSRL} matrix contains terms in a different arrangement than the IRL_{SSRL}. For example, the IRL_{GTSRL} represents the edge $IRL_{1 -> 2}$=1 and $IRL_{1->3}$=7 as uni-directional edges and $IRL_{2<->3}$=9 as a bidirectional edge.

In like manner with the Sauser approach, the TRL_{GTSRL} and IRL_{GTSRL} matrices are normalized by dividing by nine in order to convert the nine-level TRL and IRL scale values by multiplying by 1/9. This basic calculation produces normalized TRL and IRL matrices of Equations A.14 and A.15.

$$TRL_{nx1} = \begin{bmatrix} TRL_1 \\ TRL_2 \\ TRL_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.67 \\ 0.67 \end{bmatrix} \qquad (A.14)$$

$$IRL_{nxn} = \begin{bmatrix} IRL_{1,1} & IRL_{1,2} & IRL_{1,3} \\ IRL_{2,1} & IRL_{2,2} & IRL_{2,3} \\ IRL_{3,1} & IRL_{3,2} & IRL_{3,3} \end{bmatrix} = \begin{bmatrix} 0 & 0.11 & 1 \\ 0 & 0 & 0.78 \\ 0 & 0.78 & 0 \end{bmatrix} \qquad (A.15)$$

The $ITRL_{GTSRL}$ is then determined by multiplying the IRL_{GTSRL} matrix and TRL_{GTSRL} column vector using pairwise matrix multiplication. The $ITRL_{GTSRL}$ component values are represented by the i^{th} row of the $ITRL_{GTSRL}$ calculations in Equation A.16.

$$ITRL_{GTSRL} = IRL_{GTSRL} * TRL_{GTSRL} = \begin{bmatrix} 0 & 0.11 & 1 \\ 0 & 0 & 0.78 \\ 0 & 0.78 & 0 \end{bmatrix} * \begin{bmatrix} 1 \\ 0.67 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 0.74 \\ 0.52 \\ 0.52 \end{bmatrix} \qquad (A.16)$$

The individual $ITRL_{GTSRL}$ column vector components are still represented according to a [0, n] scale. The GTSRL, like the SSRL, converts these values to a [0,1] scale by multiplying each $ITRL_{GTSRL}$ vector element by a normalization factor. Unlike the SSRL method, the GTSRL approach does not consider self-integrations. Therefore, for the

GTSRL method, the normalization factor k_{GTSRL} elements count the number of adjacent edges to each of the *n* TRL components. For the system in Figure A-2, each TRL component has two integration edges. These values are placed into the k_{GTSRL} column vector that is then scalar multiplied by the $ITRL_{GTSRL}$ matrix to produce the result of Equation A.17.

$$ITRL_{GTSRL} = k_{GTSRL} * ITRL_{GTSRL} = \begin{bmatrix} \frac{0.74}{2} \\ \frac{0.52}{1} \\ \frac{0.52}{1} \end{bmatrix} = \begin{bmatrix} 0.37 \\ 0.52 \\ 0.52 \end{bmatrix} \tag{A.17}$$

The $ITRL_{GTSRL}$ mean value is then calculates as the overall GTSRL value as shown in Equation A.18.

$$SRL = \frac{1}{n}\sum_{i=1}^{n} ITRL = \frac{1}{3}\sum_{i=1}^{3} ITRL = \frac{1}{3}(0.37 + 0.52 + 0.52) = 0.30 \tag{A.18}$$

The two examples above demonstrate the basic SRL calculation methods employed by the SSRL method (Sauser et al. 2006, 2008, and 2010) and the GTSRL method of Garrett et al. (2011). Both methods employ pairwise matrix multiplication operations to calculate the ITRL vector and overall system readiness values. The primary differences between the two methods arise from the rules by which the IRL and TRL matrices are constructed and how the normalization factors are defined. The TASRL method discussed in the next section uses a similar TRL and IRL initialization as the GTSRL but employs a tropical algebra mathematical calculation approach.

A.3 TASRL Calculation Example

Tropical algebra was developed by Imre Simon and expanded by Maclagan and Sturmfels (2009) and Mikhalkin (2006). Tropical algebra is a subset of mathematical geometry in which linear objects are used to fulfill the function of classical algebraic quantities (Mikhalkin 2006). Consider the system of Figure A-2 repeated here for convenience as Figure A-3.

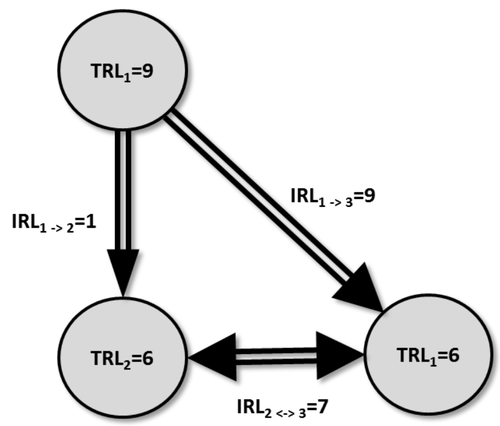


Figure A-3: TASRL calculation system example.

Figure A-3 is identical to the system employed for the GTSRL calculation example. This is a reasonable starting place because McConkie et al. (2012) and McConkie (2013) TASRL method is based upon the graph theory approach for constructing the TRL and IRL matrices. The TRL_{TASRL} and IRL_{TASRL} matrices are populated in the same manner as the GTSRL and so we have the initial matrices of Equations A.19 and A.20.

$$TRL_{TASRL} = \begin{bmatrix} TRL_1 \\ TRL_2 \\ TRL_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 6 \end{bmatrix} \tag{A.19}$$

$$IRL_{TASRL} = \begin{bmatrix} IRL_{1,1} & IRL_{1,2} & IRL_{1,3} \\ IRL_{2,1} & IRL_{2,2} & IRL_{2,3} \\ IRL_{3,1} & IRL_{3,2} & IRL_{3,3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 9 \\ 0 & 0 & 7 \\ 0 & 7 & 0 \end{bmatrix} \quad (A.20)$$

Normalizing the TRL_{TASRL} vector and IRL_{TASRL} matrix from a 9-level ordinal scale to a [0,1] scale is performed by multiplying both by 1/9 as shown in Equation A.21 and A.22.

$$TRL_{TASRL} = \begin{bmatrix} TRL_1 \\ TRL_2 \\ TRL_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.67 \\ 0.67 \end{bmatrix} \quad (A.21)$$

$$IRL_{TASRL} = \begin{bmatrix} IRL_{1,1} & IRL_{1,2} & IRL_{1,3} \\ IRL_{2,1} & IRL_{2,2} & IRL_{2,3} \\ IRL_{3,1} & IRL_{3,2} & IRL_{3,3} \end{bmatrix} = \begin{bmatrix} 0 & 0.11 & 1 \\ 0 & 0 & 0.78 \\ 0 & 0.78 & 0 \end{bmatrix} \quad (A.22)$$

At this juncture the primary difference between the GTSRL and TASRL techniques become important. Unlike the SSRL and GTSRL methods, the TASRL does not employ standard matrix multiplication calculations to calculate the ITRL vector and final readiness level values. In contrast, TASRL uses mathematical operations consistent with tropical algebra. Tropical algebra employs the operations of algebraic sum and minimum value (Michaud 2009). The algebraic sum operation, denoted by $\otimes$, is simply the sum of a given set of numbers. The minimal value operation, denoted by $\oplus$, provides minimum value of a set of numbers. Consider the following examples:

Algebraic sum: $\otimes = 2+3+5 = 10$ *(A.23)*

Minimal value: $\oplus\{2, 3, 5] = \min(2, 3, 5) = 2$ *(A.24)*

Using these mathematical operations the $ITRL_{TASRL}$ vector is calculated in the following manner as described by McConkie (2013).

$$ITRL = \begin{bmatrix} \oplus\{(IRL_{11} \otimes TRL_1), (IRL_{12} \otimes TRL_2), (IRL_{13} \otimes TRL_3)\} \\ \oplus\{(IRL_{21} \otimes TRL_1), (IRL_{22} \otimes TRL_2), (IRL_{23} \otimes TRL_3)\} \\ \oplus\{(IRL_{31} \otimes TRL_1), (IRL_{32} \otimes TRL_2), (IRL_{33} \otimes TRL_3)\} \end{bmatrix} \tag{A.25}$$

In conventional mathematical notation, Equation A.25 may be presented in a more understandable fashion of Equation A.26.

$$ITRL = \begin{bmatrix} \min\{(IRL_{11} + TRL_1), (IRL_{12} + TRL_2), (IRL_{13} + TRL_3)\} \\ \min\{(IRL_{21} + TRL_1), (IRL_{22} + TRL_2), (IRL_{23} + TRL_3)\} \\ \min\{(IRL_{31} + TRL_1), (IRL_{32} + TRL_2), (IRL_{33} + TRL_3)\} \end{bmatrix} \tag{A.26}$$

Inserting the values from the TRL_{TASRL} and IRL_{TASRL} matrices above produces the result of Equation A.27.

$$ITRL = \begin{bmatrix} \min\{(0 + 1), (0.11 + 0.67), (1 + 0.67)\} \\ \min\{(0 + 1), (0 + 0.67), (0.78 + 0.67)\} \\ \min\{(0 + 1), (0.78 + 0.67), (0 + 0.67)\} \end{bmatrix} = \begin{bmatrix} \min(1, 0.78, 1.67) \\ \min(1, 0.67, 1.45) \\ \min(1, 1.45, 0.67) \end{bmatrix} \tag{A.27}$$

The TASRL is then calculated as the minimum of the ITRL vector in order to remain consistent with the mathematical principles of tropical algebra.

$$TASRL = \min \begin{bmatrix} \min(1,0.78,1.67) \\ \min(1,0.67,1.45) \\ \min(1,1.45,0.67) \end{bmatrix} = \min \begin{bmatrix} 0.78 \\ 0.67 \\ 0.67 \end{bmatrix} = 0.67 \tag{A.28}$$

For TASRL example demonstrates a new and novel approach to calculating readiness levels of a subsystem and whole system. The TASRL, however, is yet unable to adequately deal with complex system expressed as mixed graphs or general graphs. The IMSRL model discussed in Chapter 5 solves these system representation limitations using an incidence matrix approach.

APPENDIX B: MATLAB CODE USED FOR SRL ANALYSIS

This appendix provides supplemental information regarding the computer coding and programming commands used to generate the Monte-Carlo based SRL simulation. The simulated data were generated using Matlab (Mathworks 2011) software and author-generated coding. A pseudo-code software flowchart diagram is provided in Figure C-1. The representative Matlab code pursuant to the software flowchart in Figure C-1 Is provided in individually labeled sections below.

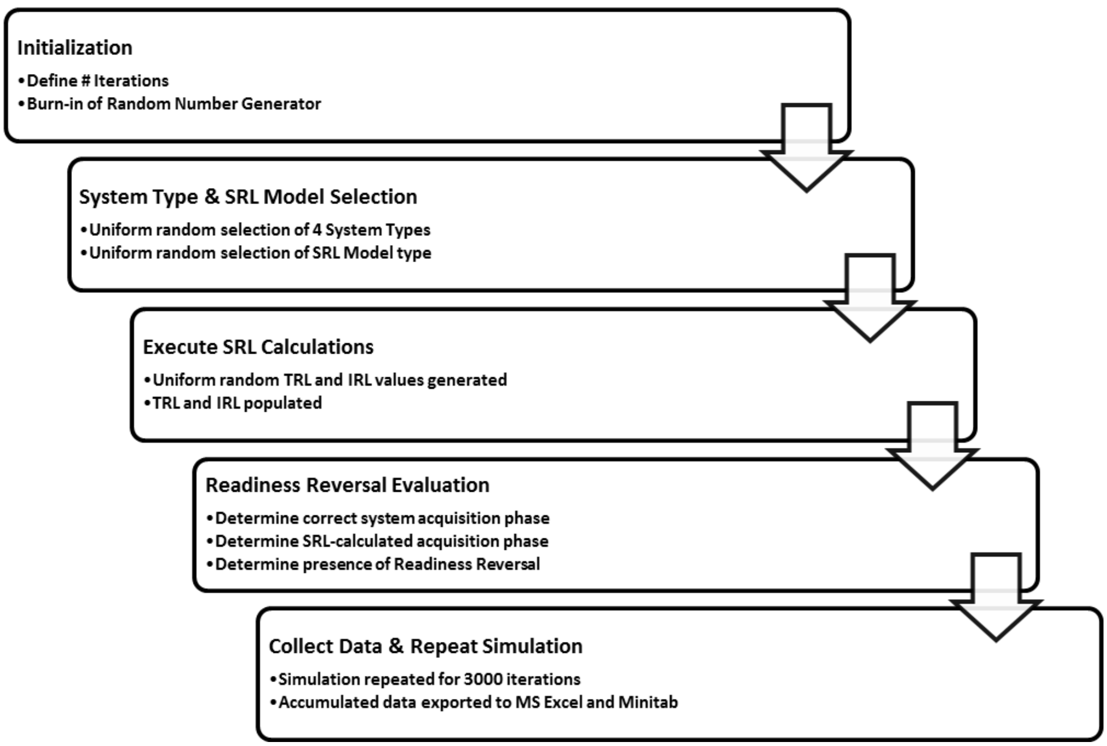


Figure B-1: Matlab code flowchart for readiness reversal Monte Carlo model.

The “Readiness Reversal Data Generation” Matlab m-file is the overarching file used to initialize global variables and call subroutines for specific calculations. All subsequent functions listed below are called from this primary m-file.

```
% READINESS REVERSAL DATA GENERATION
clear all;
clc;

NumSims=3000;

% SRL acquisition ranges defined IAW Sauser (2010) and McCOnkie (2013)
BurnIn=randi(10000);

for i=1:NumSims
    % Selects from 2-TRL, 3-TRL, or 7-TRL system
    SystemOrder=[1,2,3,7];        % selects 4 systems based on system
order
    SystemChoice=randi([1,4]);    %  randomly selects a given system
    SystemSelection=SystemOrder(1,SystemChoice);
    m=SystemSelection;        % defines # vertices

    % defines all TRL & IRL matrices for adjacency and incidence matrix
    if m==1
        n=2;                  % defines # edges in complete graph
        Cnm=n+m;              % simplified McCabe Cyclomatic complexity
        maxdeg=1;
        TRL=randi([1,9],n,1);
        values=randi([1,9],1,m);
        IRLbase=[0,values(1);values(1),0];
        Upper=triu(IRLbase);    % extracts upper triangle of matrix
        IRLssrl=Upper+Upper'+9*(eye(n));
        IRLgtsrl=Upper+Upper';
        IRLtasrl=IRLgtsrl;
        IRLimsrl=IncidenceMatrix(n,m,IRLgtsrl);
    elseif m==2
        n=3;                  % defines # edges in complete graph
        Cnm=n+m;              % McCabe Cyclomatic complexity
        maxdeg=1;
        TRL=randi([1,9],n,1);
        values=randi([1,9],1,m);
        IRLbase=[0,values(1),0;values(1),0,values(2);values(2),0,0];
        Upper=triu(IRLbase);    % extracts upper triangle of matrix
        IRLssrl=Upper+Upper'+9*(eye(n));
        IRLgtsrl=Upper+Upper';
        IRLtasrl=IRLgtsrl;
        IRLimsrl=IncidenceMatrix(n,m,IRLgtsrl);
    elseif m==3
        n=3;              % defines # edges in complete graph
        Cnm=n+m;                  % McCabe Cyclomatic complexity
        maxdeg=2;
        TRL=randi([1,9],n,1);
        values=randi([1,9],1,m);
```

```
IRLbase=[0,values(1),values(2);values(1),0,values(3);values(2),values(3
),0];
        Upper=triu(IRLbase);   % extracts upper triangle of matrix
        IRLssrl=Upper+Upper'+9*(eye(n));
        IRLgtsrl=Upper+Upper';
        IRLtasrl=IRLgtsrl;
        IRLimsrl=IncidenceMatrix(n,m,IRLgtsrl);
    elseif m==7
        n=6;                    % defines # edges in complete graph
        Cnm=n+m;              % simplified omplexity
        maxdeg=5;
        TRL=randi([1,9],n,1);   % defines TRL vector
        values=randi([1,9],1,m);    % defines set of IRL values for
matrix

IRLbase=[0,values(1),0,values(2),values(3),0;0,0,values(4),values(5),va
lues(6),values(7);0,0,0,0,0,0;0,0,0,0,0,0;0,0,0,0,0,0;0,0,0,0,0,0];
        Upper=triu(IRLbase);   % extracts upper triangle of matrix
        IRLssrl=Upper+Upper'+9*(eye(n));     % defined IRLssrl matrix
        IRLgtsrl=Upper+Upper';  % defines IRLgtsrl matrix
        IRLtasrl=IRLgtsrl;      % defines IRLtasrl matrix
        IRLimsrl=IncidenceMatrix(n,m,IRLgtsrl);    % defines IRLimsrl
matrix
    end

    % selects SRL model and calculates SRL value for each run
    Model=randi([1,4]);
    if Model==1                 % SSRL
        ModelCode=[1,0,0,0];
        SRLvalue=SSRLcalc(n,IRLssrl,TRL);
        minTRL=min(TRL);
        maxTRL=max(TRL);
        minIRL=min(min(values));
        maxIRL=max(max(values));
    elseif Model==2             % GTSRL
        ModelCode=[0,1,0,0];
        SRLvalue=GTSRLcalc(n,IRLgtsrl,TRL);
        minTRL=min(TRL);
        maxTRL=max(TRL);
        minIRL=min(min(values));
        maxIRL=max(max(values));
    elseif Model==3             % TASRL
        ModelCode=[0,0,1,0];
        SRLvalue=TASRLcalc(n,IRLtasrl,TRL);
        minTRL=min(TRL);
        maxTRL=max(TRL);
        minIRL=min(min(values));
        maxIRL=max(max(values));
    elseif Model==4             % IMSRL
        ModelCode=[0,0,0,1];
        SRLvalue=IMSRLcalc(n,m,IRLimsrl,TRL);
        minTRL=min(TRL);
        maxTRL=max(TRL);
        minIRL=min(min(values));
```

```
        maxIRL=max(max(values));
    end

    MinTRLandIRL=min([minTRL,minIRL]);
    MaxTRLandIRL=max([maxTRL,maxIRL]);
    MedTRLandIRL=median([TRL',values]);
    RangeTRLandIRL=MaxTRLandIRL-MinTRLandIRL;

[xRLphase,SRLphase,SRLreversal]=RReval(MinTRLandIRL,SRLvalue,Model);

DataList(i,:)=[n,m,Cnm,maxdeg,MinTRLandIRL,MedTRLandIRL,MaxTRLandIRL,Ra
ngeTRLandIRL,xRLphase,SRLphase,ModelCode,Model,SRLvalue,SRLreversal];

end

xlswrite('ReversalDataFinal.xls',DataList)  % exports data to Excel
```

The “IncidenceMatrix” m-file adapts a square-symmetric GTSRL adjacency matrix to an incidence matrix suitable for IMSRL calculations.

```
% INCIDENCE MATRIX IRL CALCULATION

function [IncMatrix]=IncidenceMatrix(n,m,AdjMatrix)

% n = # nodes (rows & cols) of adjacency matrix
% m = # edges in system

IncMatrix=zeros(n,m);        % initializes Incidence Matrix to (nxm)
IncColumns=0;                % sets # columns to 0

% for symmetric systems wrt the adjacency matrix diagonal
for i=1:(n-1) % evaluates ith row of adjacency matrix
    for j=(i+1):n % evaluates jth columns adjacency matrix
        EdgeValue=AdjMatrix(i,j); % extracts adjacency matrix value
        if EdgeValue ~= 0
            IncColumns=IncColumns+1; % increments columns to 1:n(n-1)/2
            IncMatrix(i,IncColumns)=EdgeValue;  % fills upper edge
            IncMatrix(j,IncColumns)=EdgeValue;  % fills lower edge
        end
    end
end


% SSRL CALCULATION
function [SSRL] = SSRLcalc(n,IRL,TRL)
    if n==6
        k=[1/4;1/6;1/2;1/3;1/3;1/2];
    else
        k=1/(n);
```

```
    end
    SSRL=mean(k.*(1/(9^2)).*(IRL*TRL));

% GTSRL CALCULATION
function [GTSRL] = GTSRLcalc(n,IRL,TRL)
    if n==6
        k=[1/3;1/5;1;1/2;1/2;1];
    else
        k=1/(n-1);
    end
    GTSRL=mean(k.*(1/(9^2)).*(IRL*TRL));


% TASRL CALCULATION
function [TASRL] = TASRLcalc(n,IRL,TRL)
    IRL=(1/9)*IRL;
    TRL=(1/9)*TRL;
    for i=1:n
        for j=1:n
            ISRLrow(i,j)=IRL(i,j)+TRL(j);
        end
        ISRLmin(i,1)=min(ISRLrow(i,:));
    end
    TASRL=min(ISRLmin);


% IMSRL CALCULATION
Function [IMSRL] = IMSRLcalc(n,m,IRL,TRL)
    % This function assumes IRL is in (n,m) format and TRL is vector
    IRLtrans=IRL';  % calculates transpose of IRL to make mxn matrix
    TRLtrans=TRL';  % calculates transpose of TRL to row vector
    for i=1:m
        IRLrow=IRLtrans(i,:); % lists out full set of IRL row values
        IRLnonzero=abs(IRLtrans(i,:)~=0); % pulls nonzero indices
        IRLrowval=IRLrow([abs(IRLnonzero)~=0]); % extracts nonzero
        TRLrowval=TRLtrans([abs(IRLnonzero)~=0]);
        IMSRLrow(i,1)=(1/9)*min([IRLrowval,TRLrowval]);
    end

    IMSRL=min(IMSRLrow);



%% READINESS REVERSAL EVALUATION CALCULATIONS
function
[TRLphase,SRLphase,RRpresent]=RReval(MinTRLandIRL,SRLvalue,ModelType)
SRLlevel=0;
TRLlevel=0;

% SRL acquisition ranges defined IAW Sauser (2010) and McCOnkie (2013)
if (ModelType==1) || (ModelType==2)
    SRLrange=[0.00,0.20,0.50,0.80,0.89,1.00]; % defines SSRL and GTSRL
    TRLrange=[4,6,7,8,9]; % defines upper TRL/IRL ranges
```

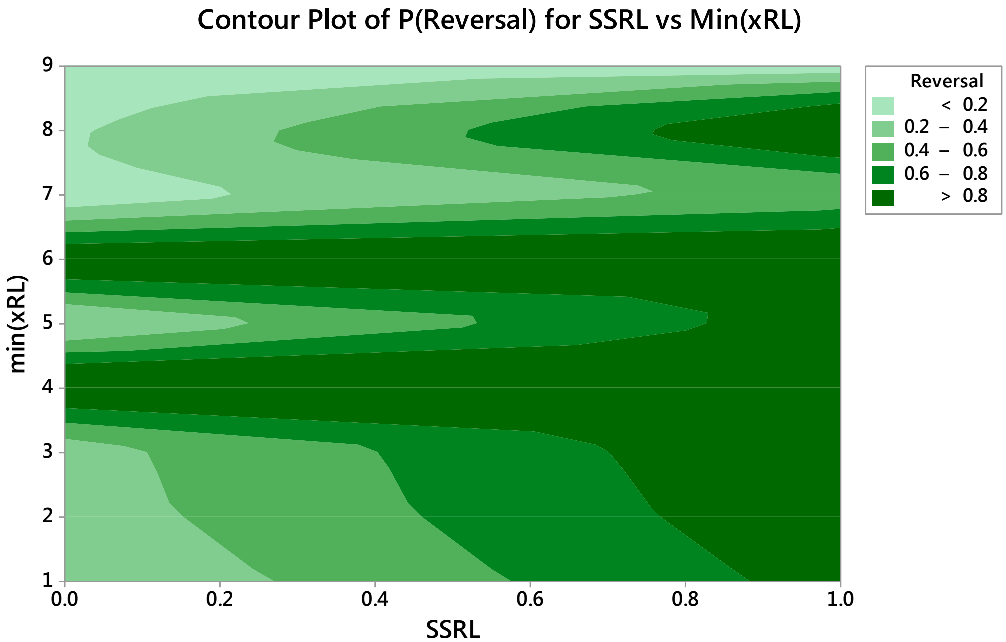
```
else
    SRLrange=[0.00,0.44,0.56,0.78,0.89,1.00]; % defines TASRL and IMSRL
    TRLrange=[4,6,7,8,9]; % defines upper TRL/IRL ranges
end


% This sectiond determines which "phase" a given SRL value falls into
% There are 9 distinct SRL value limits so there 8 interspersed phases.
   if (SRLvalue >= SRLrange(1)) && (SRLvalue < SRLrange(2))
       SRLlevel=1;
       elseif (SRLvalue >= SRLrange(2)) && (SRLvalue < SRLrange(3))
           SRLlevel=2;
           elseif (SRLvalue >= SRLrange(3)) && (SRLvalue < SRLrange(4))
               SRLlevel=3;
               elseif (SRLvalue>=SRLrange(4))&&(SRLvalue<SRLrange(5))
                   SRLlevel=4;
                   elseif
(SRLvalue>=SRLrange(5))&&(SRLvalue<=SRLrange(6))
                       SRLlevel=5;
   end
% determines phase for minimum of TRL and IRL values
% TRL(1-4)=MSA; TRL(5-6)=TD; TRL(7)=EMD; TRL(8)=Prod.&Dep.; TRL(9)=O&S
   if (MinTRLandIRL <= TRLrange(1))
       TRLlevel=1;
       elseif (MinTRLandIRL>TRLrange(1))&&(MinTRLandIRL<=TRLrange(2))
           TRLlevel=2;
           elseif
(MinTRLandIRL>TRLrange(2))&&(MinTRLandIRL<=TRLrange(3))
               TRLlevel=3;
               elseif
(MinTRLandIRL>=TRLrange(3))&&(MinTRLandIRL<=TRLrange(4))
                   TRLlevel=4;
                   else TRLlevel=5;
   end

   % determines if SRL value has reversal or not
   if (TRLlevel < SRLlevel)
       SRLphase=SRLlevel;
       TRLphase=TRLlevel;
       RRpresent = 1;
   else
       SRLphase=SRLlevel;
       TRLphase=TRLlevel;
       RRpresent = 0;
   end
```

APPENDIX C: READINESS REVERSAL CONTOUR SURFACE PLOTS

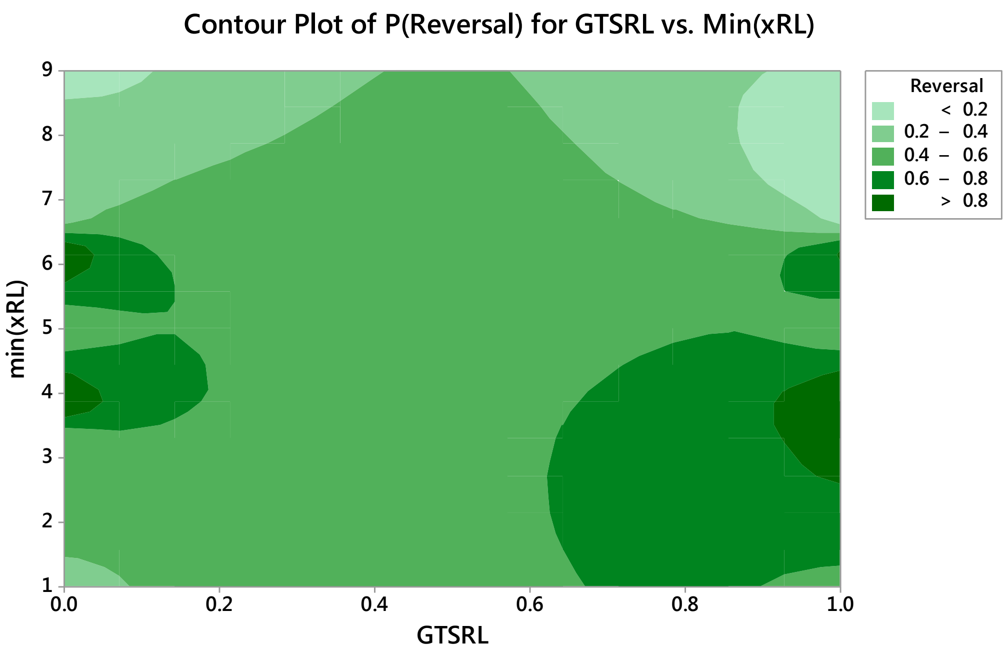
This appendix provides supplemental data plots of the SRL readiness reversal evaluation discussed in Chapter 5. The plots below are contour surface plots depicting the probability of readiness reversal for different SRL models and for different system architecture parameters. The list of contour plots is provided in Table C-1 with individual plot sets subsequently listed.

Table C-1: Readiness reversal contour plot list.

Plot Label	x-axis	y-axis	z-axis
Figure C-1(a)	SSRL	Min(xRL)	P(Readiness Reversal)
Figure C-1(b)	GTSRL	Min(xRL)	P(Readiness Reversal)
Figure C-2(a)	TASRL	Min(xRL)	P(Readiness Reversal)
Figure C-2(b)	IMSRL	Min(xRL)	P(Readiness Reversal)
Figure C-3(a)	SSRL	Med(xRL)	P(Readiness Reversal)
Figure C-3(b)	GTSRL	Med(xRL)	P(Readiness Reversal)
Figure C-4(a)	TASRL	Med(xRL)	P(Readiness Reversal)
Figure C-4(b)	IMSRL	Med(xRL)	P(Readiness Reversal)
Figure C-5(a)	SSRL	Range(xRL)	P(Readiness Reversal)
Figure C-5(b)	GTSRL	Range(xRL)	P(Readiness Reversal)
Figure C-6(a)	TASRL	Range(xRL)	P(Readiness Reversal)
Figure C-6(b)	IMSRL	Range(xRL)	P(Readiness Reversal)



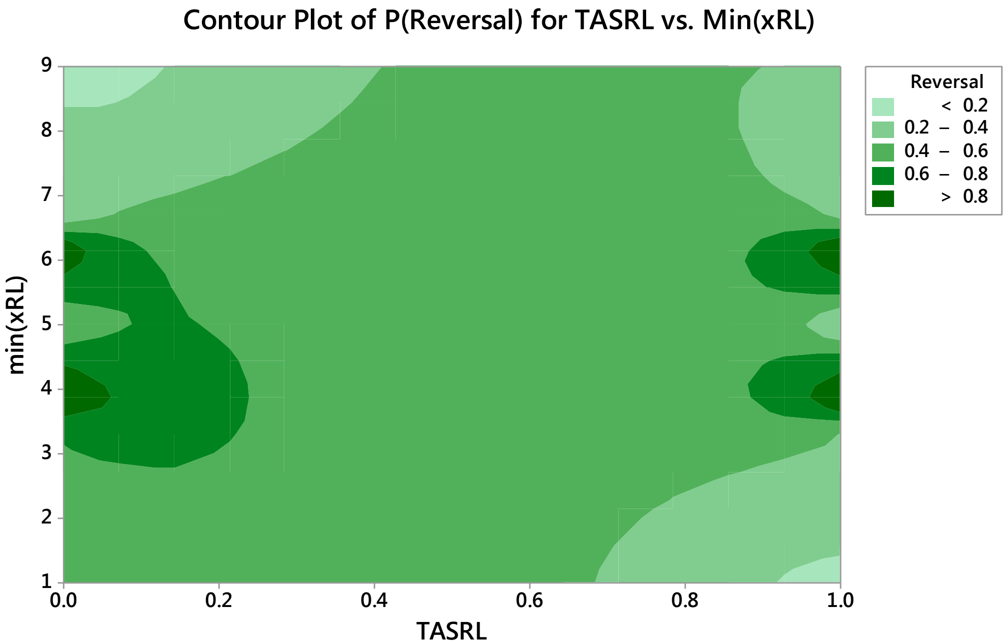
(a)



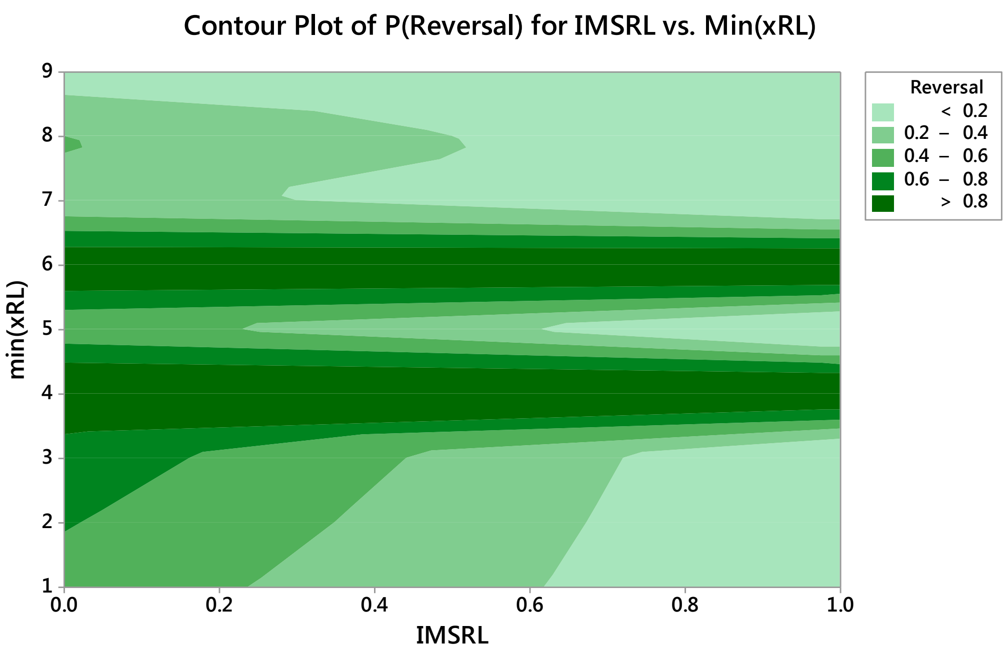
(b)

Figure C-1: Readiness reversal plots of SSRL and GTSRL vs. Min(xRL).

These plots depict the probability of readiness reversal (color scale) versus minimum TRL or IRL values (y-axis).and SRL models (x-axis): (a) SSRL and (b) GTSRL.



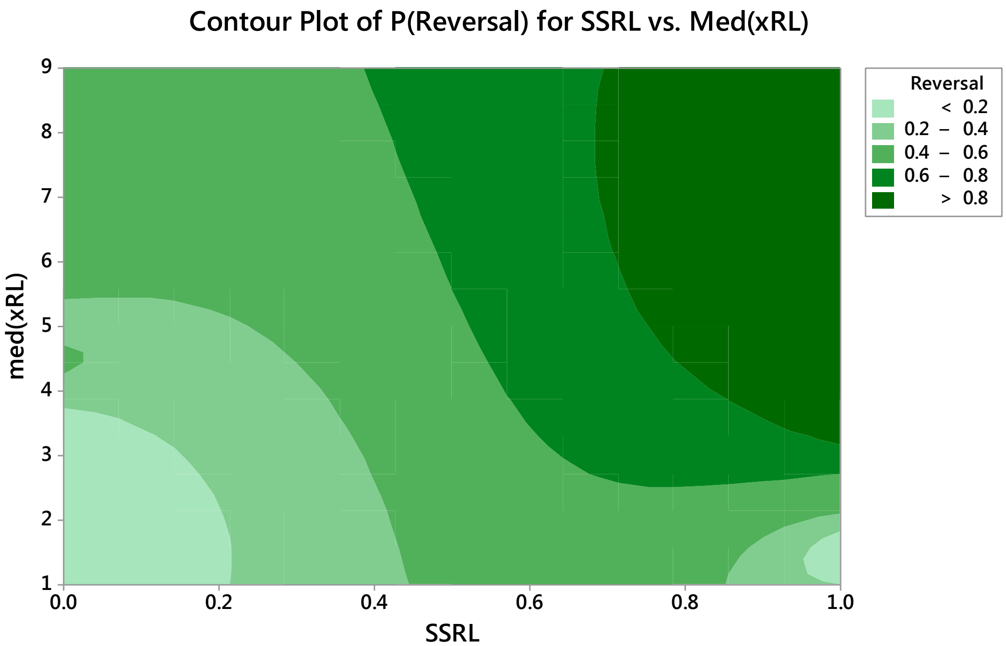
(a)



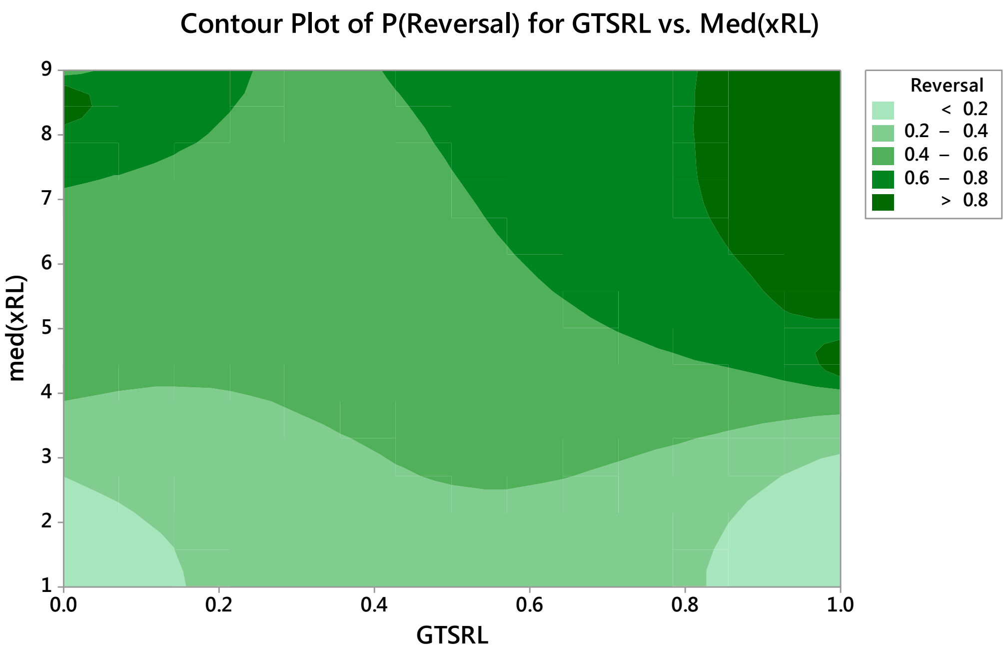
(b)

Figure C-2: Readiness reversal plots of TASRL and IMSRL vs. Min(xRL).

These plots depict the probability of readiness reversal (color scale) versus the minimum TRL or IRL value (y-axis) and SRL models (x-axis): (a) TASRL and (b) IMSRL.



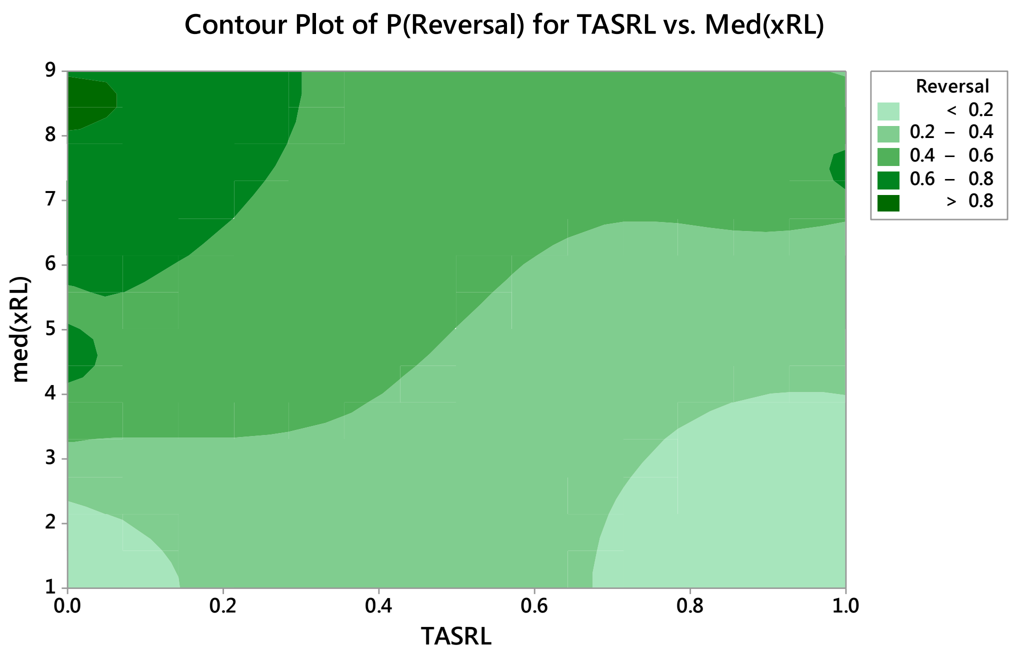
(a)



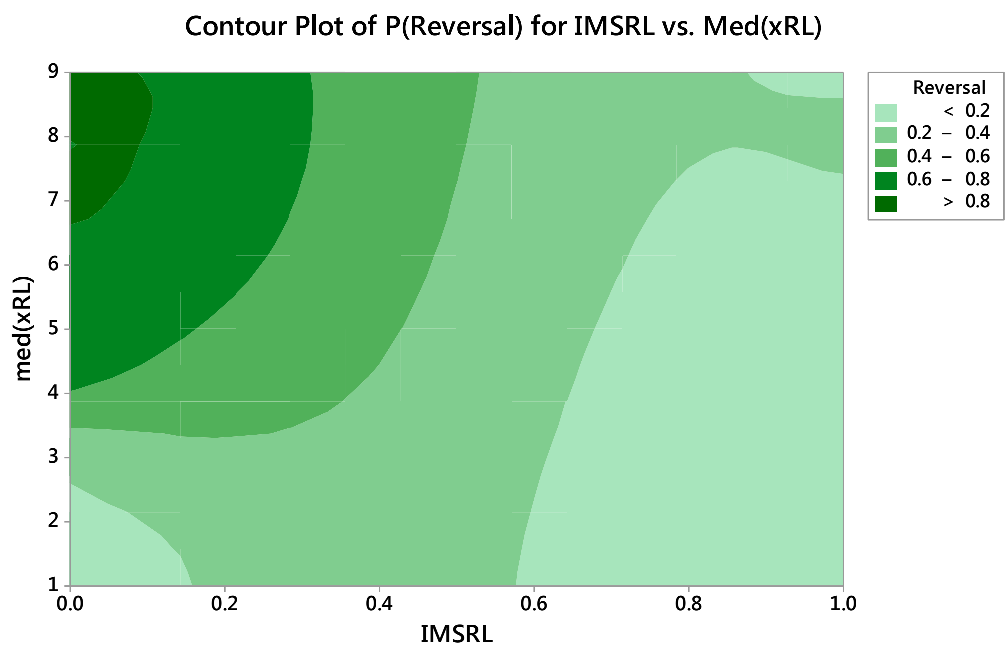
(b)

Figure C-3: Readiness reversal plots of SSRL and GTSRL vs. Med(xRL).

These plots depict the probability of readiness reversal (color scale) versus the median TRL or IRL value (y-axis) and SRL models (x-axis): (a) SSRL and (b) GTSRL



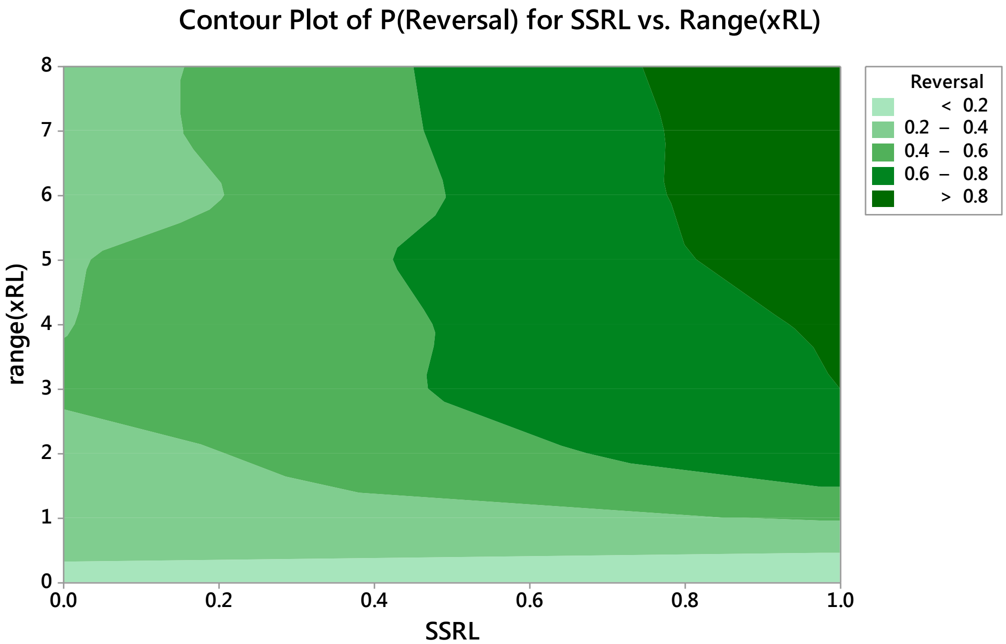
(a)



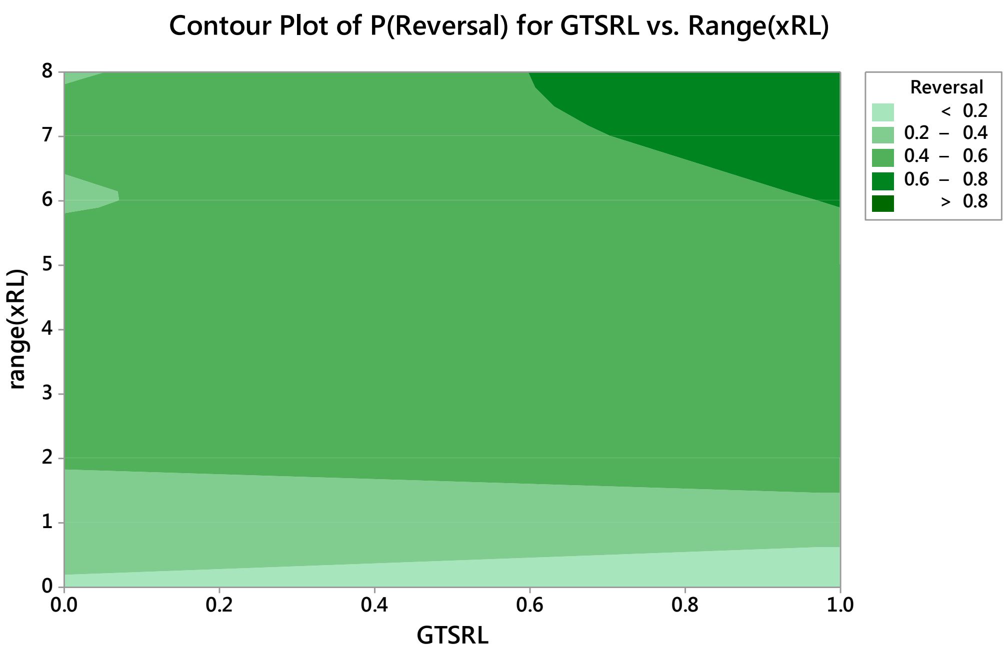
(b)

Figure C-4: Readiness reversal plots of TASRL and IMSRL vs. Med(xRL).

These plots depict the probability of readiness reversal (color scale) versus the median TRL or IRL value (y-axis) and SRL models (x-axis): (a) TASRL and (b) IMSRL.



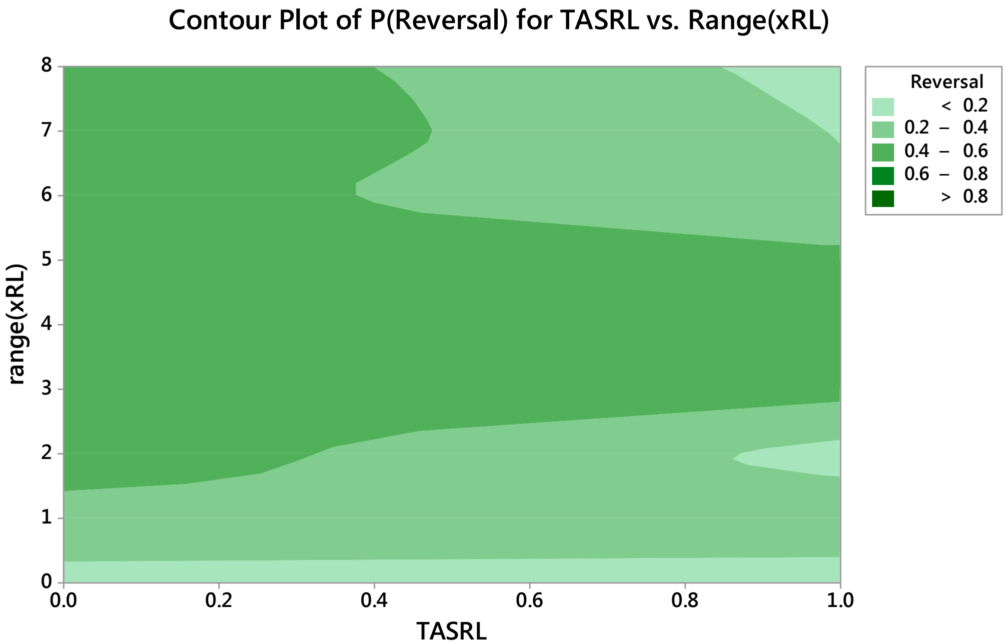
(a)



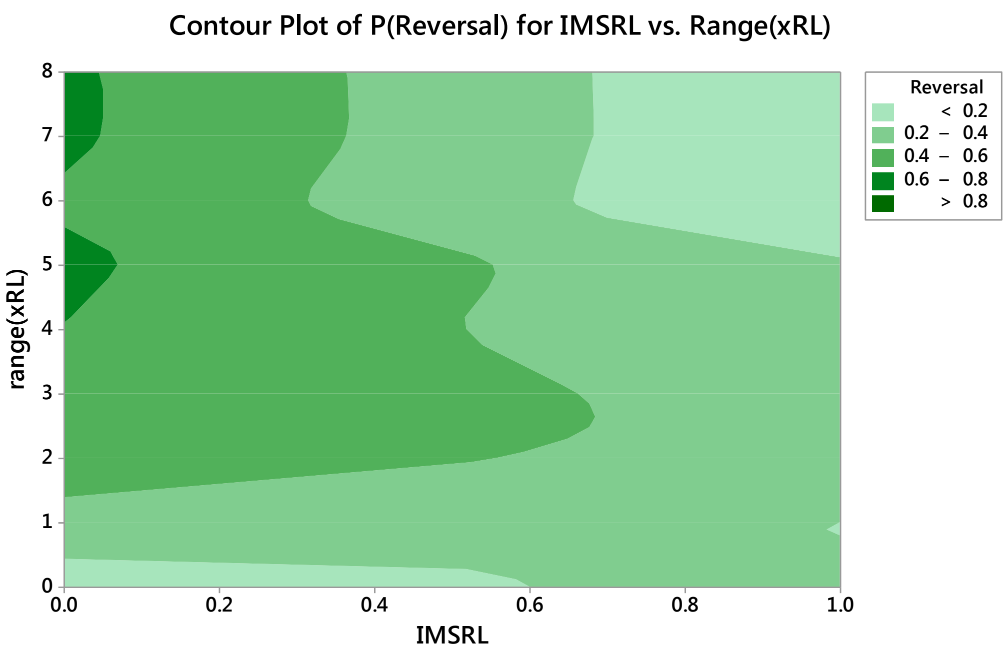
(b)

Figure C-5: Readiness reversal plots of SSRL and GTSRL vs. Range(xRL).

These plots depict the probability of readiness reversal (color scale) versus the range of TRL or IRL values (y-axis) and SRL models (x-axis): (a) SSRL and (b) GTSRL.



(a)



(b)

Figure C-6: Readiness reversal plots of TASRL and IMSRL vs. Range(xRL).

These plots depict the probability of readiness reversal (color scale) versus the range of TRL or IRL values (y-axis) and SRL models (x-axis): (a) TASRL and (b) IMSRL.