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Essays on Topics in Business Cycle Macroeconomics with Heterogeneous Agents

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Essays on Topics in Business Cycle Macroeconomics with Heterogeneous Agents

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Dedicated to my parents and grandparents

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Essays on Topics in Business Cycle Macroeconomics with Heterogeneous Agents

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This dissertation investigates several business cycle relationships when economic agents are heterogeneous. The particular focus is on the interactions between the cross-section of agents and the aggregate state of the economy.

The first chapter shows that, when occasionally binding capacity constraints limit the production of heterogeneous firms, demand shocks can endogenously generate a number of important business cycle regularities: recessions are deeper than booms are high, firm-level volatility is countercyclical, the aggregate Solow residual is procyclical and the fiscal multiplier is countercyclical. A baseline calibration of a basic New Keynesian DSGE model with capacity constraints shows that this mechanism can explain more than a quarter of the empirically observed asymmetry in output, and matches the cyclicality of firm-level profitability dispersion and of the measured Solow residual. The model implies fluctuations in the fiscal multiplier of around 0.12 between expansions and recessions.

Chapter two takes a different approach to firm level uncertainty, exploring how recessions can cause an endogenous rise in firm risk. If heterogeneous firms face real and financial frictions, then a shock to the mean of aggregate productivity endogenously leads to countercyclical profitability risk through firms' heterogeneous responses in price setting. Additionally, the mechanism endogenously generates countercyclical credit spreads and credit spread dispersion. The model explains a large share of the observed fluctuations in profitability dispersion (69%) and in credit spreads (40%) through fluctuations in aggregate TFP holding productivity risk constant. This suggests that the scope for uncertainty shocks to explain recessions may be smaller than previously thought.

The third chapter focuses on distributional effects of oil price shocks on the household side. In the model, household behavior replicates two patterns found in household-level data which show that gas consumption increases with income, but on the intensive margin gasoline consumption as a share of the household's budget decreases with income. The model includes gas consumption in household utility on top of a fixed minimum level of gas consumption. Calibrated simulations suggest that a shock to the gas price is almost twice as costly for relatively poor households than for relatively rich households.

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Chapter 1

Capacity Constraints under Demand Shocks: Business Cycle Implications (w. Chacko George)

1.1 Introduction

This paper studies how to reconcile within a simple framework four disparate business cycle facts: the asymmetry of business cycle fluctuations, the countercyclicality of productivity dispersion across firms, the acyclicality of utilization-adjusted total factor productivity, and counter-cyclical fiscal multipliers. Together, these empirical findings characterize recessions as times when output is especially low, volatility is high, and fiscal policy is particularly effective.

While previous work has considered mechanisms that can account for each fact in isolation, these potential explanations are generally at odds with other facts. For example, one can appeal to asymmetric business cycle shocks to explain the asymmetry in business cycles, but this would not, by itself, account for the observed countercyclicality in the dispersion of cross-sectional firm productivity. Rather than trying to combine all of the mechanisms that could potentially account for each fact individually into an unwieldy model, we instead show that a single mechanism —occasionally binding capacity constraints— can endogenously generate each of these business cycle facts when introduced into an otherwise standard business cycle model.

In the model, firms choose their capital capacity before the realization of idiosyncratic and aggregate demand shocks. After learning about these, they may vary their utilization of capital in a way that is increasingly costly as the utilization rate increases. When the economy experiences positive shocks to the demand for firms' products, they increase their capital utilization and output. With capital predetermined, this endogenous choice of utilization gives rise to procyclical measured total factor productivity even when business cycles are driven by shocks other than TFP. At the same time utilization-adjusted factor productivity may remain acyclical, as documented by Basu et al. (2006).

The combination of predetermined capital and convex utilization costs yields an upper bound to any individual firm's production. Large, positive aggregate shocks, then, increase the number of firms at their capacity constraint. This adds extra concavity to aggregate production as a function of demand and helps explain the three remaining business cycle facts. First, booms are "smaller" than downturns, in the sense that average deviations of output from trend are smaller in absolute value when the economy is far above trend than far below trend. In the calibrated model, capacity constraints generate around one quarter of the observed asymmetry of U.S. business cycles.

Second, capacity constraints provide a channel through which fiscal multipliers can be countercyclical. Higher government spending that increases demand for firms products will have larger effects when the economy is in a downturn than in an expansion. During downturns, few firms are capacity constrained and they can therefore readily expand production. In contrast, in booms more firms are already producing at their capacity constraint which reduces the expansionary effects of fiscal policy. While the extent of counter-cyclicality of fiscal multipliers remains a point of contention empirically, the model here suggests a difference in multipliers of about 0.12.

Third, idiosyncratic demand shocks generate a non-trivial distribution in the measured productivity of firms. The share of firms at their capacity constraint affects the variance of this distribution: Since all constrained firms look very similar in terms of their productivity, a higher share of constrained firms implies a lower variance in the distribution of productivity. Recessions, during which few firms are capacity constrained, are then periods of high cross-sectional productivity dispersion. Occasionally binding capacity constraints therefore provide a previously unexplored channel through which cross-sectional productivity dispersion can endogenously move in a countercyclical manner even in the absence of second-moment shocks. Additionally, the model predicts that this movement is concentrated in the left tail of the distribution, corresponding to empirical findings in Kehrig (2013).

Understanding the properties of recessions matters in the assessment of their welfare costs. For example, while symmetric fluctuations reduce welfare, this loss is more severe if fluctuations exhibit asymmetry and the cost of a downturn is hence concentrated in a short period of time. Increased volatility in recessions can similarly reduce the welfare of risk-averse agents, and, as recent literature has shown, can have adverse economic effects of its own. The question of how economic fluctuations originate and are transmitted also has important implications for fiscal policy because the efficacy of government spending in general depends heavily on the cause of downturns. For example, the government multiplier is generally acyclical in standard models, whereas in models of uncertainty shocks, government spending can actually be less effective in recessions than in normal times.

The main contribution of this paper is to show that capacity constraints can explain several important features of the behavior of output under few additional assumptions. Second, while the traditional Keynesian literature has long emphasized idle capacities as one likely source of high fiscal multipliers when aggregate demand is low, there has been relatively little work on integrating this mechanism into modern DSGE models. This paper provides such a model. Third, we document how much the channel of capacity constraints, in addition to being qualitatively consistent, can contribute quantitatively to the explanation of the four business cycle facts. Finally, we add some evidence to previous work on output asymmetry and find that large recessions on average deviate 30% more from trend output than large booms.

A number of papers study the effects of variable capacity utilization in general equilibrium frameworks. Work by Fagnart et al. (1999), Gilchrist and Williams (2000), Álvarez Lois (2006) and Hansen and Prescott (2005) investigates capacity constraints with heterogeneous firms. The main difference to the present paper is that they consider shocks to aggregate TFP under putty-clay technology or irreversibilities, whereas we focus on fluctuations in aggregate demand under standard Cobb-Douglas production in which capacity constraints arise endogenously rather than as an assumption on production technology. The closest models are Fagnart et al. (1999) and Álvarez Lois (2006), who explicitly model the pricing decision of monopolistically competitive firms. Fagnart et al. (1999) focus on the amplification of TFP shocks under putty-clay technology and flexible prices, whereas Álvarez Lois (2006) looks at the response of firm mark-ups when prices are set one period in advance as well as the internal propagation of the putty-clay mechanism. Gilchrist and Williams (2000) emphasize the asymmetric effects on output following large TFP shocks and the hump-shaped response that is generated through the effects of vintage capital. Hansen and Prescott (2005) generate asymmetries by including a choice along the extensive margin of operating or idling plants.

A strand of papers considers variable capacity utilization in a representative-agent framework (Greenwood et al. (1988), Cooley et al. (1995), Bils and Cho (1994), Christiano et al. (2005)). In contrast, the environment with heterogeneous firms allows us to consider occasionally binding capacity constraints, as well as price setting and demand shocks in the monopolistic competition framework. This firm heterogeneity in turn is driving several of the results in our model, as we show in section 1.5.

A recent paper that also looks at the interplay of cross-sectional and aggregate asymmetries is Ilut et al. (2014), albeit under a different mechanism. They show that under ambiguity aversion (or more generally any concave reaction of employment growth to expected profitability), news shocks can tightly link countercyclical volatility at the micro and macro level. Their explanation involving firms' decision making offers a complementary alternative to the approach in this paper focusing on firms' production technology.

The paper is structured as follows: In the next section 1.2 we review the stylized facts established by recent literature. In section 1.3 we illustrate in a stylized example how capacity constraints can generate these facts qualitatively. We embed this mechanism in a full DSGE model in section 1.4, and discuss quantitative results in section 1.5. Section 1.6 concludes.

1.2 Four business cycle regularities

In the following we review the evidence for the four business cycle facts (asymmetry in output, countercyclical profitability dispersion, strong dependence of the Solow residual's cyclicality on factor utilization, a countercyclical fiscal multiplier) that previous literature has found. Since business cycles can be "asymmetric" in many ways, we discuss the specific type of asymmetry we are interested in and then provide new evidence from US output series.

Large deviations in output from trend are likely negative The question of whether business cycles are asymmetric is fairly old. However, as noted by McKay and Reis (2008), it is also too broad to answer — there are many different ways in which business cycle asymmetry could theoretically manifest itself. As they emphasize, one should therefore be specific in exactly which way one wants to assess asymmetries. Previous literature can be loosely grouped into four ways to research this question: By looking for asymmetry in 1) output growth 2) output levels 3) employment growth 4) employment levels. It is worth recalling that asymmetry in levels and growth rates need not be associated. As discussed for example in Sichel (1993), a time series exhibits asymmetry in levels if, say, troughs are far below trend but peaks are relatively flat. Asymmetry in growth rates would be characterized by, say, sudden drops and slow recoveries. Correspondingly, these two types of asymmetry have been dubbed "deepness" and "steepness", respectively, in the literature.

Our reading of the literature is that there is no strong evidence for asymmetry in output growth rates which most papers have focused on (e.g. DeLong and Summers (1986), Bai and Ng (2005), McKay and Reis (2008)). As documented by Sichel (1993), there is some evidence for skewness in output levels. Employment tends to behave more skewed than output over the cycle: Prior work has found asymmetry in both employment growth and in employment levels (e.g. Ilut et al. (2014), McKay and Reis (2008)).

The focus of this paper is on the claim that large deviations of output from trend are more likely to be negative than positive. This means we are interested in the behavior of output *levels*, for which there is some evidence of asymmetry (Sichel (1993)).

In Table 1.1 we report a number of additional observations about the relative magnitude of "strong" booms and recessions. Specifically, we use a detrended output series to construct three measures of differences in large output deviations. For the first measure, we pick an integer N and compare the $N/2$ largest (i.e. positive) deviations with the $N/2$ smallest (i.e. negative) deviations by comparing their means. Here, if business cycles are asymmetric in levels, we would expect the mean deviation in strong recessions to be larger than the mean deviation in strong expansions. Second, in the next column we count how many of the N periods with the largest absolute deviations from trend were positive versus negative. If output is asymmetric as defined above, we would expect the number of periods with negative output deviations to be larger. As a third measure we report the overall skewness of the series (using all periods), defined as the sample estimate of $E\left[(x-\mu)^3/\sigma^3\right]$. This is a less direct measure of only large output deviations, but all else equal we would expect the coefficient of skewness to be negative.

We construct these measures for a range of specifications in which we vary the time-series representing "output", the length of the series, the trend filter, as well as the number N of extreme periods considered. The baseline specification uses HP-filtered postwar data. HP filtering often constitutes the weakest case in terms of differences between expansions and recessions since at the edges of the sample this detrending method tends to attribute parts of the cyclical movement into the trend. For almost all specifications in Table 1.1 we see that large deviations from trend are more likely to be negative.[1] On average across all specifications, recessions appear around 30% deeper than

[1] In fact the only specification in which negative output deviations are not larger than positive deviations is for annual GDP when we start the series in 1929 and use an HP filter which, at the beginning of the sample, picks up the Great Depression as part of the trend.

booms are high.

In section 1.5 we calibrate our model to an HP-1600-filtered quarterly US GDP series, corresponding to the quarterly baseline specification in Table 1.1. The model will yield trend deviations of 3.24% in an expansion and −3.45% in a recession and thus covers a little more than a quarter of the observed asymmetry under the baseline specification.

Cross-sectional measures of firm productivity are countercyclical The second fact is connected to a range of findings in the literature that associate recessions with increased cross-sectional dispersion among firms along several dimensions. Eisfeldt and Rampini (2006) show that capital productivity is more dispersed in recessions. Bloom (2009) and Bloom et al. (2012) include empirical evidence associating times of low aggregate production to higher dispersion in sales growth, innovations to plant profitability, and sectoral output. Directly related to levels of firm productivity, Kehrig (2013) finds that the distribution of plant revenue productivity becomes wider in recessions; Bachmann and Bayer (2013b) reach a similar result for innovations to the Solow residual in a dataset of German firms. Kehrig (2013) notes that it is mainly the bottom tail of the distribution that moves over the cycle. This variation in the cross-sectional skewness of profitability will be replicated in our model where the firms at the top of the distribution, which are near their capacity, all look very similar in terms of their profitability.

Broadly, there have been two, not mutually exclusive, approaches to

Table 1.1: Strong recessions larger than strong expansions

Specification	Mean pos vs neg	# pos vs neg	Skewness
Quarterly GDP			
Baseline	2.73% vs −3.43%	16 vs 24	−0.46
$N = 20$	3.12% vs −4.33%	6 vs 14	−0.46
$N = 80$	2.28% vs −2.87%	40 vs 40	−0.46
Until 2007	2.71% vs −3.36%	18 vs 22	−0.46
Linear filter	7.99% vs −12.70%	6 vs 34	−0.81
Rotemberg filter	4.19% vs −5.68%	6 vs 34	−0.33
Rot. filter, $N = 80$	3.74% vs −5.14%	29 vs 51	−0.33
Annual GDP			
Baseline	3.20% vs −4.40%	3 vs 7	−0.35
$N = 6$	3.37% vs −4.83%	0 vs 6	−0.35
$N = 20$	2.99% vs −3.55%	13 vs 7	−0.35
Until 2007	3.20% vs −4.41%	4 vs 6	−0.35
From 1929	16.69% vs −11.61%	6 vs 4	+1.00
Linear filter	7.29% vs −12.51%	2 vs 8	−0.88
Linear filter from 1929	20.50% vs −31.08%	3 vs 7	−0.91
Rotemberg filter	6.23% vs −13.50%	1 vs 9	−0.87
Rot. filter from 1929	16.15% vs −36.95%	1 vs 9	−1.22
Monthly industrial production			
Baseline	4.52% vs −5.90%	50 vs 70	−0.65
$N = 40$	5.48% vs −7.57%	7 vs 33	−0.65
$N = 240$	3.71% vs −4.45%	124 vs 116	−0.65
Until 2007	4.39% vs −5.58%	56 vs 64	−0.65
From 1919	11.35% vs −13.59%	54 vs 66	−0.55
Linear filter	17.03% vs −22.69%	33 vs 87	−0.52
Rotemberg filter	7.47% vs −11.23%	46 vs 74	−0.62

Notes: "Mean pos vs neg": Mean of the $N/2$ largest periods vs mean of the $N/2$ smallest periods. "# pos vs neg": Out of the N periods with largest absolute value, how many were positive and how many were negative. "Skewness": Coefficient of skewness defined as $E\left[(x-\mu)^3/\sigma^3\right]$.
For all three series in the baseline, N corresponds to a little less than 1/6 of observations, series were HP filtered and starting date is January 1949. "Quarterly GDP": $N = 40$, end date 2014:4, HP(1600)-filtered. "Annual GDP": $N = 10$, end date 2013, HP(100)-filtered. "Monthly industrial production": $N = 120$, end date 2014/02, HP(10,000)-filtered. Alternative specifications differ from respective baseline only along listed dimensions.

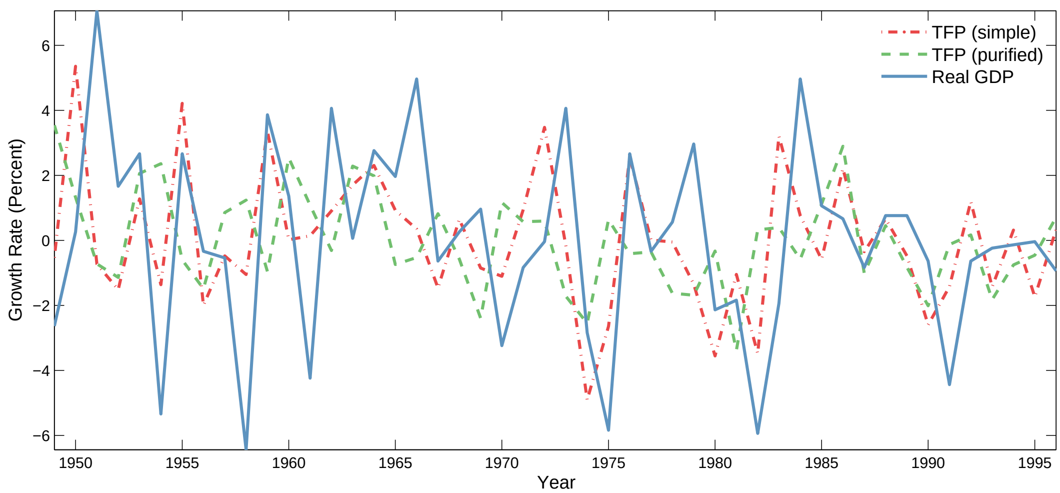
explain the negative correlation of profitability risk with output. One fruitful strand of literature starting with Bloom (2009) investigates the causal effect of exogenous increases in aggregate, cross-sectional, or policy uncertainty on economic conditions. A different set of papers has considered the opposite direction of causality, studying under which conditions a bad aggregate state can cause firm-level dispersion to increase endogenously; examples include Bachmann and Sims (2012), Decker et al. (2014), and Kuhn (2014).

The simple Solow residual is procyclical, but much less so if corrected for factor utilization For this stylized fact we draw on Basu et al. (2006) who discuss ways to improve the measurement of aggregate productivity. In particular, they construct a measure for aggregate technology that accounts for potentially confounding influences of returns to scale, imperfect competition, aggregation across sectors and (especially relevant here), utilization rates of factor inputs. Their uncorrected productivity measure, the Solow residual, is strongly procyclical: Correlation between output growth and simple TFP is 0.74. The corrected measure does not exhibit this strong association with aggregate production, as the correlation of purified TFP with (contemporaneous) output growth is 0.02. Figure 1.1 visualizes Basu et al. (2006)'s results.

Since the mechanism considered in this paper hinges strongly on the effect of adjustment in factor input utilization, we recalculate the above correlation coefficients using data provided by John Fernald[2] (see Fernald (2012))

[2]Data available at www.frbsf.org/economic-research/economists/jfernald/quarterly_tfp.xls

Figure 1.1: GDP and TFP measures from Basu et al. (2006)



Notes: Annual series for growth rates of GDP (blue solid line), simple TFP as measured by the Solow residual (red dash-dotted line), and purified TFP as constructed by Basu et al. (2006) (green dashed line). Data from Basu et al. (2006). Correlation between output growth and simple TFP growth is 0.74, correlation between output growth and purified TFP growth is 0.02.

which corrects *only* for intensity of capital and labor utilization. This allows us to check if utilization is indeed responsible for the difference in cyclicality between the simple and the purified productivity measure (or if instead the difference stems mainly from the other 'purifying' steps taken by Basu et al. (2006)). Additionally, this dataset spans 15 more years at the end of the sample and is at a quarterly frequency. Again, simple TFP is strongly procyclical with a correlation of 0.83 whereas utilization-corrected TFP has a coefficient of -0.03.

Our takeaway from this finding is that not correcting for factor input utilization strongly increases the relationship between measured aggregate productivity and output. While we do not want to weigh in on the question of which type of shocks drive business cycles, we focus on demand shocks in order to take the extreme stance of constant physical productivity. This allows us to assess how much cyclicality in *measured* TFP can be generated even when the model's correlation of output with *physical* TFP is 0.

As suggested by Wen (2004) and Basu et al. (2006), demand shocks under variable capacity utilization are a possible explanation of this fact. Alternatively, Bai et al. (2012) provide an example of a search model in which demand shocks can show up as productivity shocks when search effort is a variable margin.

The government spending multiplier is countercyclical The cause of asymmetries in the business cycle in our model is directly relevant for the effectiveness of policy. Our contribution about capacity constraints and business cycle asymmetries thus complements the literature on cyclical fiscal multipliers. Empirically estimating the level and cyclicality of the government multiplier is difficult because of severe endogeneity issues. Nevertheless, recent estimates have found significant cyclicality in fiscal multipliers, although the exact size of fluctuations is not identified very precisely. On one end of the spectrum, Auerbach and Gorodnichenko (2012) estimate the fiscal multiplier in a regime-switching model and find large swings over the cycle ranging from around 0 during a typical boom to around 1.5 during a typical recession, albeit with large confidence intervals. Other papers identifying the multiplier in structural VARs are Mittnik and Semmler (2012) and Bachmann and Sims (2012) who also find significant cyclicality. Auerbach and Gorodnichenko (2011), Ilzetzki et al. (2013) and Corsetti et al. (2012) all find evidence for state-dependence of the fiscal multiplier in cross-country comparisons. Nakamura and Steinsson (2014) use regional variation in the US to identify a positive relationship between the local spending multiplier and the unemployment rate. Ramey and Zubairy (2014) find that the estimated magnitude of multiplier fluctuations over the cycle is sensitive to the exact specification of the employed empirical model.

Not too much is known about the particular transmission channel through which aggregate conditions affect the multiplier. As Sims and Wolff (2014) point out, several papers model the difference between government spending when interest rates are at the zero lower bound and spending during normal times. Historically however, episodes at the zero lower bound have been relatively rare; and the empirical estimates go beyond these times indicating that the fiscal multiplier also fluctuates with the business cycle when interest rates are positive. Sims and Wolff (2014) explicitly consider multiplier fluctuations over the business cycle in a medium-scale RBC model. Their mechanism is based on households' higher willingness to supply additional labor in recessions. The model by Michaillat (2014) generates a labor multiplier, in which a search friction causes overall employment to respond stronger to government hiring in recessions than in booms. Here, we focus on the effect of underutilized capacity which complements mechanisms in these papers. Our calibrated model implies average fluctuations of the fiscal multiplier of around 0.12, with the fiscal multiplier increasing with the size of recessions.

1.3 Numerical Illustration

We now illustrate the aggregate effects of capacity constraints in a framework of heterogeneous firms by looking at a stylized example. Firms choose their capacity before their random demand is realized. A given capacity is associated with an upper bound to production, so that if a firm's demand is greater than this bound, that firm will be constrained and produce just at capacity.

Formally, there is a continuum of ex-ante identical firms indexed by $i \in [0,1]$. Each firm can rent capital (or "capacity") k_i at a real rental price of R at the beginning of the period. A firm's production y_i is a function of *utilized* capital $\tilde{k}_i$, which for simplicity is specified as linear. Capital utilization is free here, however it is subject to the constraint that utilized capital is less than capacity $y_i = \tilde{k}_i$ s.t. $\tilde{k}_i \leq k_i$. Finally, a firm faces random demand b_i which is distributed according to a cumulative distribution function $F(b)$.

A firm's sales after realization of b_i will then be $y_i = \min\{b_i, k_i\}$. The firm uses this fact when deciding on the amount of capacity to rent in order to maximize expected profits. The problem can be written as

$$\max_k -Rk + \int_0^k b \, \mathrm{d}f(b) + [1 - F(k)] \, k.$$

The resulting choice for k_i (if interior) requires $1 - F(k_i) = R$, such that for any firm there is a chance of $1 - R$ that the capacity constraint binds. Denote the cutoff value for b_i at which the firm just produces at capacity as $\bar{b}_i = k_i$.

Since all firms face the same problem, they choose the same $k_i = k$ and of course have the same cutoff $\bar{b} = k$. The demand shocks b_i then induce a distribution over y_i with a point mass $1 - F(\bar{b})$ at mass point $\bar{b}$.

We can now look at what happens in response to aggregate fluctuations modeled as unexpected shifts in the mean of the distribution $F(b)$. To see the effects in this example, consider the case of a uniform$(0, 1)$ distribution over b, such that the optimally chosen capacity is $k = 1 - R$.

Output fluctuations and fiscal multiplier: Aggregate output in this case is $Y = \int_0^{1-R} b \, \mathrm{d}b + R(1 - R) = \frac{1}{2}(1 - R^2)$. Now there is an unexpected fluctuation in the mean by ϵ, and b is now distributed uniform $(\epsilon, 1 + \epsilon)$. But then aggregate production is $Y = \frac{1}{2}(1 - R^2) + \epsilon(1 - R) - \frac{1}{2}\epsilon^2$. This exemplifies the second-order nature of output fluctuations: for small values of ϵ, positive and negative output changes are about the same size, while a large positive ϵ has a smaller output effects than a large negative one due to the increasing importance of the quadratic term.

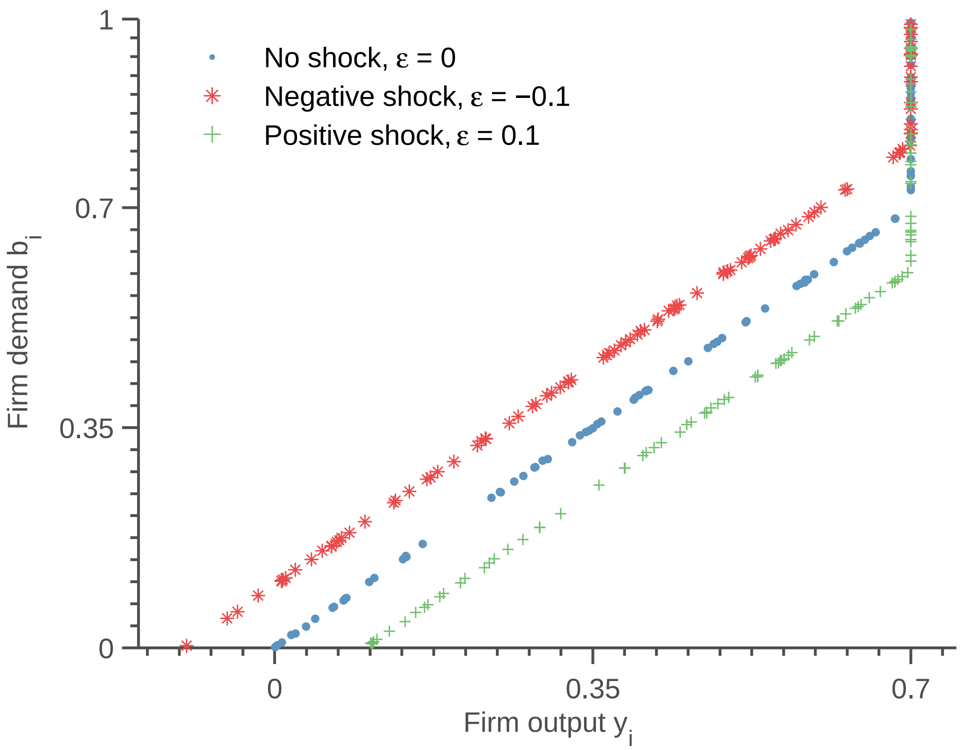
Similarly we can look at the effect of a *marginal* increase in ϵ. This could represent government policy that unexpectedly increases demand by just a little and so is a measure of the (marginal) fiscal multiplier. With the second derivative $\mathrm{d}^2Y/\mathrm{d}\epsilon^2 = -\epsilon$ an additional small increase in aggregate demand affects output less, the higher aggregate demand already is.

The government multiplier and the asymmetry in output are therefore closely related. They are not quite measuring the same thing however. The difference between a large boom and recession is given by the average effect of an increase in demand (that is, the difference in output *between* aggregate states), while the multiplier is determined by the marginal effect (that is, the effect of a small demand shock on output *at* different aggregate states).

Figure 1.2 displays the mapping from demand shocks b_i into output y_i for an interest rate of 0.3 such that the implied capacity constraint is at 0.7. The three sets of points represent the case without aggregate shock ($\epsilon = 0$) as well as aggregate shocks of $\epsilon = \pm 0.1$.

Profitability dispersion: The example illustrates that while aggregates are asymmetric, the cycle affects differences between firms as well. An individual firm's profitability can be measured as $y_i/k_i = y_i/(1 - R)$. Since the factor input cost R is the same for all firms this means that the relative cross-sectional

Figure 1.2: Distribution of y_i in numerical illustration



Notes: The figure plots simulated output levels y_i (X-axis) for a sample of 100 firms depending on their respective realized demand b_i (Y-axis). Blue $\bullet$: no aggregate shock, firm output uniformly distributed between 0 and 0.7, and a mass point at 0.7. Green $+$: For a positive demand shock $\epsilon = 0.1$, additional firms get pushed into their capacity constraint. Output expands less than proportionally, dispersion in output (and profitability) decreases, aggregate capacity utilization and Solow residual increase. Red $*$: The opposite is true for a negative demand shock $\epsilon = -0.1$. The left tail of the distribution becomes wider and the mass of firms at capacity decreases.

variance in profitability at any point is equal to the relative variance in output. While the analytic expression for $\mathrm{Var}(y_i)$ as a function of ϵ is somewhat involved, the intuition is straightforward: the greater the mass of firms at the capacity constraint, the smaller the variance in profitability of the overall distribution. In the extreme case of a very large negative shock (corresponding to $\epsilon < -0.3$ in the example), no firm would be at the constraint and thus dispersion would be greatest. This mechanism is consistent with the data: Kehrig (2013) finds that it is predominantly movement in the left tail that drives changes in firms' profitability distribution.

Measured aggregate productivity: Simple measured aggregate TFP is $Y/K = \frac{1}{2}(1+R)+\epsilon-\frac{1}{2}\epsilon^2/(1-R)$, hence it increases with ϵ due to more intensive use of installed capacity. Measured aggregate TFP is hence endogenously procyclical while TFP corrected for utilization is trivially given by $Y/\tilde{K} = 1$ by definition of the production function.

This example illustrates how capacity constraints can qualitatively generate deep recessions along with meek booms, countercyclical fiscal multipliers and a more dispersed productivity distribution in recessions. All of these features come from a simple shock structure that is perfectly symmetric over time and across firms.

1.4 Model

We now embed capacity constraints in a New-Keynesian model of aggregate demand shocks to look at the effects in general equilibrium. While the intuition from the previous section about their qualitative implications fully carries through, only a general-equilibrium model will be able to inform us about the size of asymmetries generated by capacity constraints quantitatively.

The main difference relative to the example in the previous section is that the capacity constraint now arises endogenously due to convex capital utilization costs. Such convex utilization costs can easily be justified by empirically relevant features such as overtime pay or increased depreciation. In particular, a firm's maximal production is given by its willingness to supply goods rather than an assumed technological constraint. For this, in the model firms not only choose their capacity, but also their goods price at the beginning of the period before any shocks are realized. Labor constitutes a second flexible factor of production in addition to utilized capital, and an individual firm's demand now comes from a standard final goods aggregator. Finally, there is a central bank setting nominal interest rates.

There are several reasons why we model firms as setting their price in advance. First, it keeps the model tractable since all firms face the same environment at the time of their decision and hence choose the same price. Second, it will allow us to endogenize capacity constraints as the quantity firms are willing to supply at the set prices. Third, in this context it provides a convenient way of introducing price rigidities which allow preference shocks to affect output through changes in relative prices, as is usual in New Keynesian models.[3]

[3]Kuhn (2014) shows that in general it is important to model firms' pricing behavior

In order for firm supply to constitute an upper bound to production we will specify that, when supply and demand do not coincide at the set price, quantity traded is given by the minimum of supply and demand, and hence determined by the 'short' market side. This rule differs in particular from an alternative in which the price setter is required to satisfy the other market side's demand or supply at the given price. Fagnart et al. (1999) use a similar setup and discuss the implications for planned and traded quantities in more detail.

1.4.1 Timing

The timing within a period is as follows:

1. Households enter a period t with an amount of aggregate capital K_t. At the beginning of the period, before any shocks are realized, a capacity rental market opens where households supply K_t and firms rent their capacity for this period, k_{it}. Simultaneously, firms choose their price p_{it}. (Later in equilibrium, because all firms are the same at the beginning of the period, $k_{it} = K_t$ and $p_{it} = p_t$.)

2. All idiosyncratic and aggregate shocks are realized.

3. The remaining markets open: Firms make their decisions about labor demand and capacity utilization; households decide on their labor supply

explicitly when considering cross-sectional profitability measures: Differences in pricing can prevent firms' profitability from tracking their physical productivity, as highlighted by Foster et al. (2008).

and desired savings in capital and bonds. Households also receive firm profits and pay taxes. The monetary authority sets the nominal interest rate as a function of inflation. The period ends.

1.4.2 Final goods aggregator

The final good Y is assembled from a continuum of varieties indexed by $i \in [0,1]$ according to a standard CES function with parameter σ measuring the elasticity of substitution between intermediate goods

$$Y = \left(\int b_i^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}} .$$

The weights $\{b_i\}$ are realizations of *iid* random variables with mean 1.

The perfectly competitive final goods aggregator takes intermediate goods prices as given. It has a nominal budget of $I \geq \int p_i y_i \, \mathrm{d}i$, where p_i is an intermediate variety's nominal price. The aggregator also takes into account the capacity constraint that limits the supply of some varieties. Denoting this upper limit[4] by $\bar{y}$, it therefore has to consider a continuum of inequality constraints $y_i \leq \bar{y} \; \forall i$. The problem can then be expressed as

$$\max_{\{y_i\},\lambda,\{\mu_i\}} \left(\int b_i^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}} + \lambda \left(I - \int p_i y_i \, \mathrm{d}i \right) + \int \mu_i \left(\bar{y} - y_i \right) \mathrm{d}i.$$

After taking first-order conditions (see appendix 1.1), one has

$$y_i^d = b_i \frac{I_U P_U^{\sigma-1}}{p_i^{\sigma}}$$

[4]In equilibrium the upper bound $\bar{y}$ is going to be equal to the intermediates' maximum supply dictated by costly capacity utilization y^s and indeed the same for all firms. One could solve the aggregator's problem more generally using a variety-specific $\bar{y}_i$ at the cost of more notation, but considering a $\bar{y}$ constant across varieties is enough here.

with $I_U \equiv \int_{y_i<\bar{y}} p_i y_i \, \mathrm{d}i$ the budget spent on unconstrained varieties and $P_U^{1-\sigma} \equiv \int_{y_i<\bar{y}} p_i^{1-\sigma} \, \mathrm{d}i$ a price index over unconstrained varieties.[5]

1.4.3 Firms

As in the simple example, we are going to solve the firm's problem backwards: We first determine a firm's optimal utilization and labor input given its realization of b_i and chosen capacity and price, and then the optimal k and p choices that maximize expected profits.

Technology The intermediate goods firms' production function is $y = \tilde{k}^{\alpha} l^{1-\alpha}$, where l is the hired labor input.[6] There is a quadratic real cost of utilizing capital which depends on the utilization rate $\tilde{k}/k$ and total capacity k given by

$$\mathrm{cu}\left(\frac{\tilde{k}}{k}, k\right) = \frac{\chi}{2}\left(\frac{\tilde{k}}{k}\right)^2 k.$$

This formulation ensures that the utilization costs scale linearly with k and hence the optimal utilization rate is going to be independent of firm size. There is also a quadratic real cost of adjusting the nominal price p which depends on the relative change p/p_{-1} through

$$\mathcal{C}\left(\frac{p}{p_{-1}}\right) = \frac{\xi}{2}\left(\frac{p}{p_{t-1}} - 1\right)^2.$$

[5]As noted by Fagnart et al. (1999) the demand function for the constrained varieties is undefined, and y^d denotes demand for the unconstrained varieties.

[6]In this section the firm index i is suppressed to save notation. It will reappear in the section on aggregation below.

We employ this cost because it is the simplest possible way of introducing nominal rigidities — its tractability in the context of this model stems from the fact that all firms choose the same price in equilibrium. Additionally, the price adjustment cost adds an intertemporal dimension to the firm's problem and thus generates some internal propagation of shocks (if $\xi = 0$ the firm's problem is reduced to an infinite sequence of one-shot problems).

Cost function The cost function describes the cheapest way for a firm to produce a fixed output level y given the marginal cost of the input factors which are in turn determined by the level of capacity k and the real wage w. It is given by

$$C(y) = \min_{\tilde{k},l} wl + \frac{\chi}{2}\left(\frac{\tilde{k}}{k}\right)^2 k \qquad \text{s.t. } \tilde{k}^{\alpha} l^{1-\alpha} \geq y.$$

The first-order conditions give optimal input factor quantities as

$$\tilde{k} = \left(\frac{\alpha}{1-\alpha}\frac{w}{\chi}k\right)^{\frac{1-\alpha}{\alpha+2(1-\alpha)}} y^{\frac{1}{\alpha+2(1-\alpha)}} \tag{1.1}$$

$$l = \left(\frac{1-\alpha}{\alpha}\frac{\chi}{w}k^{-1}\right)^{\frac{\alpha}{\alpha+2(1-\alpha)}} y^{\frac{\psi}{\alpha+2(1-\alpha)}} \tag{1.2}$$

such that the cost function is

$$C(y) = \frac{\alpha+2(1-\alpha)}{2\alpha}\left[\chi^{\alpha}\left(\frac{\alpha}{1-\alpha}w\right)^{2(1-\alpha)} y^2 k^{-\alpha}\right]^{\frac{1}{\alpha+2(1-\alpha)}}.$$

Supply function and cutoff $\bar{b}$ The firm considers the level of output y^s that maximizes profits given its price and cost function, but ignoring the demand curve. In other words, the firm thinks about how much it would produce if there was infinite demand for its variety. With $\mathcal{P}$ denoting the nominal price of the final good, it considers its maximal operating profits

$$\max_y \frac{p}{\mathcal{P}} y - C(y)$$

which is solved by

$$y^s = \left(\frac{\alpha}{\chi}\right)\left(\frac{1-\alpha}{w}\right)^{\frac{2(1-\alpha)}{\alpha}} \left(\frac{p}{\mathcal{P}}\right)^{\frac{\alpha+2(1-\alpha)}{\alpha}} k. \tag{1.3}$$

The convexity of the capital utilization cost function ensures that supply given $w, p/\mathcal{P}$ and k is finite.

As mentioned above, there are no contractual arrangements that would require firms to produce more than they desire, so that actual quantity traded is given by

$$y = \min\left\{y^d, y^s\right\}. \tag{1.4}$$

This defines a cutoff value $\bar{b}$ for the idiosyncratic demand shock at which $y^s = y^d$ as

$$\bar{b}\frac{I_U P_U^{\sigma-1}}{p^\sigma} \equiv \left(\frac{\alpha}{\chi}\right)\left(\frac{1-\alpha}{w}\right)^{\frac{2(1-\alpha)}{\alpha}} \left(\frac{p}{\mathcal{P}}\right)^{\frac{\alpha+2(1-\alpha)}{\alpha}} k.$$

Any firm with $b > \bar{b}$ will be constrained due to costly utilization, while firms with $b < \bar{b}$ just satisfy demand. An algebraically useful implication is that y^d can be written as

$$y^d = (b/\bar{b})y^s. \tag{1.5}$$

Operating profits, expected profits, and value function Depending on realized demand b, operating profits as a function of p and k are given by

$$\pi(p,k,b) = \begin{cases} \frac{p}{\mathcal{P}} y^d(p,b) - C\left(y^d(p,b);k\right) & \text{if } b \leq \bar{b} \\ \frac{p}{\mathcal{P}} y^s(p,k) - C\left(y^s(p,k)\right) = \frac{p}{\mathcal{P}} y^s(p,k)\frac{\alpha}{2} & \text{if } b > \bar{b} \end{cases}$$

At the beginning of the period the firm can compute expected profits by integrating over b:

$$E\left[\pi(p,k,b)\right] = \int_0^{\bar{b}} \frac{p}{\mathcal{P}} y^d(p,b) - C\left(y^d(p,b);k\right) \,\mathrm{d}f(b) + \int_{\bar{b}}^{\infty} \frac{p}{\mathcal{P}} y^s(p,k)\frac{\alpha}{2}\,\mathrm{d}f(b).$$

The firm can now choose its price and capacity at the beginning of the period in order to maximize expected operating profits minus the rental cost of capacity and the (expected discounted sum of future) costs of price adjustment. In fact, only the price adjustment cost makes the firm problem truly dynamic. The problem is summarized in the firm's value function

$$V(p_{-1}) = \max_{p,k} E\left[\pi\left(p,k\right)\right] - \left[R-(1-\delta)\right]k - \frac{\xi}{2}\left(\frac{p}{p_{-1}}-1\right)^2 + \beta E\left[V\left(p\right)\right]. \quad (1.6)$$

1.4.4 Households

There is a price-taking representative household. She maximizes lifetime utility given by

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_t - \varphi_t\frac{L_t^{1+\varepsilon}}{1+\varepsilon}\right)\right]$$

where C_t is consumption and L_t is hours worked in period t. There is a random weight φ_t shifting the relative preference of consumption and leisure and which will serve as an aggregate demand shock. This formulation of preferences is consistent with existence of a balanced growth path. Separability between consumption and leisure precludes the concavity in the household's labor supply function that drives the results in Sims and Wolff (2014) which helps us isolate the effects of variable capacity utilization on the firm side.

Besides working, the household also earns income from renting capital K_t to firms as well as from holding one-period bonds issued by the central bank. Her real bond demand in t is denoted with S_t, and central bank pays a nominal interest rate of $\mathcal{R}_t$ on these bonds. The household also collects all profits from firms $\widetilde{\pi}_t \equiv \int \pi_{it} - [R - (1-\delta)]\, k_{it} - \frac{\xi}{2}\left(\frac{p_{it}}{p_{i,t-1}} - 1\right)^2 \mathrm{d}i$ and finances any government spending with a lump-sum transfer of G_t. Combining all these payments in units of final goods yields her real flow budget constraint

$$C_t + S_t + K_{t+1} = \frac{\mathcal{R}_{t-1}}{\Pi_t} S_{t-1} + R_{t-1} K_t + w_t L_t + \pi_t - G_t.$$

The variable $\Pi_t \equiv \mathcal{P}_t / \mathcal{P}_{t-1}$ denotes inflation.

Her optimality conditions are the labor supply equation

$$w_t = \varphi_t L_t^{\varepsilon} C_t, \tag{1.7}$$

the Euler equation

$$\frac{1}{C_t} = \beta \mathcal{R}_t E\left[\frac{1}{C_{t+1}\Pi_{t+1}}\right], \tag{1.8}$$

as well as a no-arbitrage condition between nominal assets and capital

$$\mathcal{R}_t E\left[\frac{1}{C_{t+1}\Pi_{t+1}}\right] = E\left[\frac{R_t}{C_{t+1}}\right]. \tag{1.9}$$

1.4.5 Central bank and government

The central bank sets nominal interest rates in accordance with a simple Taylor rule such that inflation fluctuates around its long-run mean of zero:

$$\log\left(\mathcal{R}_t\right) = \log\left(1/\beta\right) + CB_{rf}\log\left(\Pi_t\right). \tag{1.10}$$

The parameter CB_{rf} determines how strongly the central bank reacts to inflation.

A government undertaking fiscal policy is the second part in the public sector, and also kept very simple. It can buy goods G_t from the final goods firm which it then consumes. It runs a balanced budget and collects lump-sum taxes G_t from the household. We do not explicitly model a (stochastic) process for government spending. Instead, we fix $G_t = 0$ unless we are specifically interested in the effects

1.4.6 Aggregation and equilibrium

Firms use their first-order necessary conditions from maximization of their value (1.6) to determine optimal price and capacity (p_{it}, k_{it}) at the beginning of the period. Since, before realization of period t shocks, all firms share the same state variables, they choose identical prices and capacities such that $p_{it} = p_t$ and $k_{it} = k_t$ $\forall i$. Additionally, firms' decisions about utilization and labor in (1.1) - (1.2) and quantity traded in (1.4) are monomial in $\min\left\{b_i/\bar{b}, 1\right\}$. This makes integration over i straightforward and gives aggregate capital utilization costs and labor demand as

$$CU = \frac{\alpha}{2}\frac{p}{\mathcal{P}}y^s\left(\int_0^{\bar{b}}\left(\frac{b}{\bar{\bar{b}}}\right)^{\frac{2}{2-\alpha}}df\left(b\right)+\left[1-F\left(\bar{b}\right)\right]\right) \tag{1.11}$$

$$L^d = \frac{1-\alpha}{w}\frac{p}{\mathcal{P}}y^s\left(\int_0^{\bar{b}}\left(\frac{b}{\bar{\bar{b}}}\right)^{\frac{2}{2-\alpha}}df\left(b\right)+\left[1-F\left(\bar{b}\right)\right]\right) \tag{1.12}$$

and final goods supply using the aggregator's production function as

$$Y = \bar{b}^{\frac{1}{\sigma-1}}y^s\left\{\left[\int_0^{\bar{b}}\frac{b}{\bar{\bar{b}}}df\left(b\right)+\int_{\bar{b}}^{\infty}\left(\frac{b}{\bar{\bar{b}}}\right)^{\frac{1}{\sigma}}df\left(b\right)\right]\right\}^{\frac{\sigma}{\sigma-1}}. \tag{1.13}$$

In equilibrium, the final goods price $\mathcal{P}_t$ as well as the producer price p_t are not determined in levels. These prices, however, only matter relative to each other or their respective values from the previous period. We therefore define the real price of intermediate goods as $\overline{rp}_t = p_t/\mathcal{P}_t$, inflation as $\Pi_t = \mathcal{P}_t/\mathcal{P}_{t-1}$, and producer price inflation as $\Pi_t^{\mathrm{ppi}} = p_t/p_{t-1}$. These relative prices in turn are related according to

$$\Pi_t^{\mathrm{ppi}} = \Pi_t\frac{\overline{rp}_t}{\overline{rp}_{t-1}} \tag{1.14}$$

as can easily be derived from their definition.

Equilibrium then is defined in the usual way using agents' optimality conditions and clearing of aggregate markets. Notably, the clearing of *aggregate* markets is unaffected by the fact that predetermined prices prevent intermediate goods markets from clearing. Specifically, we define as equilibrium a sequence of prices $\left\{R_t, \mathcal{R}_t, w_t, \overline{rp}_t, \Pi_t, \Pi_t^{\mathrm{ppi}}\right\}_{t=0}^{\infty}$, and of quantities $\left\{Y_t, C_t, CU_t, L_t^d, y_t^s, k_t, K_t, L_t\right\}_{t=0}^{\infty}$ and cutoffs $\left\{\bar{b}_t\right\}_{t=0}^{\infty}$ that satisfy the firms' two optimality conditions derived from (1.6), their supply (1.3), aggregate factor demands and final goods supply (1.11)-(1.13), the household's optimality conditions (1.7)-(1.9), the Taylor rule (1.10), the definition of producer price inflation (1.14), as well as market clearing for labor and capital, an aggregate resource constraint, and the aggregator's zero-profit condition. Note that for this definition we have already imposed $y^s = \bar{y}$.

Appendix 1.2 collects these equilibrium conditions.

1.5 Calibration and results

1.5.1 Calibration

In the following we simulate the model and show that the qualitative results from the example hold up in general equilibrium. Table 1.2 summarizes the calibration of model parameters in two groups: The first group contains parameters that have direct empirical interpretations, whereas the second group consists of parameters that are specific to the model.

The first group of parameters is set to conventional values found in the literature. Capital's share of income α is set to 1/3, and capital depreciation is $\delta = 2.6\%$ implying an annual rate of 10%. Based on estimates of the average mark-up between around 10% and 30% , the macroeconomic literature uses values for the elasticity of substitution between goods σ between 4 as for example in Bloom et al. (2012) and 10 as for example in Sims and Wolff (2014). We hence choose an interior value of 6. Households have a discount factor of $\beta = 0.99$ such that the annual steady-state interest rate is around 4%. The

Table 1.2: Baseline Calibration

Parameter	Value	Meaning	Calibration
Standard parameters			
α	$\frac{1}{3}$	$y_i = \tilde{k}_i^{\alpha} l_i^{1-\alpha}$	Capital share
β	0.99	Hh discount factor	Standard (quarterly)
δ	0.026	Capital depreciation	Standard (quarterly)
ε	$\frac{1}{2}$	Inv. Frisch E. of labor	Standard
σ	6	E. of S. intermediates	Lit: $\sigma \in [4, 10]$)
ρ_φ	0.9	Shock persistence	Standard (quarterly)
σ_φ	0.004	Shock variance	$\text{sd}(Y_t) = 1.8\%$
ξ	75	Scale price adj. cost	Ireland (2001)
CB_{rf}	1.75	Taylor rule	Sims and Wolff (2014)
Model-specific parameters			
χ	1	Scale utiliz. cost	See text
σ_b	0.67	sd idiosync. shocks	$\text{sd}(\Delta\text{TFP}_i) = 0.185$

parameter ε set to 1/2 targets a Frisch elasticity of labor supply of 2 which is also a standard value in macroeconomic models. The aggregate shock follows an AR(1) process in logs such that $\log(\varphi_t) = \rho_\varphi \log \varphi_{t-1} + u_\varphi$ where u_φ is a mean-zero normal random variable with variance σ_φ^2. We set the persistence parameter $\rho_\varphi = 0.9$. The standard deviation of innovations $\sigma_\varphi = 0.004$ is chosen to match the empirical standard deviation of quarterly postwar US GDP of 1.8% when detrended with an HP(1600) filter. The price adjustment cost parameter ξ is set to 75, corresponding to the estimate in Ireland (2001). The coefficient measuring how the central bank reacts to inflation is set to 1.75 as in Sims and Wolff (2014).

The second group of parameters describes the utilization cost function and the variance of idiosyncratic shocks. We assume the distribution of the *iid* idiosyncratic shock b_i to be log-normal and set the parameter σ_b governing its variance to match the variance of innovations to firm profitability in the data. In particular, both Syverson (2011) and Ilut et al. (2014) find a standard deviation of innovations to the log of firm TFP of around 0.185. We match this to average growth rates in the firms' measured TFP in the model. Unfortunately we are unaware of direct empirical estimates for the parameter χ. Moreover, varying the parameter over the admissible range for determinacy implied by the Blanchard-Kahn conditions changes quantitative results only minimally – the fact that the utilization cost parameter is not very well identified by the model can also be observed in other papers, see for example Christiano et al. (2005). We therefore set the parameter χ to unity.

A central feature of the model is that the fluctuating share of capacity constrained firms generates extra concavity in aggregate production. This causes effect sizes to increase with the magnitude of aggregate fluctuations. For example, if aggregate shocks are small, the response of output to a positive shock is similar to the response to a negative shock. Relative differences between booms and recessions increase as the aggregate shock becomes larger. Model results are therefore somewhat sensitive to the variance σ_φ^2 of innovations to φ. In the baseline calibration we take a conservative stance by detrending the empirical GDP series with an HP-1600 filter, which implies a relatively moderate standard deviation of 1.8% for its cyclical component. If, on the other hand, the underlying growth trend of the empirical series were better described by a linear trend, then the time-series standard deviation of the cyclical component is 4.7%, which significantly amplifies output asymmetry and the fiscal multiplier in our results. We consider this alternative calibration in section 1.5.2.5.

1.5.2 Results

1.5.2.1 Impulse response functions

We simulate business cycles by a shock to the household's preference weight φ governing her relative taste for consumption and leisure. While we acknowledge many other possible shocks that can cause aggregate fluctuations, as discussed above we focus on this preference shock as a simple way to generate demand-side effects through distorted relative prices, which in turn allows us to assess how much movement in the measured Solow residual is generated even by a non-technology shock. The model is solved with a second-order approximation around the non-stochastic steady state using the software package Dynare (see Adjemian et al. (2011)). An approximation of at least second order is necessary here since we want to account for the non-linearities generating differences between positive and negative shocks. Under linearization these differences would be lost.

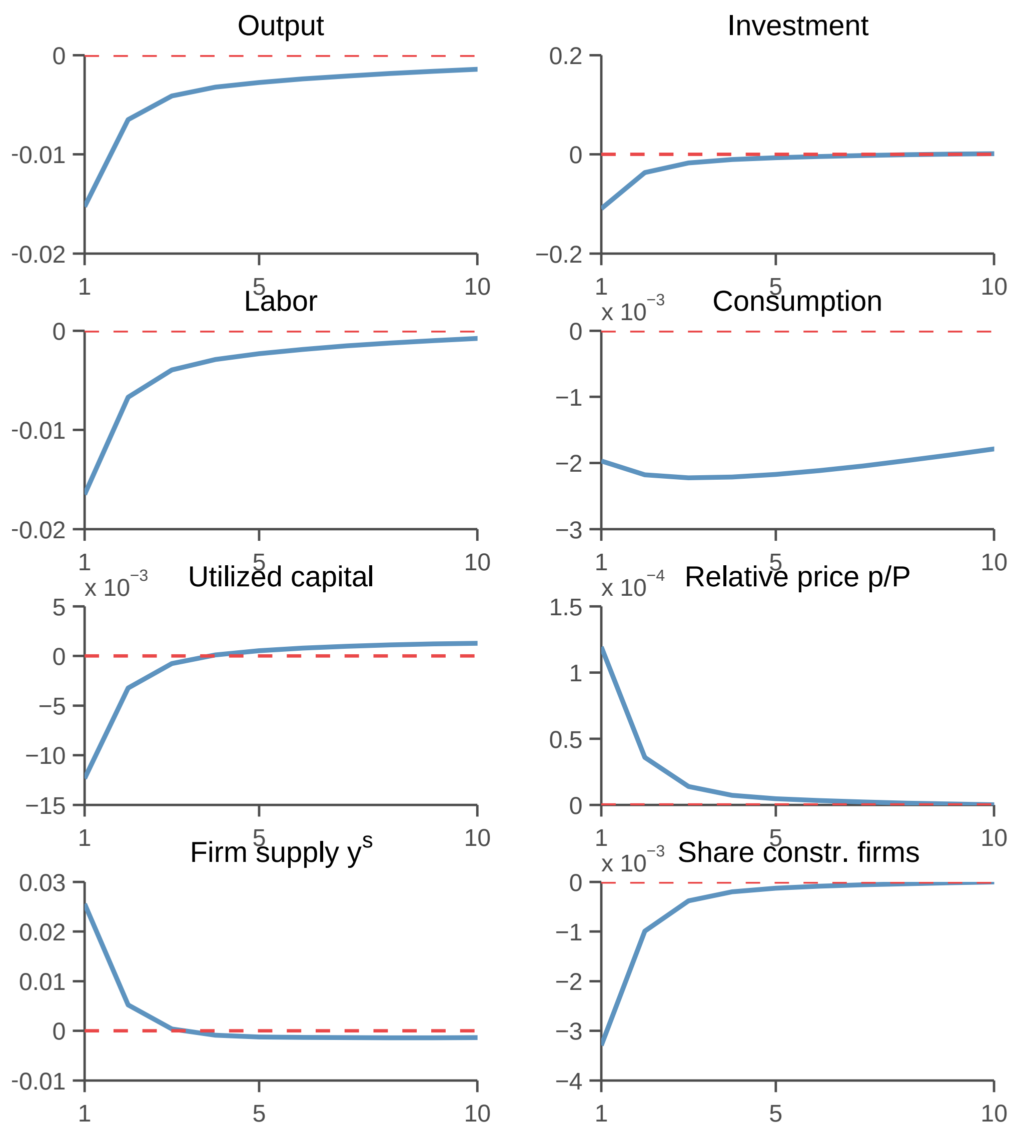
Figure 1.3 displays simulated impulse response functions following a 1-standard-deviation increase in the leisure preference of households φ. For approximations of order higher than 1 the effect size of a shock will in general depend on the state of the economy at the time of impact. The standard way of computing impulse responses in such a case is through simulation, which approximates an 'average' effect of the shock across many simulated states.[7]

Most notable is the strong reaction to the shock on impact in period 1. With prices set one period in advance the usual "New Keynesian" effect of demand shocks via relative prices is fully concentrated in period 1. What remains of the shock in periods 2 and later is primarily driven by the supply side effect of reduced household willingness to work and reduced capital stock from period 1, as well as the fact that firms' price adjustment costs prevent a full alignment of relative prices in period 2.

As expected, capacity utilization drops along with aggregate output. The share of firm below their capacity constraint $F\left(\bar{b}\right)$ decreases as well. This is not only due to the reduction in demand for intermediates, but also due to the increase in firms' willingness to supply their respective variety: With nominal intermediate goods prices fixed at p, the decrease in the aggregate price level $\mathcal{P}$ leads to a temporarily high relative price.

[7] More precisely, one chooses an appropriate 'burn-in' period and a large number I of simulations indexed by i. For each simulation one simulates the model forward such that the model economy is at some random point $S_{i,0}$ of its ergodic state set. Next, one draws a sequence of aggregate shocks $\{Z_{i,t}\}_{t=1}^{T}$ of length T equal to the desired time horizon of the impulse response, and simulates the model forward twice starting from $S_{i,0}$: Once, using only the shocks $\{Z_{i,t}\}$, and once using the same shocks where for $Z_{i,1}$ an additional 1-sd shock the exogenous state variable has been added. The simulated impulse response is then just the difference between the two simulations, averaged over all I repetitions. For more details see, for example, Adjemian et al. (2011).

Figure 1.3: Impulse Response Functions



Notes: Simulated impulse response functions for a positive 1-sd shock to the leisure preference φ_t in period 1. Y-axes show log-deviations from the non-stochastic steady state. A description of the simulation procedure is given in footnote 7.

1.5.2.2 Output asymmetry

We now turn to an assessment of the implications for the stylized facts in general equilibrium. Quantitatively, the model explains around 1/4 of the observed asymmetry in output, and explains fluctuations in the fiscal multiplier of around 0.12.

For the difference between large positive and negative deviations in output, following the approach from the empirical section, we choose an integer N of around 1/6 of the observations ($N = 1666$ out of $10,000$ simulated periods) and compare the mean of the $N/2$ periods with highest output to the $N/2$ periods where output is lowest. As shown in Table 1.3, the average large recession in that sense is -3.45% below trend, whereas the average large expansion is 3.24% above trend. Output is also negatively skewed with a coefficient of -0.11.

Comparing this to the empirical equivalents in Table 1.1, the differences between positive and negative output deviations in the model cover around a quarter of those in the data. In the model, recessions are 0.21 percentage points (or a bit more than 6%) deeper than expansions. As the model was calibrated to match the standard deviation of HP(1600)-filtered, the closest comparable measure is the first row of Table 1.1 showing a relative difference of 23%, or 0.7 percentage points.

Regarding the other aggregate time series also listed in Table 1.3, the model generates asymmetry in levels of investment as well as levels of hours

Table 1.3: Asymmetry of Output Levels and Other Variables

	Model		Data	
Variable	Mean pos vs neg	Skewness	Mean pos vs neg	Skewness
Levels				
Output Y	3.24% vs −3.45%	−0.11	2.73% vs −3.43%	−0.46
Labor L	3.35% vs −3.54%	−0.12	3.11% vs −3.99%	−0.40
Investment I	18.97% vs −23.12%	−0.43	13.08% vs −17.12%	−0.53
Consumption C	1.46% vs −1.47%	−0.02	2.23% vs −2.24%	−0.00
Growth rates				
Output ΔY	3.16% vs −3.15%	0.01	2.30% vs −2.25%	0.19
Labor ΔL	3.45% vs −3.43%	0.01	1.63% vs −2.03%	−0.55
Investment ΔI	23.32% vs−23.04%	0.02	10.00% vs −11.30%	−0.21
Consumption ΔC	0.36% vs −0.37%	−0.06	1.94% vs −1.85%	0.38

Notes: Measures of asymmetry as defined in Table 1.1 (baseline specification). "Levels" measured in log-deviations from simulation mean (model) or from HP-1600 trend (data), respectively. "Growth rates" measured as log-differences. Data for Output, Investment and Consumption from BEA NIPA tables (Real gross domestic product, personal consumption expenditures, gross private domestic investment", respectively). Data for labor from BLS statistics as hours of all persons in the nonfarm business sector. All data are quarterly.

worked, but not for the level of consumption nor the growth rate of output — all these patterns are consistent with empirical findings discussed in section 1.2 and replicated in the Table. The simulation does not exhibit asymmetry in growth rates of employment, even though there is some empirical evidence for this (e.g McKay and Reis (2008)). The reason here is that in the model with its perfectly flexible labor markets the employment and output series move together very closely.

1.5.2.3 Cross-sectional volatility

To assess the correlation of profitability dispersion and output, we consider the cross-sectional standard deviation of $\log(\text{profitability}_i)$. Profitability is measured as firm i's priced Solow residual $p_i SR_i = p_i y_i/(k_i^{\alpha} l_i^{1-\alpha})$ which has the interpretation of "revenue in dollars per input factor basket". As discussed above, this measure uses rented capacity as a measure of capital input — of course firms' true physical productivity $y_i/(\tilde{k}_i^{\alpha} l^{1-\alpha})$ is constant by definition. It can then be shown that a firm's profitability is only a function of its price and demand shock (see appendix 1.3). Since all firms choose the same price, profitability dispersion only depends on the variance of realized demand up to capacity $\min\left\{b_i, \bar{b}\right\}$ with

$$\mathrm{Var}\left(\log(p_i SR_i)\right) = \left(\frac{\alpha}{2-\alpha}\right)^2 \mathrm{Var}\left(\log\left(\min\left\{b_i, \bar{b}\right\}\right)\right).$$

This means that profitability dispersion is only a function of the cutoff level $\bar{b}$, such that its correlation with output will mirror the correlation of $\bar{b}$ with output. In the simulations the correlation $\mathrm{corr}(\mathrm{sd}(\log(SR_i))_t, Y_t) = -0.93$ is correspondingly strong, and indeed higher than the -0.4 to -0.5 that have been measured in Kehrig (2013) and Bloom et al. (2012). This high correlation in the model results from the close comovement between aggregate output and the level of constrained firms we saw in the impulse response functions.

The mechanism of binding capacity constraints has implications for further measures of aggregate uncertainty. In particular, we can look at conditional volatility both at the aggregate and the firm level.

Turning to the aggregate level first, we construct a measure of aggregate volatility from the simulated output series. For this, we look at the variance in the growth rates of output in recessions and expansions, respectively. Specifically, we compute $\mathrm{sd}(\log(Y_{t+1}/Y_t))$ conditional on Y_t being in its lowest or highest quintile. We expect the variance of output growth to be large in recessions: The more firms are far away from their capacity constraint, the stronger the output effects a shock of a given size has. In the model here there actually exists a dampening effect in that firms can adjust their capacity levels and prices quickly in response to an aggregate shock. This allows firms to lower their capacity after the realization of a bad shock, which in turn increases the number of firms at their constraint. If it took firms longer to react, say with a 'time to build' of two periods instead of one, we would expect to see a significantly stronger movements in aggregate conditional volatilities.

Table 1.4 lists the volatility of several model time-series in the first two columns. Going from boom to recession, the standard deviation of output growth increases from 1.49% to 1.66%. The model's investment and labor series exhibit countercyclical conditional volatilities as well, whereas consumption volatility stays constant over the cycle. In the model, aggregate risk as measured by the volatility of output increases by 10.8%. We also construct the empirical analogues of the volatility measures using US data, which are shown in columns 3 and 4 of Table 1.4. As in the baseline empirical specification of Table 1.1, we consider as recessions the 20 quarters since 1949 in which detrended output was lowest. In the data, output volatility in a recession is 39.5% higher in recessions than in booms and thus fluctuates a bit stronger than in the model. Additionally, in the US series both investment and consumption exhibit cyclical volatilities, whereas in our model households are generally able to smooth consumption very well as they do not face any frictions.

Using different empirical strategies, Bloom et al. (2012) find that recessions are associated with a 23% higher standard deviation of output (compared to normal times), and Bachmann and Bayer (2013b) obtain a difference of around 35% between booms and recessions, in line with the empirical values found here. Based on these estimates, the model covers between a third to a quarter of observed fluctuations in aggregate output volatility.

Table 1.4: Aggregate risk: Conditional Volatilities of Aggregate Variables

	Model		Data	
Variable	Expansion	Recession	Expansion	Recession
Output Y	1.49%	1.66%	0.95%	1.41%
Labor L	1.59%	1.77%	0.68%	1.08%
Investment I	9.66%	11.97%	5.21%	5.91%
Consumption C	0.19%	0.20%	1.08%	0.68%

Notes: Standard deviation of growth rate in expansion/recession in the model. For time series X, conditional volatility in recession is computed as the standard deviation of growth rates following a recessionary quarter; i.e. we compute sd($\log X_{t+1} - \log X_t | X_t$ in recession). Analogous for expansions. Recessions and expansions as defined in Table 1.1 (baseline specification) and section 1.5.2.2; in particular output is among the lowest/highest 20 periods (data) and lowest/highest 833 periods (simulated model series).

We proceed in a similar way to look at the variance in growth rates at the firm level. Specifically, we draw idiosyncratic demand shocks for a panel of 1000 simulated firms and follow them for the full duration of the simulation. Keeping track of their levels of profitability (as seen above, firm level output is directly linked to profitability) we again compute the respective growth rate as log difference, and then assess how much the cross-sectional variance in growth rates[8] varies conditional on being in a recession or expansion. The numerical effects here are small, as the standard deviation of profitability growth (in log-differences) is 18.57% in a recession versus 18.43% in a boom. The intuition behind this is the following: Given a constant variance of the idiosyncratic shock, the only thing that affects the variance of growth rates is the likelihood

[8] Due to the *iid* nature of the idiosyncratic demand shocks it actually does not matter much whether one considers the time-series mean of the crosss-sectional variance in growth rates between firms, or the cross-sectional mean in the time-series variance of one firm.

$F\left(\bar{b}_t\right)$ of running into the capacity constraint. Because $F\left(\bar{b}_t\right)$ only moves little over the cycle, individual firms' growth rates do not vary much either.

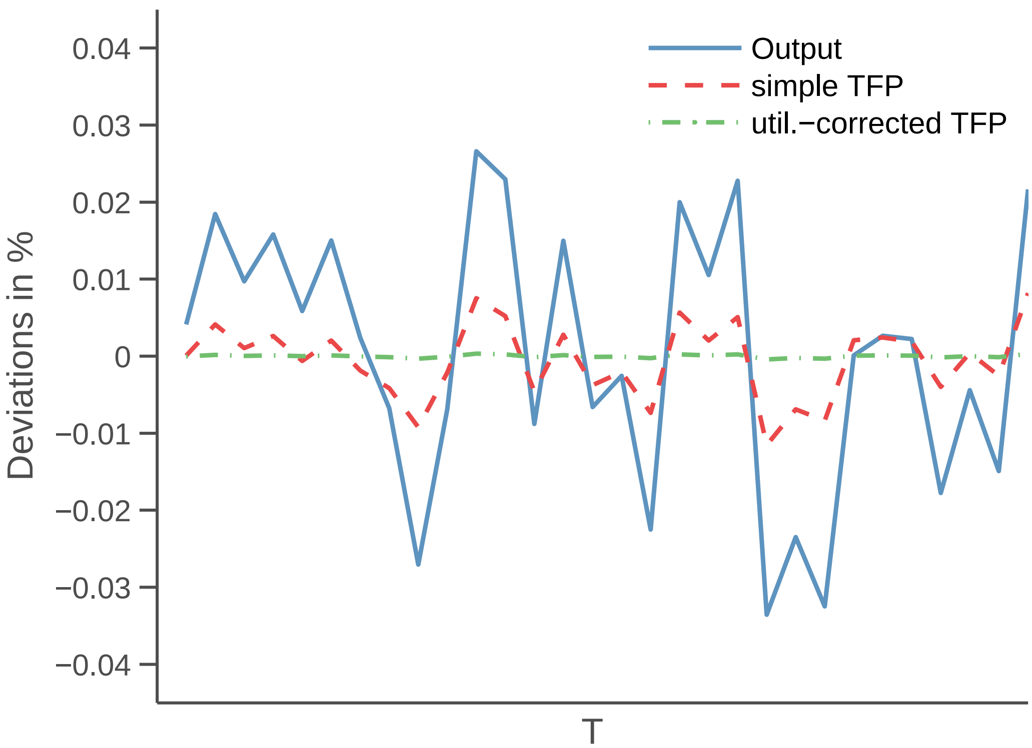
1.5.2.4 Aggregate Solow residual and fiscal multiplier

We construct the aggregate Solow residual in a similar way as its firm-level equivalent. We compute the uncorrected Solow residual as $SR_{\text{simple},t} = Y_t / \left(K_t^{\alpha} L_t^{1-\alpha}\right)$ using aggregate capital in the denominator, and the corresponding version corrected for utilization as $SR_{\text{corr},t} = Y_t / \left(\tilde{K}_t^{\alpha} L_t^{1-\alpha}\right)$ where $\tilde{K}_t = \int_i \tilde{k}_{it} \mathrm{d}i$ is defined as the aggregate utilized capital. Figure 1.4 displays the log deviations from the mean for output as well as both Solow residual for a subset of the simulated periods. As in Basu et al. (2006) and Fernald (2012), the correlation between the simple TFP measure with output is strong with a value of 0.76. Utilization-corrected productivity on the other hand barely moves over the cycle.[9] The standard deviation of simple TFP growth in the simulated series is 0.52%. This value is a little smaller than the corresponding measure in John Fernald's quarterly dataset where the uncorrected Solow residual grows with a standard deviation of 0.87%.

Finally, we consider the cyclicality of the contemporaneous fiscal multiplier $\mathrm{d}Y_t/\mathrm{d}G_t$. In constructing it we follow Sims and Wolff by averaging the state variables over those periods in which production is in its lowest quintile.

[9]Strictly, even utilization-corrected TFP fluctuates over time because of changes in the composition of input factors and their allocation between firms. Since corrected TFP has a very small variance (it has a standard deviation of 0.00018), however, even a tiny amount of noise —like measurement error— renders it acyclical.

Figure 1.4: Output and TFP measures



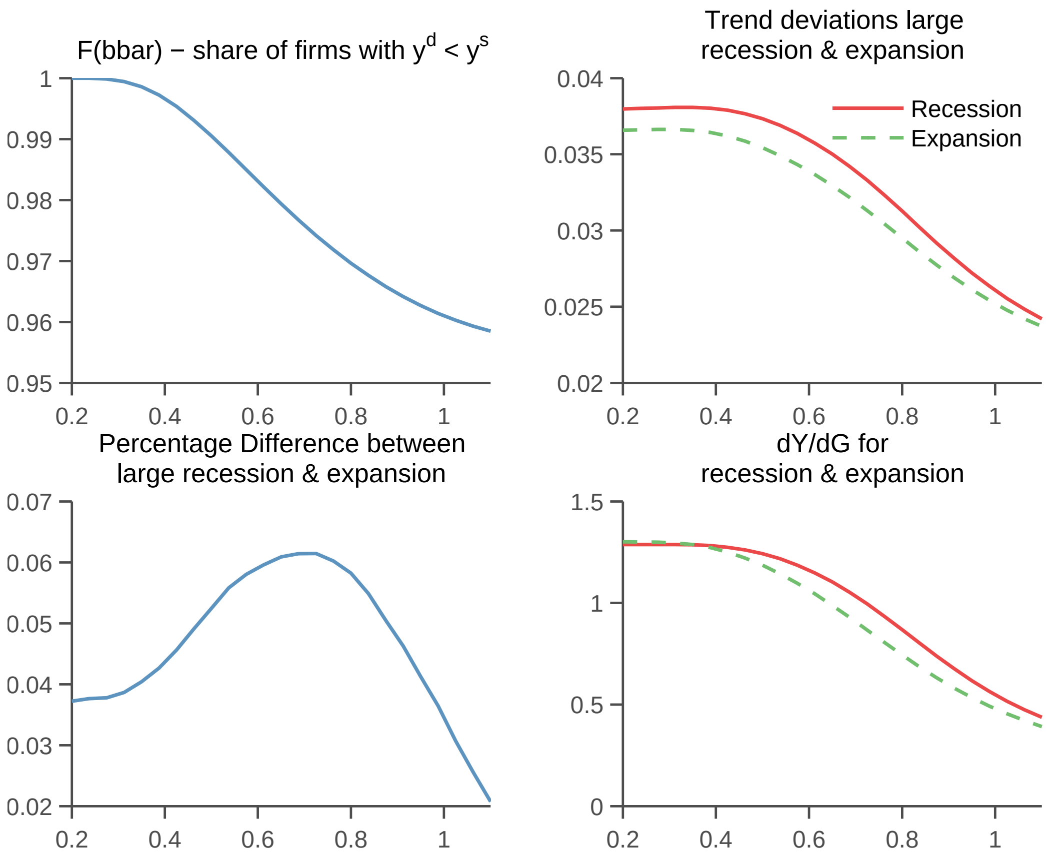
Notes: Output (solid blue line) is Y_t, simple TFP (red dashed line) is measured as $Y_t/(K_t^\alpha L_t^{1-\alpha})$, corrected TFP (green dash-dotted line) is measured as $Y_t/(\tilde{K}_t^\alpha L_t^{1-\alpha})$. Y-axis displays log-differences from non-stochastic steady state. X-axis displays a window of 100 periods out of the $10,000$ simulation periods.

We compare output in this "average bad state" to output in the same state, but with an additional small positive shock to government spending. More formally, if S is the aggregate state, and $S + \Delta G$ the aggregate state after small fiscal spending shock ΔG, the government multiplier is computed as $\left(Y^{S+\Delta G} - Y^{S}\right)/\Delta G$. The value of the multiplier when output is in its top quintile is computed the same way. We obtain values of 1.07 for the multiplier in a recession, and 0.95 for a multiplier in a boom.

1.5.2.5 Role of heterogeneity, discussion and sensitivity

Variance of idiosyncratic shocks The variance of idiosyncratic demand shocks, parameterized by σ_b, directly influences how many firms are capacity constrained. It is instructive to consider how model results depend on this parameter. Figure 1.5 shows this for several outcomes. The graph in upper left displays the share of constrained firms in steady state. Unsurprisingly, the wider the distribution of idiosyncratic demand shocks, the more firms face a level of demand exceeding their capacity. The next two graphs show output deviations from steady state for booms and recessions (top right), and the relative size of these deviations to each other (bottom left), respectively. Notably, output asymmetry is non-monotonic in σ_b. Why is this? What matters is the average change in the share of constrained firms over the cycle, and not its absolute level. Those differences in $F\left(\bar{b}_t\right)$ between expansion and recessions are largest for an interior value of σ_b. At a low value of 0.3 there are practically no constrained firms in equilibrium, and recessions are around

Figure 1.5: Varying Idiosyncratic Shock Variance σ_b



Notes: X-axes display value for σ_b. On Y-axes: Top left – Fraction of unconstrained firms $F(\bar{b})$ in the non-stochastic steady state. Top right – Absolute log deviations of recessions and expansions from non-stochastic steady state. Bottom left – Log difference between absolute deviations in recession and expansion (i.e. log difference of the curves in top right). Bottom right – Government multiplier in recession and expansion.

3.5%, or 0.13 percentage points, larger than expansions. (Even when there is no heterogeneity between firms there is some concavity in production through the convex capacity utilization cost.) Increasing the standard deviation σ_b to around 0.75 makes recessions more than 6% larger than expansions. For high values of σ_b, output asymmetry is reduced again because, despite a larger share of constrained firms in steady-state, the *change* in this share over the cycle is smaller.

A similar pattern can be observed for the fiscal multiplier in the bottom right graph of Figure 1.5. When virtually no firms are capacity constrained, the timing of government spending does not matter for its effect on output — all firms can increase their production in response to government demand. The cyclicality of the multiplier is strongest when the fluctuations in $F\left(\bar{b}_t\right)$ over the cycle are large. In this case comparatively many firms have idle capacities in a recession and can respond to an increase in government demand.

Summarizing, the firm heterogeneity causing capacity constraints to bind occasionally matters in this model because it *generates* cyclicality in the fiscal multiplier and the cross-sectional profitability dispersion, and it *amplifies* the deepness of recessions.

Effect size Is it possible for the same mechanism to deliver stronger effects? One can think of several factors potentially affecting the results.

First, the model is only solved locally, i.e. any effects of aggregate fluctuations are captured by evaluation of the first and second derivative of the equilibrium conditions at the steady state. Any higher-order concavity in the relation between shock size and output is lost when moving away from the steady-state and could only be recovered through a global solution method.

Second, the model has little internal propagation due to the one-period-ahead choices of prices and capacity. Firms are thus very quick to adjust to aggregate shocks, such that it is hard for individual shocks to "add up" over time. In fact it is predominantly the *innovation* to the aggregate state variable φ_t that matters for chance of binding capacity constraints. Since the model is solved up to a second-order approximation, effect sizes increase linearly in the size of the aggregate shock. As an illustration, if one detrends quarterly GDP since 1949 with a linear filter (instead of the HP(1600) filter used in calibration) this implies a considerably higher standard deviation of the detrended series of 4.7% instead of 1.8%. In the model this means correspondingly stronger effects of output deviations of -9.57% and 8.12% for average recessions and expansions, respectively; and values of 1.17 and 0.85 for the recessionary and expansionary fiscal multiplier, respectively. Similar effects can be expected by increasing the "time to build" (and price-set) from one period to a longer horizon.

Finally, the mechanism may have relevant interactions with further types of frictions often considered within the framework of New Keynesian models. Including the mechanism in a richer environment (which may also allow for estimation of the model parameters) is the subject of work in progress.

1.6 Conclusion

This paper includes occasionally binding capacity constraints in a DSGE framework under demand shocks and shows that the model replicates several stylized facts of US output: Recessions are deep; they are times of high cross-sectional and aggregate volatility; and they are times when fiscal policy is particularly effective. Since firms choose their capital utilization after capacity has been installed, the mechanism also generates a fourth stylized fact, namely an endogenously procylical Solow residual.

A calibrated New Keynesian model yields differences in output between booms and recessions of around 0.21 percentage points, such that the model explains more than a quarter of the 0.7 percentage-point difference we find empirically. While the empirical literature has not settled on the size of fluctuations in the government spending multiplier over the cycle, in our basic model we find a multiplier of on average 0.95 in booms and 1.07 in recessions. The multiplier increases with the severity of recessions.

Possible extensions of the model include the addition of frictions that are likely to interact with the capacity mechanism, like time-to-build and wage rigidities. A different direction of further research is to try to expand the amount of firm heterogeneity in the model which is currently limited by our choice of a second-order approximation as solution technique. In particular, in equilibrium all firms choose the same price and capacity before uncertainty is realized. It would be interesting to allow firms' responses to differ along these dimensions, which would likely require global solution methods. This could add a stronger intertemporal component to the model and would allow a more explicit look at aggregate non-linearities.

Chapter 2

Endogenous profitability risk

2.1 Introduction

In recent years, the empirical finding that recessions are associated with an increase in volatility of firm profitability has been widely documented. A number of papers has since shown that this correlation can be driven by the effects of exogenous movements in firm risk (fittingly called "uncertainty shocks", e.g. Bloom (2009)). A smaller strand of the literature has examined the other direction of causation, investigating how cross-sectional dispersion among firms can increase following a worsening of aggregate conditions. In both cases, most papers have focused on firms' physical productivity as a measure of firm profitability, and how this cross-sectional productivity dispersion interacts with the business cycle.

This paper explicitly models firms' pricing decisions to account for differences in physical productivity (TFP) and revenue productivity (TFPR or "profitability")[1]. The aim is to assess how much of the measured profitability dispersion can potentially arise from conventional first-moment shocks to TFP

[1]Foster et al. (2005) have shown that this distinction can be important for firm dynamics for the case of entry and exit decisions.

even when the underlying TFP dispersion is constant. To do so, the model combines the standard simple pricing behavior derived from monopolistic competition with two frictions often used in uncertainty-shock models: non-convex adjustment costs and costly financial intermediation. I choose these two frictions in order to make the model comparable to models that rely on uncertainty shocks to generate dispersion in TFPR. Moreover, the inclusion of a financial channel allows me to assess consistency of the model with data on the financial state of firms which is frequently interpreted as a channel through which firm risk operates.

The model's profitability distribution is driven by firms' heterogeneous investment policies when responding to an aggregate shock. The particular frictions to generate this heterogeneity are fixed costs of capital adjustment and a borrowing friction in the style of Gilchrist et al. (2013). In a recession, firms with high idiosyncratic TFP reduce their output stronger than low-TFP firms. As a consequence, low-TFP firms' prices fall, lowering their profitability further relative to high-TFP firms. This mechanism spreads out the profitability distribution, such that its dispersion is countercyclical. Additionally, credit spreads and credit spread dispersion rise because default risk increases.

In simulations, the calibrated model generates around two-thirds (69%) of the time-series volatility in profitability dispersion found by Kehrig (2013). It also creates around 40% of fluctuations in credit spreads as measured by the GZ credit spread index (Gilchrist and Zakrajšek (2011)). Thus the model gives an alternative explanation for the increase of firm risk in recessions, suggesting that exogenous uncertainty shocks needed to explain the remainder may be smaller than previously thought.

The countercyclicality of a number of cross-sectional dispersion measures has emerged as a fairly robust empirical finding. Eisfeldt and Rampini (2006) document that capital productivity is more dispersed in recessions. In seminal papers, Bloom (2009) and Bloom et al. (2012) show that the distribution of stock returns, firm sales growth, shocks to plant TFPR, and sectoral output all become wider when aggregate output is low. Kehrig (2013) establishes that dispersion in the level of profitability is countercyclical, especially for unproductive firms.

If one takes increases in firm risk as exogenous, there are two prominent ways in which these uncertainty shocks can generate recessions. The first channel is a real-options effect: If it is costly to adjust production inputs it may be worth holding off doing so until the economic environment is less uncertain (Bachmann and Bayer (2013b) have dubbed this the "wait-and-see effect"). In this way high firm risk can reduce investment and hiring, thereby lowering output in subsequent periods. This is the mechanism used in Bloom (2009), Bloom et al. (2012), and Bachmann and Bayer (2013a). A second channel is through financial intermediation. If firms need to borrow funds in order to invest, then high firm risk will make it more likely that the firm ends up defaulting on its debt. This drives up the risk premium which in turn dampens investment. Models utilizing this effect include Christiano et al. (2013), Arellano et al. (2012) and Gilchrist et al. (2013).

There are also a variety of papers that endogenize countercyclical productivity dispersion. Kehrig (2013) shows that in a model of entry and exit under overhead costs the marginal entrant's productivity can be procyclical, implying a tighter productivity distribution in a boom. Cui (2013) builds a vintage model of capital in which capital reallocation is procyclical and consequently recessions are times when relatively unproductive machinery is still being utilized. In Decker et al. (2014), intangible capital investment is needed to access markets. Firms find it optimal to invest more heavily in intangibles during a boom, giving them access to a larger number of markets such that market-specific risk smoothes out. Finally, Bachmann and Moscarini (2012) provide a model of demand uncertainty in which firms are unsure if a drop in demand is due to weak aggregate demand, which is transient, or due to weak private demand, which is permanent. In order to learn about their permanent demand elasticity, firms find it profitable to experiment by setting a higher price in recessions when the opportunity cost of foregoing profits is low. This in turn increases dispersion in sales among firms.

This paper is similar to Bachmann and Moscarini (2012) in the sense that to my knowledge they are the only two papers explicitly modeling prices in a framework of endogenous firm risk. In the present paper, however, firms face idiosyncratic TFP shocks, allowing me to investigate the relationship between physical productivity and profitability, whereas in Bachmann and Moscarini firms differ in their (constant) demand elasticity and an idiosyncratic demand shock. This paper is also close to Gilchrist et al. (2013) in the sense that their paper includes both real adjustment costs and costly financial intermediation (in fact the mechanism of the financial friction here is taken from their paper). Their focus however is on the effect of second-moment shocks on aggregate outcomes. While they do consider aggregate TFP shocks, these shocks can not generate revenue productivity dispersion since firms all produce the same output good and thus no pricing mechanism exists.

The outline of the paper is as follows: The next section 2.2 describes the model setup. Section 2.3 aims to give the intuition how adjustment costs generate endogenous TFPR dispersion. In section 2.4 I discuss calibration and results, and section 2.5 concludes.

2.2 Model

2.2.1 Households

The model's household and final goods sectors are kept as simple as possible. The representative household works for a fixed amount of hours and owns the firms in the other sectors. Hence, in any period her only decision is between consumption C_t and saving S_t. Savings are deposited in the representative bank discussed below. Normalizing her labor supply to one, the household's labor income is equal to the wage w_t. She also earns capital income from past savings $R_{t-1}S_{t-1}$, where R_{t-1} is the risk-free real interest rate determined last period. Finally, she receives all profits Π_t from the final goods, banking, and intermediate goods sectors, respectively, such that $\Pi_t \equiv \pi_t^f + \pi_t^b + \pi_t^i$. Consequently her flow budget constraint is

$$C_t + S_t = w_t + R_{t-1}S_{t-1} + \Pi_t$$

Subject to the sequence of budget constraints, the household maximizes

$$U\left(\{C_t\}_{t=0}^{\infty}\right) = E_t\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\psi} C_t^{1-\psi}\right]. \tag{2.1}$$

2.2.2 Final goods sector

The final goods sector is represented by a single competitive firm functioning as a standard CES aggregator. It uses as input a continuum of intermediate goods $\{y_{it}\}_{i\in[0,1]}$, where i indexes the variety of the good. Production according to a function F yields output $Y_t = F\left(\{y_{it}\}\right)$ which can be used for consumption and investment. All prices in the economy are expressed in units of the final good, i.e its price is normalized to 1. As usual for this type of model the production function F is given by

$$F\left(\{y_{it}\}\right) = \left(\int_{i=0}^{1} y_{it}^{\frac{\sigma-1}{\sigma}}\,\mathrm{d}i\right)^{\frac{\sigma}{\sigma-1}}$$

where the parameter σ measures the elasticity of substition between intermediate goods. Taking prices $\{p_{it}\}$ of the intermediates goods as given, the standard inverse demand curve for input i follows from the final goods firm's maximiation problem as

$$p_{it}(q_{it}) = \left(\frac{Y_t}{q_{it}}\right)^{\frac{1}{\sigma}}.$$

2.2.3 Banks

The banking sector constitutes the only source of financing for the intermediate goods firms. By assumption, firms can not raise equity and instead have to rely on external finance. These loans are risky, because there is a chance that a firm will default and trigger a state verification process which generates additional costs to the lender. Because of this friction, which will be discussed in detail in the next section, each individual loan may be repaid to the bank either fully, partially, or not at all.

There is a large number of atomistic, perfectly competitive banks that each hold a fully diversified portfolio of loans and deposits — this is equivalent to modeling the sector by a representative bank operating under a zero-expected-profit condition. The bank receives deposits S_t from the households on which it pays the risk-free interest rate R_t determined this period. On the other side of the budget constraint stand loans to intermediate goods firms, and their aggregate repayments. The latter include the sum of all full and partial repayments minus the state-verification costs that are due in the case of default. Finally, while the bank absorbs all idiosynratic firm risk through the law of large numbers, it cannot hedge against the risk of aggregate shocks which influence the default rate. This risk is passed on to the bank's owners, the households, via positive or negative profits. These accounting profits are given by

$$\pi_t^b = S_t + \mathrm{Rep}_t - R_{t-1}S_{t-1} - \mathrm{Loans}_t$$

where Repayments and Loans will be defined as aggregates below.

By the assumption of perfect competition embedded in the representative bank, the risk-neutral bank takes loan rates that it can charge firms as given. In particular, if a firm approaches a bank requesting a loan of size b, the bank's optimization problem is simply to decide whether to grant the loan or to walk away. As discussed below, this implies that in equilibrium the bank's expected return on a loan to a firm is just equal to the risk-free interest rate.

2.2.4 Intermediate goods producers

The intermediate goods sector is where the two main frictions, nonconvex adjustment costs and borrowing under costly state verification, are built into the model. Firms accumulate capital and are subject to idiosyncratic and aggregate productivity shocks. They have the option of borrowing from a bank. The interest rate of the loan is firm-specific; that is, the firm borrows against its future profits and capital stock, and the loan rate reflects the size of that collateral and the risk of default.

Production and adjustment cost Production is assumed to follow the Cobb-Douglas form and is given by

$$f(z, A, k, l) = zAk^{\alpha}l^{1-\alpha}.$$

Capital k is quasi-fixed and can be changed only with a lag of one period via investment whereas labor input l is fully flexible and can be hired as needed every period.[2] Physical productivity evolves exogenously according to an idiosyncratic and an aggregate shock process $\{z_{jt}\}_t$ and $\{A_t\}_t$, respectively. Both will be specified as log-normal AR-1 processes. The firm takes into account the demand function for its goods from the final goods sector.

Next period's capital k' is determined today by the standard accumulation rule

$$k' = (1-\delta)k + I,$$

where I represents investment in the current period. Finally, as far as production technology goes, there is a fixed cost ϕ to capital adjustment. I assume the adjustment cost has to be paid if $k' \neq (1-g)k$ where $g \ll \delta$ is a small positive number (and matters only to avoid an indeterminate steady state). Intuitively, if the firm wants to change its capital stock at all (besides the small change through g), the fixed cost is due. This assumption aims to make capital adjustment upward and downward approximately symmetric.

Borrowing problem In order to introduce a financial dimension to the firm problem I assume that for undertaking investment the firm has to rely on external finance. This loan market is subject to costly state verification in case of default, a commonly used type of financial friction[3]. The particular setup

[2] In order to keep the model's state space small, I do not consider the case of additional adjustment frictions in the labor input of the firm. This is despite the fact that, for example in Bloom et al. (2012) it is actually the labor friction that has the largest impact on aggregate dynamics. Allowing for stronger frictions on input factors would most likely strengthen the results here, as firms will find it even harder to achieve their desired revenue product which leads to more dispersion.

[3] See Carlstrom and Fuerst (1997) and Bernanke et al. (1999) for seminal papers.

is taken from Gilchrist et al. (2013). Their version of the borrowing problem allows for substantial firm heterogeneity (in particular admitting persistent differences in physical productivity), and yields endogenous credit spreads while remaining computationally tractable.

Firms can sell bonds b at a price of q that entitle the buyer to 1 unit of tomorrow's consumption good. The firm-specific price of the bond determines the implicit interest rate the firm faces, and will be determined by the marginal risk of default. Default risk exists because, by assumption, there is a minimum amount of net worth denominated $\bar{n}$, that is not enforceable for repayment. In other words, lenders can enforce repayment only up to the point where the borrower's net worth is just $\bar{n}$, so that if $b > n - \bar{n}$ default occurs (where n denotes the firm's net worth just before repayment is due). If the firm is unable to repay the lender in full, default is partial if $n > \bar{n}$ and total if $n \leq \bar{n}$.

Besides the loss of the principal, there is an additional state-verification cost that the lender has to bear in case of default which represents his cost of determining the borrower's remaining net worth. As a simplifying assumption, the default cost is assumed to be proportional to the size of the original loan. A further assumption is that the cost is always due in case of default — in particular, there is no decision to be made by the lender about whether it might be worth to walk away from the loan entirely and avoid paying the state-verification cost.

A final element of the friction is a discount parameter γ which represents the firms' preference for dividend payments over future profits. It measures to which extent the intermediate goods firms discount future income stronger than households. The presence of this parameter with a value strictly less than 1 is a standard way to prevent firms from saving their way out of the borrowing constraint.

Timing The period starts with draws of aggregate productivity A_t and the set of idiosyncratic productivity states $\{z_{it}\}$. With the resolution of aggregate uncertainty all aggregate variables in period t are determined, and the economy's aggregate state is given by $\Sigma \equiv (A, \mu)$. The function μ is the density of the distribution of firms over (z, k, b). Next, intermediate goods firms hire the optimal amount of labor l_{it} given their capital stock k_{it} and productivity z_{it}, regardless of their level of debt. After production occurs at the intermediate and final goods level, intermediate goods firms consider their net worth $n_{it} = (1 - \delta)k_{it} + \pi(z_{it}, k_{it}, \Sigma_t)$ composed of undepreciated capital and revenue net of wages. As described in the previous section a firm then defaults if $n_{it} - \bar{n} < b_{it}$ or otherwise repays its debt b_{it} in full. In either case the firm is left with an end-of-period net worth $\tilde{n}_{it}$, and since there is no 'punishment' for default in terms of an exclusion from credit markets, the firm's state after production is captured by the end-of-period net worth together with the undepreciated capital stock and physical productivity. The firm now has to pick its desired levels of investment and borrowing which, through the budget constraint, also determines dividend payments to its owners.

Optimal choices and firm value With labor hired on the spot it is straightforward to write down a function for revenue net of wages as

$$\begin{aligned}\pi(z,A,k) &\equiv \max_{l} f(z,A,k,l)^{-\frac{\sigma-1}{\sigma}} Y^{\frac{1}{\sigma}} - wl \\ &= C(w)\left[(Azk^{\alpha})^{\sigma-1} Y\right]^{\frac{1}{1-\alpha+\alpha\sigma}}.\end{aligned}$$

where $C(w) \equiv \frac{1-\alpha+\alpha\sigma}{\sigma}\left[\frac{(\sigma-1)(1-\alpha)}{\sigma w}\right]^{\frac{(\sigma-1)(1-\alpha)}{1-\alpha+\alpha\sigma}}$. The maximizing labor input follows as

$$l(z,A,k) = \left\{\left[\frac{(\sigma-1)(1-\alpha)}{\sigma w}\right]^{\sigma}(Azk^{\alpha})^{\sigma-1} Y\right\}^{\frac{1}{1-\alpha+\sigma\alpha}}. \tag{2.2}$$

Of course with capital quasi-fixed, with its choice of labor the firm simultaneously picks production y, price p and its revenue product pz.

The firm-specific bond price follows directly as a no-arbitrage constraint from the bank's zero-profit condition. In expectation, a loan to the firm will yield the risk-neutral bank the same return as an investment at the safe interest rate R_t. The bond price as a function of the loan size, next period's capital, and this period's firm productivity is then given by

$$\begin{aligned}q_b(z,k',b',\Sigma) =& \frac{1}{R}E_A\Bigg\{1-F(\bar{\epsilon})\,(1+\chi)+\left[F(\bar{\epsilon})-F(\underline{\epsilon})\right]\left[\frac{(1-\delta)\,k-\bar{n}}{b'}\right]+ \\ &+\frac{C\,(w')\left[(A'z^{\rho_z}k'^{\alpha})^{\sigma-1}\,Y'\right]^{\frac{1}{1-\alpha+\alpha\sigma}}}{b'}\frac{e^{\sigma_\nu^2/2}}{2}\times \\ &\times\left[\operatorname{erf}\left(\frac{\sigma_\nu^2-\log\left(\underline{\nu}\right)}{\sqrt{2}\sigma_\nu}\right)-\operatorname{erf}\left(\frac{\sigma_\nu^2-\log\left(\bar{\nu}\right)}{\sqrt{2}\sigma_\nu}\right)\right]\Bigg\}.\end{aligned} \tag{2.3}$$

where $\bar{\epsilon}, \underline{\epsilon}, \bar{\nu}$ and $\underline{\nu}$ are cutoff values for productivity shocks that in turn lead to the threshold productivities $\bar{z}$ and $\underline{z}$ for partial and total default, respectively;

erf denotes the error function. For details see appendix 2.1 and Gilchrist et al. (2013).

The firm's optimal borrowing and investment choices are then represented by the firm's value function. The firm's value depends on the idiosyncratic state variables (z, k, b) and on the aggregate state Σ, and the choice variables are next period's capital k' and debt b'. The choice of capital can be thought of as two sequential decisions: A discrete one whether to adjust at all, and, if the answer is yes, an unconstrained choice about the level of investment. Representing the investment decision as this two-step process makes it easy to write down the value function as the maximum of the value of adjusting and the value of not adjusting capital, denoted by V_a and V_n respectively. Denote the binary decision whether to adjust $\mathbb{1}_{\text{adj}}(z, k, b, \Sigma)$.

$$V(z, k, b, \Sigma) = \max_{\mathbb{1}_{\text{adj}} \in \{0,1\}} \mathbb{1}_{\text{adj}} V_a(z, \tilde{n}(z, k, b, \Sigma), \Sigma) + [1 - \mathbb{1}_{\text{adj}}] V_n(z, k, b, \Sigma) \tag{2.4}$$

If the firm chooses adjustment and the fixed cost has been paid its current capital stock relevant only in so far as it contributes to the firm's net worth. Its value is therefore a function of just productivity and net worth

$$V_a(z, \tilde{n}, \Sigma) = \max_{k', b'} \tilde{n} + q(z, k', b', \Sigma) b' - k' + \gamma E_{z,\Sigma}[d(\Sigma', \Sigma) V(z', k', b', \Sigma')]. \tag{2.5}$$

The policy functions for next period's capital and debt conditional on adjustment are denoted $k'_a(z, k, b, S)$ and $b'_a(z, k, b, \Sigma)$, respectively.

On the other hand if the firm has made the decision to save the adjustment cost and stay at its current level of capital, then both k and b remain relevant state variables. while the firm's choice is now only over the trade-off debt/dividend payments.

$$\begin{aligned} V_n(z,k,b,\Sigma) = \max_{b'} \tilde{n} + (z,k,b,\Sigma)\, q\,(z,(1-g)k,b',\Sigma)\, b' - (1-g)k \\ +\gamma E_{z,\Sigma}\left[d(\Sigma',\Sigma)V\,(z',(1-g)k,b',\Sigma')\right] \end{aligned} \tag{2.6}$$

Denote the firm's bond supply choice conditional on not adjusting the capital stock $b'_n(z,k,b,\Sigma)$. The corresponding policy function for capital is trivially given as $k'_n(z,k,b,S) = (1-g)k$. The function $d(\Sigma',\Sigma)$ represents the household's stochastic discount factor.

Finally, from (2.4)-(2.6) the unconditional policy functions follow as

$$k'(z,k,b,\Sigma) = \mathbb{1}_{\text{adj}}(z,k,b,\Sigma)\, k'_a(z,k,b,\Sigma) + \left[1 - \mathbb{1}_{\text{adj}}(z,k,b,\Sigma)\right] k'_n(z,k,b,\Sigma)$$

and

$$b'(z,k,b,\Sigma) = \mathbb{1}_{\text{adj}}(z,k,b,\Sigma)\, b'_a(z,k,b,\Sigma) + \left[1 - \mathbb{1}_{\text{adj}}(z,k,b,\Sigma)\right] b'_n(z,k,b,\Sigma)\,.$$

2.2.5 Aggregation and Recursive Equilibrium

There are three aggregate markets in the economy: the market for final goods, the market for labor, and the market for loans. Aggregate supply and demand on each of these markets are:

$$
\begin{aligned}
L^s &= 1 \\
L^d &= \int_{(z,k,b)} l\left(z, k, \Sigma\right) d\mu\left(z, k, b\right) && (2.7) \\
Y^s &= \left[\int_{(z,k,b)} \left(y^s\left(z, k, \Sigma\right)\right)^{\frac{\sigma-1}{\sigma}} d\mu\left(z, k, b\right)\right]^{\frac{\sigma}{\sigma-1}} \\
Y^d &= C + I + \mathrm{Mon} + \Phi && (2.8) \\
\mathrm{Loan}^s &= S \\
\mathrm{Loan}^d &= \int_{(z,k,b)} q_b\left(z, k'(z,k,b,\Sigma), b'(z,k,b,\Sigma), \Sigma\right) \times && (2.9) \\
&\qquad b'\left(z, k, b, \Sigma\right) d\mu\left(z, k, b\right).
\end{aligned}
$$

In equation (2.8), aggregate monitoring cost and aggregate adjustment cost paid are defined as

$$\mathrm{Mon} = \chi \int_{(z,k,b)} b \mathbb{1}_{z<\bar{z}(k,b,\Sigma)} d\mu\left(z, k, b\right)$$

and

$$\Phi = \phi \int_{(z,k,b)} \mathbb{1}_{\mathrm{adj}}\left(z, k, b, \Sigma\right) d\mu\left(z, k, b\right),$$

respectively.

A recursive equilibrium is composed of a set of value functions, policy functions, pricing functions, as well as a law of motion that are consistent with agent optimization, market clearing, and rational expectations.

Specifically,

- the functions $S\left(\Sigma\right), C\left(\Sigma\right)$ are the policy functions derived from the household's problem (2.1) and $U^*\left(\Sigma\right)$ is maximized lifetime utility; the functions $V(z,k,b,\Sigma)$, $V_a(z,\tilde{n},\Sigma)$, $V_n(z,k,b,\Sigma)$ are an intermediate goods firm's value functions (2.4)-(2.6) and $l(z,k,b,\Sigma)$, $k'(z,k,b,\Sigma)$, $b'(z,k,b,\Sigma)$ are the corresponding policy functions;

- Wages are given by $w(\Sigma)$, interest rates by $R(\Sigma)$, bond prices by $q_b(z,k',b',\Sigma)$
- Markets in equations (2.7) - (2.9) clear, i.e. $L^s = L^d$, $Y^s = Y^d$, and $\text{Loan}^s = \text{Loan}^d$.
- The law of motion $\mu'(\mu, A)$ for the evolution of distribution over (z,k,b) follows from the firms' policy functions and the exogenous process for z.

2.3 Determinants of the profitability distribution

The aim of this section is to give an intuition for how cross sectional dispersion in revenue productivity can arise when one includes firm pricing. I will first discuss the general TFPR distribution and how it interacts with the real friction, and then turn to the financial friction.

Non-convex adjustment costs The general mechanism is that, if there are frictions, firms' investment need not react symmetrically in response to shocks. Considering the example of fixed adjustment costs, physically productive firms are more likely to adjust, since for these firms the relative benefit of adjustment is larger. This in turn will reset their revenue product towards the level chosen if there were no adjustment costs. The non-adjusters on the other hand will tend to be physically unproductive firms. Not adjusting fully to a, say, negative shock in physical productivity levels will leave these firms with too much factor inputs and cause them to overproduce relative to the frictionless case. This drives down the price and therefore revenue productivity. Hence the left tail of the TFPR distribution becomes wider in a recession, and conversely it becomes shorter in a boom.

Turning to the specifics of the model, we can use the fact that labor is a fully flexible input factor and is chosen optimally as given by (2.2). One can then write down an analytic expression for revenue productivity pAz as being proportional the productivity states and capital:

$$pAz \sim (Az)^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} \, k^{-\frac{\alpha}{1+\alpha(\sigma-1)}}. \tag{2.10}$$

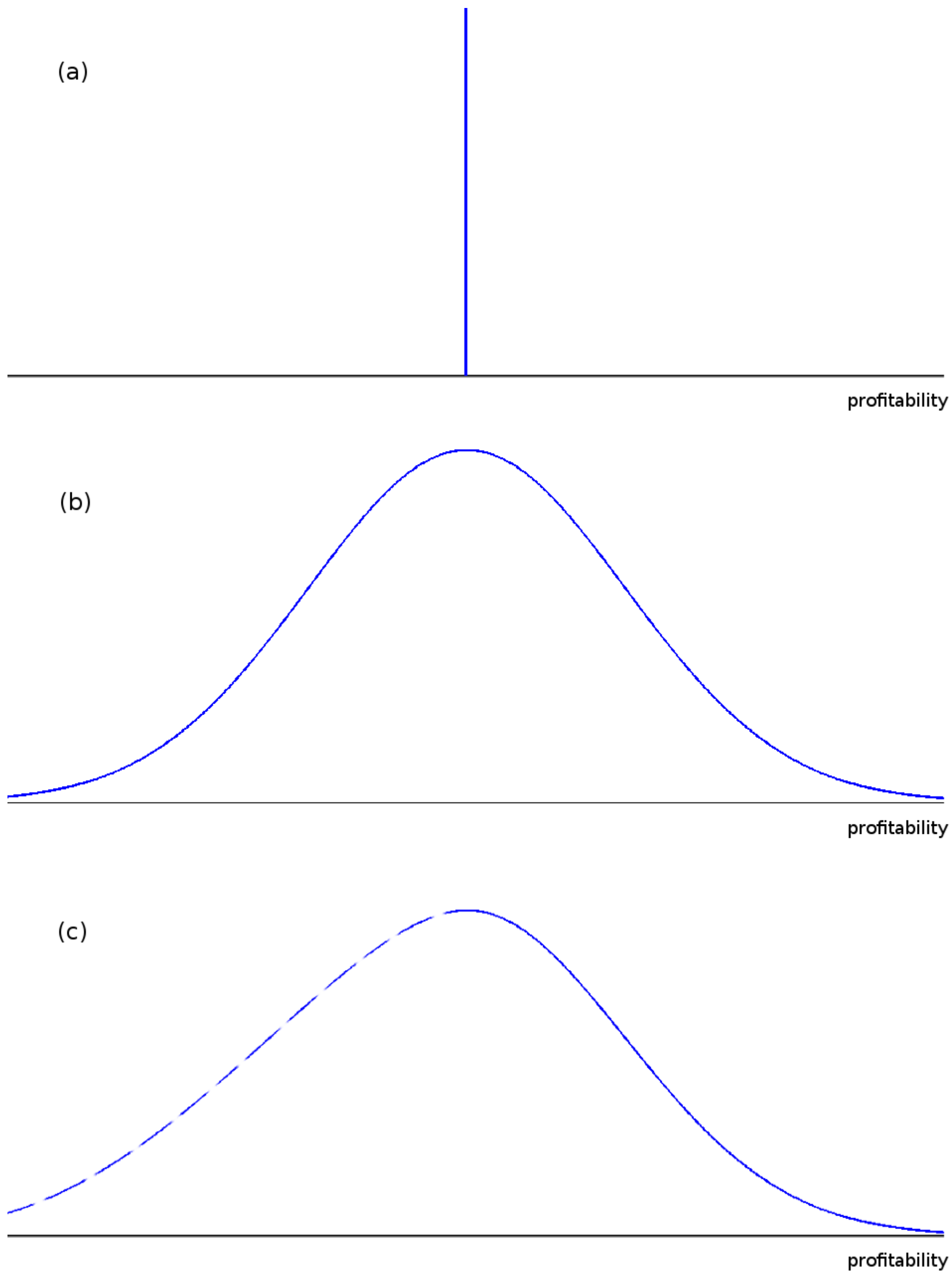
Given capital, TFPR increases in physical productivity; and given productivity, TFPR decreases in the amount of capital held by the firm.

It can be informative to consider this relationship under some extreme cases. First, if A and z are fixed (the variance of the respective shocks is set to 0) and there are no adjustment costs, then firms will choose the same k equalizing their revenue product. This also corresponds to the setup often used in models where production is linear in the fully flexible factor labor and the TFPR distribution is degenerate (case a in figure 2.1). Second, if there is variation in A and z while still holding adjustment costs at 0, then using the firm's profit function it can be shown that firms pick k in a way that equalizes their expected revenue product. This means that any dispersion in TFPR will be caused by contemporanous shocks to idiosyncratic productivity z, implying a symmetric profitability distribution (case b in figure 2.1). It is important to note that the firms' ranks in the TFP distribution are preserved in the TFPR distribution: a higher physical productivity implies a higher revenue productivity. Additionally, for a given variance of shocks to z, dispersion is increasing in α (the more the quasi-fixed factor is used in production, the more dispersion) and increasing in σ (the higher the price elasticity of demand, the more dispersion).

Third, say the level of idiosyncratic TFP z displays some persistence and that there are positive adjustment costs. As discussed above, assume they are such that firms with high z find it profitable to adjust their capital stock in response to a given change in A, while firms with low z do not. Then next period's capital is going to be correlated with productivity: For example, following a negative shock to A, physically unproductive firms will have relatively much capital, and therefore a low revenue product (case c in figure 2.1) . Underlying this is of course the fact that these firms produce more than they would have if capital adjustment were free, and therefore drive down the price of their good.

Financial friction As just outlined, the effect of the aggregate state on the profitability of firms depends on the response of investment and the associated marginal costs of production in the following period. The question is therefore whether a recession more strongly affects productive firms or unproductive

Figure 2.1: Profitability distribution in special cases



firms. Under the borrowing friction investment is influenced by the firm-specific credit spread; so what matters is whether unproductive firms' loan price reacts more to a change in A than a productive firm's loan price. Consider a case without adjustment costs: A firm with low expected z will invest little, therefore requiring only a small loan. Of course its expected revenues are also small. Together these effects determine the firm's default risk and its credit spread.

If a recession drives up productive firms' credit spreads more than that of unproductive firms, productive firms will have less capital and higher prices next period, increasing dispersion in profitability. On the other hand it is possible that a recession mainly affects unproductive firms default risk. In this case by the same argument the TFPR distribution would become narrower making its dispersion procyclical. Analytically it is not clear which one of the cases applies, and in general this will depend on the specific model environment. The calibration used below shows countercyclical TFPR dispersion when only the financial friction is active, indicating that the effect on productive firms dominates.

2.4 Simulation and Results

2.4.1 Simulation

Calibration For most parameter values I take existing estimates from the literature as a baseline case. In particular, I draw on Khan and Thomas (2008)'s model of fixed adjustment costs for a range of technology parameters, as do Bloom et al. (2012). Table 2.1 summarizes the baseline parameter choices.

Table 2.1: Parameter values

Parameter		Value	Target / Source
Household and final goods sector			
Rate of time preference	β	0.96	Real interest rate of 4%
EIS	ψ	1	Log utility
Price elasticity of demand	σ	4	Bloom et al. (2012)
Technology			
Persistence of aggregate TFP	ρ_A	0.86	Khan and Thomas (2008)
SD of innovations to A	σ_A	0.027	Volatility of output 2.2%
Persistence of idiosyncratic TFP	ρ_z	0.86	Khan and Thomas (2008)
SD of innovations to z	σ_z	0.022	Khan and Thomas (2008)
Capital share	α	0.2	Labor income 60%
Depreciation rate	δ	0.1	Standard value
Frictions			
Adjustment cost	ϕ	0.04	2.6% spike adjusters
Depreciation when not adjusting	g	0.01	Small value
Verification cost in default	χ	0.10	Gilchrist et al. (2013)
Firms' dividend preference	γ	0.95	Gilchrist et al. (2013)
Borrower's protected net worth	$\bar{n}$	0.0	Gilchrist et al. (2013)

The model is calibrated to an annual frequency in order to facilitate computation. The household's rate of time preference β is set to 0.96 generating an average annual interest rate of around 4%. The household is given an elasticity of intertemporal substitution of 1 corresponding to a log utility function. As in Khan and Thomas (2008). I set the persistence of the aggregate productivity process $\rho_A = 0.86$. For the standard deviation of shocks to A I choose a value of $\sigma_A = 0.027$ which targets the detrended time-series volatility of annual output in the United States of 2.2%. The elasticity of substituion in the final goods sector is chosen to be $\sigma = 4$ implying a price elasticity of demand for intermediate goods of -4. While this is on the lower end of estimates in the literature it corresponds to the value implied by Bloom et al. (2012)'s choice of decreasing returns to scale on the firm level.[4]

Turning to the intermediate goods firms, I again follow Khan and Thomas (2008) in choosing a persistence for the idiosyncratic productivity process that is equal to the one of the aggregate process and set $\rho_z = 0.86$. There is a range of estimates for the variance of innovations to firm TFP in the literature. For example, Khan and Thomas (2008) use a σ_z of 2.2%, and Bloom et al. (2012) use 4%. As will be seen below, the relative TFPR dispersion over the cycle in this model is somewhat sensitive to the choice of σ_z. For now I stick to the calibration by Khan and Thomas (2008) and will discuss higher values of σ_z below. As is standard in the literature, the annual depreciation

[4]Bloom et al. point out that the source of decreasing returns of their firms' production function could be derived from monopolistic competition.

rate is set to $\delta = 0.10$. For the parameter g that describes capital shrinkage in the case of non-adjustment I ad-hoc pick 0.01 as a 'small value'. The elasticity of intermediate firm output with respect to capital is set to $\alpha = 0.2$. Given demand elasticity σ above, this value matches a 60% labor share of output as α and σ determine the monopolistic firms' profits jointly.

What remains is to set the parameters governing the model's frictions. For the financial friction I follow Gilchrist et al. (2013) in setting the default cost to $\chi = 0.10$, the preference rate for dividend payments to $\gamma = 0.95$ and the level of protected net worth to $\bar{n} = 0$. The final parameter that needs to be chosen is the fixed adjustment cost ϕ. These costs are usually calibrated to match a moment of the distribution of investment rates; oftentimes this is the share of 'spike adjusters' whose investment rates exceed 20%. For the US this value is around 15% for equipment capital and around 10% for all types of capital (including structures). These numbers however include replacement investment. Since in the model depreciation is paid by all firms every period (unless they adjust downward) I target a fraction of adjusters significantly lower between 2% and 3%. For this parameter, too, I consider alternative specifications below.

Numerical approximation I solve the model approximately using standard techniques for the computation of heterogeneous agent DSGE models. I briefly outline the procedure here and put additional description into appendix 2.2.

The state space is discretized using a uniformly spaced grid for the endogenous variables k and b. The exogenous state variables z and A and their evolution over time are approximated as discrete Markov chains using Tauchen's method. I use a particularly dense grid for z since the realization of the idiosyncratic shock is essential for the default decision. Aggregate TFP A is modeled as a process over three discrete states standing for recessions, normal times, and booms. The distribution μ of capital and debt among firms as endogenous aggregate variable is approximated using a grid for the first moment of the marginal distribution of capital, K.

Agents in the economy are assumed to use the aggregate state (A, K) to forecast other aggregate variables. Making household's marginal utility the numéraire as in Khan and Thomas (2008), the variables that need to be forecast are next period's capital stock K', the wage w, the price of the final good (in utils), as well as the output of the final good Y. The latter is relevant because the intermediate goods firms' output decisions and aggregate output are interdependent through final goods firm's demand function. With agents thus taking all aggregates as given functions of the state variables, I use value function iteration to derive the monopolists' policy functions for k' and b'. The economy is simulated over a long time horizon. This procedure is then repeated, updating the forecast rules iteratively until forecasts match simulated prices and quantities as closely as possible.

2.4.2 Results

This section discusses the model economy's response to shocks in three ways. First, I compare average booms and recessions to get an idea of the cyclicality of central variables. Then I graph impulse response functions as a way to capture the response of the economy to an isolated shock. Finally I look at unconditional volatilities and the correlations of aggregates over the cycle to assess the magnitude and cyclicality of the measures of interest quantitatively.

Average booms and recessions I now consider fluctuations in the level of aggregate TFP. With TFP A modeled as a three-point process, table 2.2 presents model statistics for the recessionary and expansionary state of the economy relative to the 'neutral' state in which A is normalized to 1, respectively.

TFPR dispersion is measured as the coefficient of variation of cross-sectional revenue productivity.[5] The mean credit spread is the average of the differences of firm-specific interest rates as implied by their bond price and the risk-free interest rate in the same period. Finally, dispersion of credit spreads is again measured as the cross-sectional coefficient of variation.

These statistics were generated by simulating the full economy, and then simply averaging over all periods in which A was, say, low. The table qualitatively confirms that the model displays countercyclical TFPR dispersion and worsening credit conditions: Revenue productivity is on average 9.7% more

[5] That is, $sd(p_i A z_i)/mean(p_i A z_i)$. The coefficient of variation is useful in this context because it is a scale invariant dispersion measure.

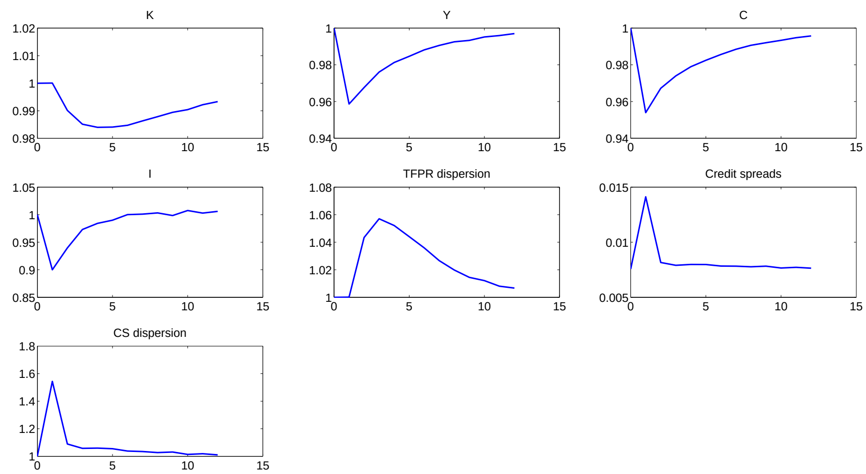
Table 2.2: Recession vs boom

	Recession	Normal times	Boom
Mean of...			
TFP A_t	-3.92%	1	3.92%
Output Y_t	-2.44%	0.99	2.61%
TFPR dispersion	3.7%	0.019	-4.4%
Mean credit spread	$+22$ bps	88 bps	-24 bps
Credit spread dispersion	21.5%	0.016	-17.9%

dispersed comparing recessions to booms, and credit spreads are 46 basis points higher as well as 39% percent more dispersed. This table does not tell us much about the timing of the responses to shocks, nor do we readily observe A in the real world. Next I therefore consider impulse response functions, and will afterwards compare the unconditional volatilities of the aggregates to the data.

Impulse response functions I obtain simulated impulse response functions by manually holding the aggregate shock A at its long-run mean of 1 and simulating the economy for enough periods that all aggregate variables become approximately constant. The firms are still being hit by idiosyncratic shocks, and in contrast to the non-stochastic steady state, they expect movements in A according to its regular distribution (which simply don't materialize). A large number of economies is then seeded to this 'neutral' aggregate state as period 0. In period 1 all economies are hit by a negative shock, i.e. A is set to its low value. The economies are then simulated forward, differing in how long they remain in the recessionary state. Specifically, for each economy, the chance of remaining in the low state for one more period is given by the transition probabilities of A,

Figure 2.2: Impulse response functions to negative shock in A



and once an economy gets a draw that would put it into the ‘neutral’ or ‘high’ TFP state, its A is set back to 1 for the remainder of the simulation. This part of the procedure ensures that all aggregate variables eventually return to their pre-impact levels and there is no sampling randomness. The impulse response functions for several variables are shown in figure 2.2.

The dispersion measures react as implied by theory. Revenue productivity dispersion increases by around 6%, credit spreads by around 50 bps and credit spread dispersion by 50%. Because the profitability distribution is only affected through the capital stock, TFPR dispersion does not react on impact, but increases as firms carry out their different investment policies in response to the shock. Since loan prices are determined in the period in which investment takes place, credit spreads and credit spread dispersion react immediately, shooting up and flattening back out quickly.

Time series volatility I now compare the time-series variance of the model-simulated aggregates to the ones measured in the data. Table 2.3 displays second moments of the dispersion measures as well as the main business cycle aggregates.[6]

The empirical moments in table 2.3 are compiled from several sources. The data on revenue productivity dispersion is calculated using Kehrig (2013)'s annual time series on the median sectoral coefficient of variation of TFPR fro 1972 to 2010.[7] Similarly, the information on the level of the average credit spread uses the GZ-credit spread index released by Gilchrist and Zakrajšek (2011).[8] This time series ranges from 1973 to 2012. While information on the volatility of the GZ-credit spread dispersion is not readily available, Gilchrist and Zakrajšek (2011) report the correlation of output with both mean credit spreads and the cross-sectional standard deviation of credit spreads. Finally, the numbers for output, consumption and investment are derived from FRED[9] using HP(100)-filtered data from 1950 to 2013. I apply the same HP(100)-filter to the model-generated data for output, consumption, investment, and TFPR dispersion.

[6]In case of the dispersion measures (TFPR dispersion and credit spread dispersion), the time-series variance is the longitudinal second moment of a cross-sectional second moment.

[7]The data provided by Kehrig (2013) is HP(100)-filtered. Data available for download at https://sites.google.com/site/matthiaskehrig/research

[8]Data available at http://people.bu.edu/sgilchri/Data/data.htm

[9]http://research.stlouisfed.org/fred2/

Table 2.3: Time-series volatility and cyclicality

	Model	Data
Standard deviation of		
TFPR dispersion	3.24%	4.7%
Credit spreads	36 bps	92 bps
Credit spread dispersion	24.6%	n.a.
Y	2.2%	2.2%
C	2.0%	1.8%
I	6.0%	8.3%
Cyclicality		
$corr(Y, \text{TFPR disp})$	-0.32	-0.40
$corr(Y, \text{credit spreads})$	-0.53	-0.46
$corr(Y, \text{CS disp})$	-0.60	-0.25

Consumption is a bit too volatile in the model, at the expense of an undershoot in the variance of investment (with the volatility of Y being targeted in the calibration). Looking at profitability dispersion, I find that the model can explain a little more than two thirds of the empirically observed volatility (3.24% versus 4.7%), and exhibits a similar correlation coefficient with output as in the data (-0.32 versus the observed -0.40). The model generates close to 40% of the fluctuations in the level of credit spreads (36 bps versus 92 bps in the GZ index), and it matches the empirical cyclicality fairly closely. Finally, the model delivers significant swings in credit spread dispersion. This suggests that the model's first-moment shocks generate the empirically observed negative relationship between recessions and financial indicators of firm risk. Moreover, in order to generate a larger amplitude in credit spreads additional financial frictions may be needed. For example, Gilchrist et al. (2013) in their uncertainty-shock model supplement the borrowing friction with frictions on raising and lowering equity (issuing stocks and paying dividends, respectively).

Contribution of individual frictions Figure 2.3 displays the impact of frictions separately. For this exercise, I compare four simulations: First, the model in the baseline calibration as before; second, a model with only adjustment costs where firms can borrow on a frictionless credit market (there is no default, i.e. $\bar{n} = -\infty$); third, a calibration with only financial frictions (adjustment costs $\phi = 0$); and fourth, a simple model of monopolistic competition without real nor financial friction.

The most notable feature is that the paths of real variables between the case of both frictions and only adjustment costs are very similar. In particular, as long as the adjustment cost is present the responses generate a significant increase in profitability dispersion. The financial friction on its own, however, only leads to a small increase. Moreover, including adjustment costs raises the baseline level of credit spreads and increases their response to a negative shock compared to the case of the borrowing friction alone.

Table 2.4 confirms this finding: Adding the borrowing friction to a model of non-convex adjustment costs does not change the implications for the profitability distribution much. Conversely the adjustment cost improves the fit compared to only the financial friction.[10] Notably, the borrowing friction

[10]The excessive TFPR dispersion appears to be result of a nonlinearity: In the simulation, a more than proportionate jump in credit spreads occures when the aggregate state quickly switches from boom to recession without spending much time in the intermediate state. Without these episodes volatility in TFPR dispersion is small.

Figure 2.3: Impulse response functions of frictions separately

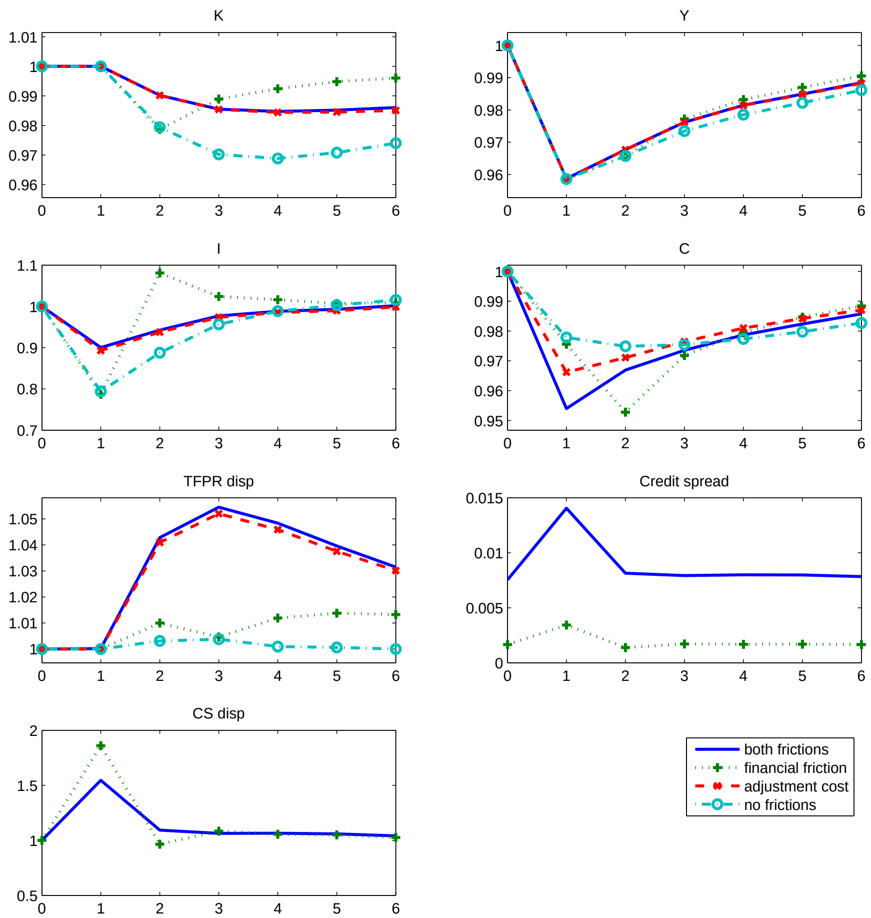


Table 2.4: Impact of individual frictions

Friction	Both	Financial	Real	None
Standard deviation of				
TFPR dispersion	3.24%	8.04%	3.16%	0
Credit spreads	36 bps	17 bps	-	-
Credit spread dispersion	24.6%	45%	-	-
Cyclicality				
$corr(Y, \text{TFPR disp})$	-0.32	-0.17	-0.31	0
$corr(Y, \text{credit spreads})$	-0.53	-0.06	-	-
$corr(Y, \text{CS disp})$	-0.60	-0.41	-	-

itself does not generate the observed countercyclicality of credit spreads, a point also found by Gilchrist et al. (2013).

Sensitivity to parameters I consider alternative specifications for two central parameters; namely the adjustment cost ϕ as well as the standard deviation of idiosyncratic productivity shocks σ_z. Table 2.5 contains results for a few different parameter values.

Overall, the model economy does not respond too strongly to changes in the adjustment cost. Lower ϕ mainly causes a larger share of firms to adjust each period, generating higher volatility in aggregate investment. Volatility of the profitability distribution reacts only mildly in a non-monotonic way except for very low values of the parameter.

Increasing σ_z reduces the magnitude of swings in TFPR dispersion. Intuitively, a high variance of firm-specific shocks makes the aggregate state less important to the firm in making its investment decisions. These decisions

Table 2.5: Alternative parameter values

	Time-series volatility (sd)					*(mean)*
	TFPR disp	Cred. spread	Y	C	I	Frac. adjusters
Baseline						
	3.2%	36bps	2.2%	2.0%	6.0%	2.5%
ϕ						
0.03	3.8%	35bps	2.2%	2.0%	5.7%	3.0%
0.02	3.6%	32bps	2.2%	1.9%	6.3%	4.3%
0.015	2.6%	35bps	2.2%	1.8%	8.1%	8.4%
0.01	7.8%	26bps	2.2%	2.0%	14.7%	75%
$\sigma_{\mathbf{z}}$						
0.04	2.7%	22bps	2.2%	1.8%	6.8%	2.8%
0.08	0.8%	22bps	2.2%	1.5%	9.0%	7.3%

are now mainly driven by the firm's idiosyncratic state. For example, when using the parameter value from Bloom et al. (2012) with $\sigma_z = 0.04$, the standard deviation of TFPR dispersion is 2.7% or 55% of the empirical estimate, and decreases further for higher σ_z.

2.5 Conclusion

This paper shows that profitability dispersion among firms can arise endogenously in a response to a change in aggregate production levels. Therefore recessions can look like times of increased firm risk even when underlying productivity risk is constant over the cycle. In general, this result comes from heterogeneity in how firms' pricing responds to an aggregate shock. The particular structure chosen in this paper demonstrates that this differential response can result under the same setup used in models of uncertainty shocks, employing non-convex adjustment costs and costly financial intermediation. The baseline calibration generates two-thirds of the empirical observed cyclicality in levels of revenue productivity as well as procylicality in the financial health of firms. These results suggest that accounting for the difference between productivity and profitability is relevant when assessing firm risk over the business cycle and calibrating uncertainty shocks to cross-sectional moments.

Chapter 3

Gasoline consumption and the welfare effects of oil price shocks

3.1 Introduction

In this paper I examine how oil price shocks can directly affect household welfare, and how these effects vary with household income. To this purpose I focus on households' gasoline consumption which is the main end to which households purchase (processed) oil. I evaluate the relationship between gas expenditures and household income empirically using data from the Consumer Expenditure Survey. In this dataset I find that among gasoline-using households there is a robust negative relationship between income and the share of a household's budget that is spent on gasoline. I replicate this finding in an equilibrium model with two types of households who differ in their labor income. In particular, the decreasing propensity to consume gasoline is introduced via a fixed minimum quantity of gasoline that must be consumed by all households. This inelastic part of a household's gasoline consumption can be interpreted as required for commuting to work. Any quantity of gasoline consumed beyond this minimum level enters households utility as a complement to an output good which represents the remainder of the consumption basket.

The model is then calibrated to match the differences in household consumption and gasoline expenditures between the top and the bottom half of the income distribution. I then examine the welfare effects of a shock to the gasoline price on either type of household by comparing a one-time gasoline price shock to a permanent change in the steady-state labor tax. Poor households' welfare is almost twice as sensitive to the gasoline price as the welfare of rich households: For example, a temporary increase of the gasoline price from $2 to $3 is equivalent to a permanent labor tax hike of 0.5 percentage points, whereas for poor households it is equivalent to a hike of 0.95 percentage points.

There exists an ample literature about the effects of oil price shocks on output, inflation and the conduct of monetary policy. Extensive reviews of this literature can be found, for example, in Barsky and Kilian (2004) and Hamilton (2003). As emphasized by Edelstein and Kilian (2009) and Barsky and Kilian (2004), there is some evidence that an important channel for the effect of the oil price on output is through aggregate demand and, specifically, consumer spending. Several New Keynesian models have taken this approach by combining a rolre for oil in firm or household demand with price rigidities and fluctuations in the oil price, e.g. Blanchard and Riggi (2013) or Kilian and Vigfusson (2014).

A second strand of the literature tends to use a more reduced-form approach to investigate the reaction of consumer spending on energy-related and other items in response to oil-price shocks. For example, there are many estimates of gasoline demand elasticity in the 1970s and 1980s (see Brons et al. (2008) for a meta-analysis of such studies). More recently, Schmalensee and Stoker (1999) employ semi-parametric methods to estimate gasoline consumption as a function of household characteristics, whereas Edelstein and Kilian (2009) and Hughes et al. (2008) infer the price elasticity of oil demand from aggregate data. Bento et al. (2009) examine the effect of gasoline taxes on vehicle purchase decisions in a dynamic discrete choice model.

This paper adds to the literature by including the direct effect of oil price shocks on household welfare in a dynamic setting. Additionally, it takes into account household heterogeneity in gasoline usage. Finally, because of the welfare considerations, it focuses not on households' absolute gasoline consumption, but rather on the share of gasoline expenditures on total expenditures.

3.2 Household data

In this section I use data from the Consumer Expenditure Survey (CEX) to investigate how gasoline consumption varies with household income. Throughout this section the main focus is going to be on the "gasoline budget", i.e. gasoline expenditures as a share of total household expenditures or of household income in the given period. This means in particular that I do not refer to absolute levels of gasoline consumption unless mentioned explicitly.

The main result is that gasoline consumption on the intensive margin is negatively related to income. This is in contrast to the unconditional household gasoline budget which is a hump-shaped function of income. However, the increase in the gasoline budget at low income levels is driven entirely by the extensive consumption margin, i.e. the decreasing likelihood that households consume zero gasoline: Restricting the sample to households with positive gasoline usage, or households owning a positive number of automobiles yields a negative relationship between income and gasoline budget. I then focus on this intensive-margin relationship between income and gasoline consumption and estimate its linear regression coefficient on the set of households that choose positive gasoline consumption.

There are two main reason why in the structural model below I focus on the gasoline-using households (the "intensive-margin" households). The first is that this abstracts from two discrete margins and simplifies the analysis: Not owning a car is well predicted by low household income and non-employment. This suggests that households are constrained by both their discrete employment status and the non-divisibility of cars. Second, and relatedly, while gas prices may well influence the households discrete decision of owning a car (and hence may affect non-owners' welfare), this channel is hard to identify in the CEX given that car purchase decisions are relatively rare and the dataset's panel dimension is relatively short.

3.2.1 Dataset

The CEX is a rotating panel of households in which households remain up to 4 consecutive quarters. In any given quarter, there are observations for around $6,800$ households, and in this analysis I use the publicly available part of the dataset ranging from 1999 to 2013[1]. The interview data contains information on quarterly expenditures in a large number of categories, of which the main item of interest is expenditures for "Gasoline and Motor Oil". The CEX also contains data for the household's annual income before and after taxes as well as its income rank (among the set of CEX households). The income data is collected only in the first and in the fourth interview, such that for the second and third interview the income data from the first interview is used. There is a considerable number of missing values (around 27%) for the income data.

I construct a measure for gasoline usage (the "gas budget") by dividing quarterly gasoline expenditures by the households total quarterly expenditures, which will be used as the main dependent variable below. Information on household income is mainly used as an independent variable, although I also consider the total expenditures as a proxy for income as a regressor because of the aforementioned concerns about the availability of the income data. Because of the limited number of observations per household along the time dimension, I pool all observations. Identification hence overwhelmingly comes from cross-sectional income differences.

3.2.2 Summary statistics and distribution of gasoline consumption

Figure 3.1 displays density plots of gasoline consumption. Most notably is the mass of observations for which households did not have any gasoline

[1]Data available at http://www.bls.gov/cex/pumdhome.htm

expenditures in the previous quarter. Out of all 446,114 household-quarter observations, zero gasoline expenditures are reported in 46,813 cases or 10.4% of observations.

Figure 3.2 then excludes the 0 observations and replots both gasoline budgets as well as their log-transforms. Somewhat interestingly, not only is gasoline consumption in gallons considerably right-skewed, so are the shares of income and total expenditures spent on gasoline. The log transforms of the budget shares are distributed more symmetrically.

This can also be observed in table 3.1 which lists conditional and unconditional summary statistics for the two gasoline budgets: For the log-transformed gasoline budgets, mean and median are very close.

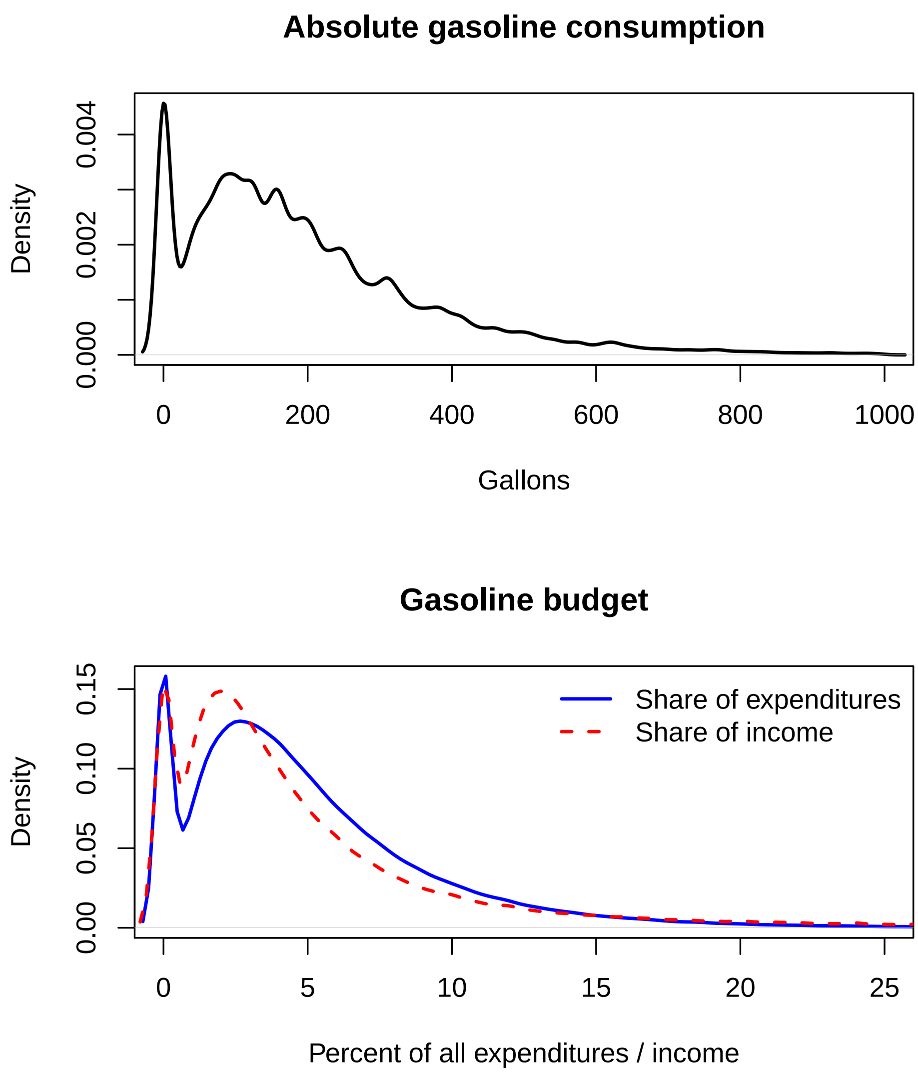
Table 3.1: Summary statistics for gasoline consumption

Statistic	N	Mean	St. Dev.	Median
Consumption, in Gal	446,114	202.3	195.6	157.3
" (positive usage)	399,301	226.0	193.4	177.2
Expenditure budget, in pct	446,025	5.0	4.6	4.0
" (positive usage)	399,281	5.6	4.5	4.4
" (log-transformed)	399,281	−3.2	0.8	−3.1
Income budget, in pct	324,902	6.5	11.0	3.4
" (positive usage)	291,268	7.3	11.4	3.9
" (log-transformed)	291,268	−3.2	1.0	−3.2

3.2.3 Conditional nonparametric bivariate relationships

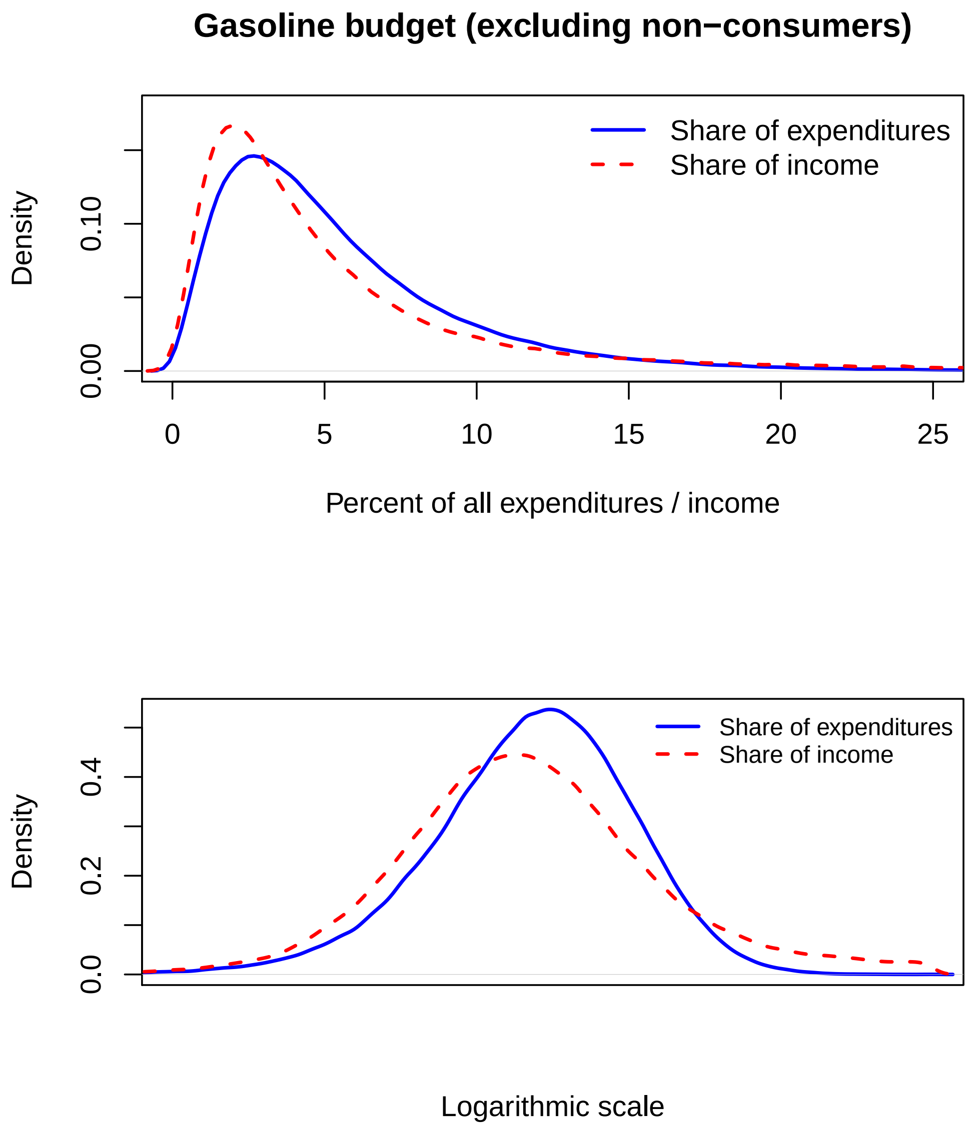
In this subsection I display the (bivariate) relationship of the gasoline budget with household income and a few other variables. As mentioned in

Figure 3.1: Distribution of gasoline consumption and expenditures



Density estimates of household gasoline consumption and gasoline budget shares pooled across households and quarters. Top panel: Approximate absolute gasoline consumption derived as quarterly gasoline expenditures divided by gasoline price. Bottom panel: Gasoline budgets derived as quarterly gasoline expenditures divided by quarterly total expenditures and divided by a quarter of annual income, respectively.

Figure 3.2: Distribution conditional on positive usage



Gasoline expenditures as share of total expenditures and income and their log transforms, respectively.

the data section 3.2.1 in the analysis I mainly focus on gas expenditures as a share of total expenditures rather than as a share of income because total expenditures are observed every quarter and income only annually. Additionally it allows us to directly focus on how the household splits up its consumption basket between gasoline and other items.[2]

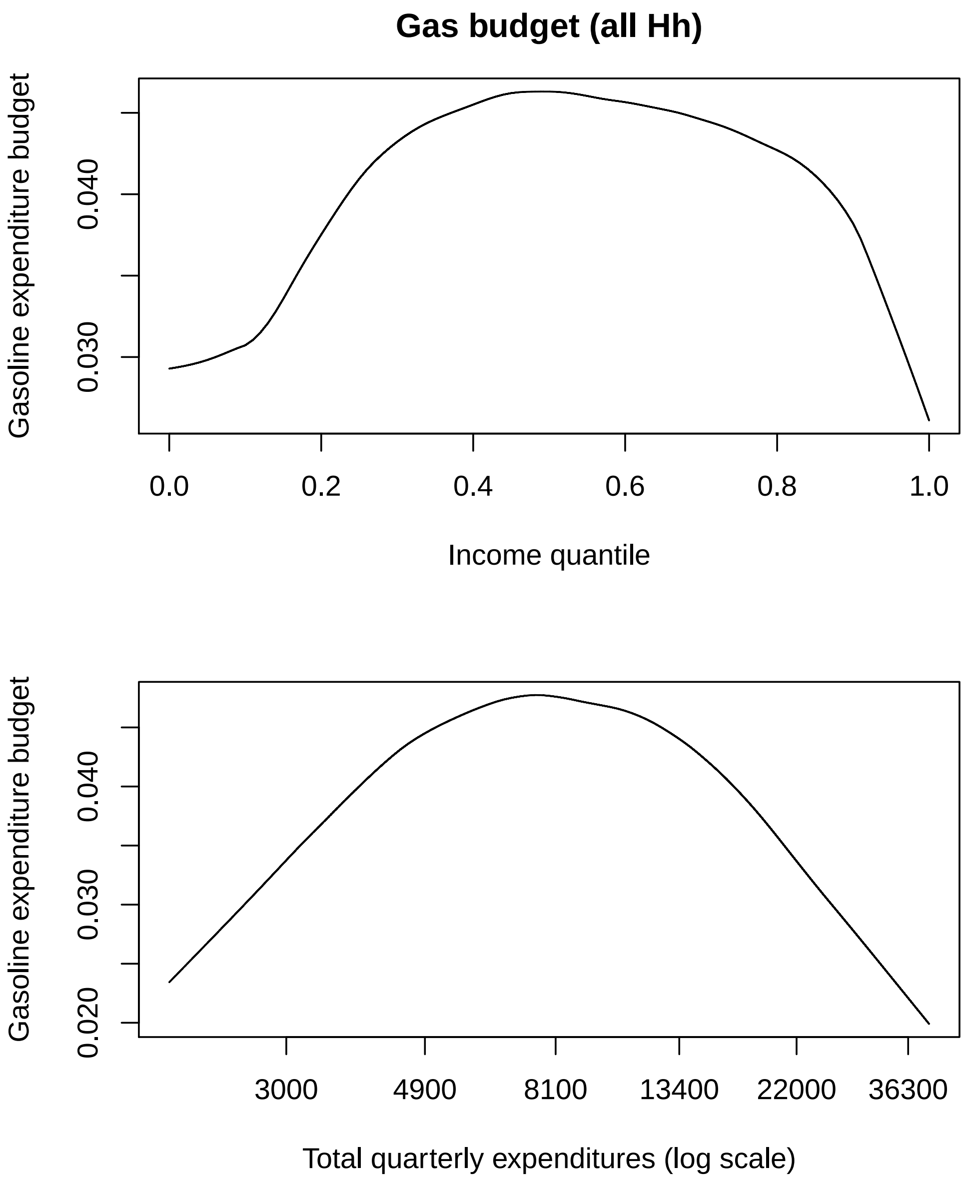
Figure 3.3 displays a nonparametric relationship between the gasoline budget along the y-axis and income quantile and total expenditures, respectively, along the x-axes. Most notable is the graphs' hump shape: For low levels of income, the conditional expectation of gas expenditures increases with income. For incomes higher than approximately median or more than around $\$8,000$ of quarterly expenditures, the relationship reverses such that further increasing the income level predicts a lower gasoline budget.

The initial increase in the gas budget for low income levels appears to be driven entirely by the extensive margin, i.e. households that do not to buy any gasoline. Excluding such households, as is done in figure 3.4, leads to a consistently negative relationship in the nonparametric estimation. This is because, as one may expect, non-usage is heavily concentrated among low-income households.

That the incidence of no gasoline expenditures is located in the left tail of the income distribution also becomes evident if one considers several subsamples with higher rates of positive gasoline usage, as is done in figure

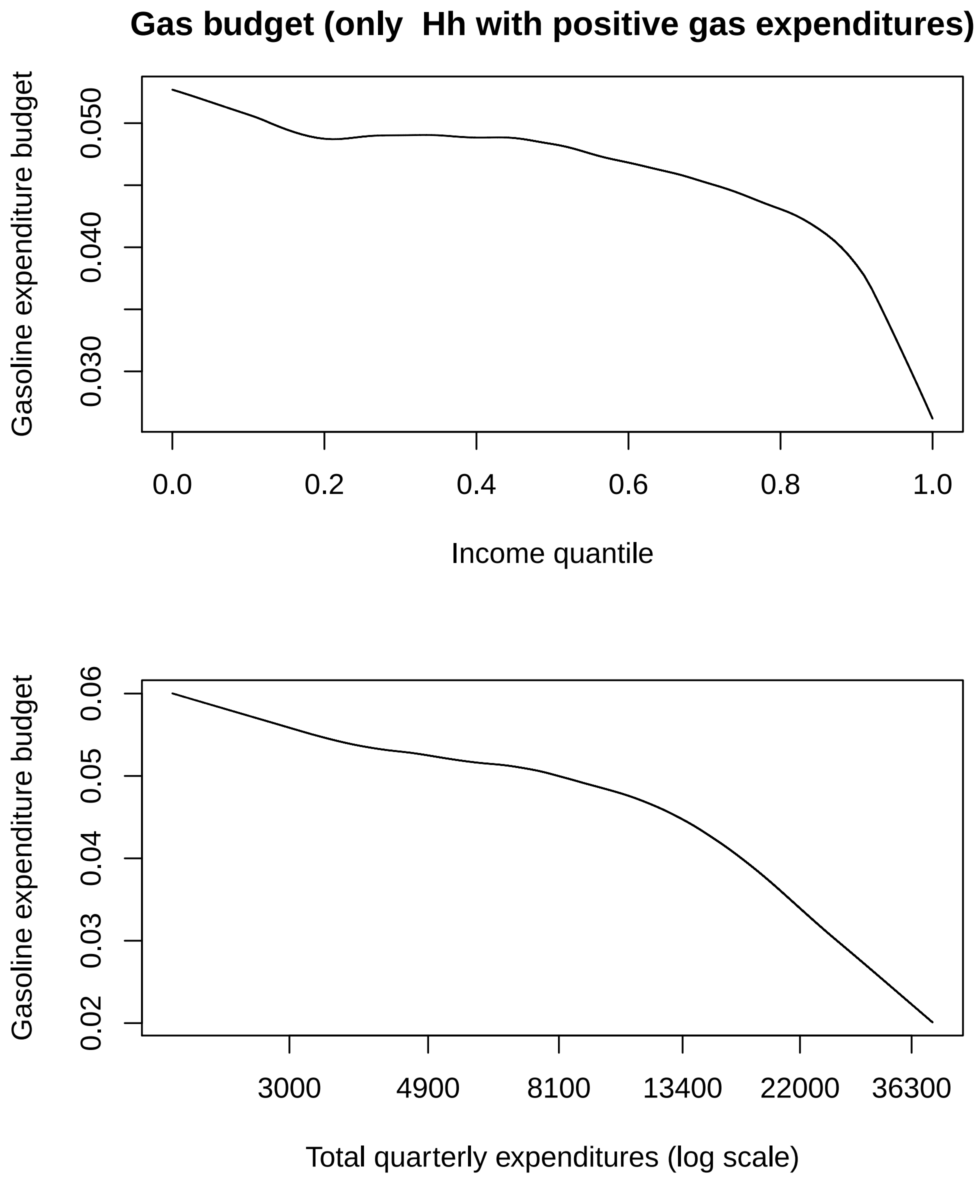
[2]This can reduce bias if households' consumption expenditures on gas and all other items are less volatile over time than their incomes.

Figure 3.3: Gas budget as function of income / total expenditures



Nonparametric estimation of expected gas budget conditional on income quantile (top panel) and total expenditures (bottome panel) using a lowess algorithm.

Figure 3.4: Gas budget conditional on positive usage



Nonparametric estimation of expected gas budget for households with positive usage conditional on income quantile (top panel) and total expenditures (bottome panel) using a lowess algorithm.

3.5. The figure displays the relationship conditional on properties like being of working age, having a positive number of workweeks over the past year, and owning a car, all of which are correlated with gasoline usage. These conditions strongly change the predicted gasoline budget for the lower three income deciles, but little in the right part of the income distribution.

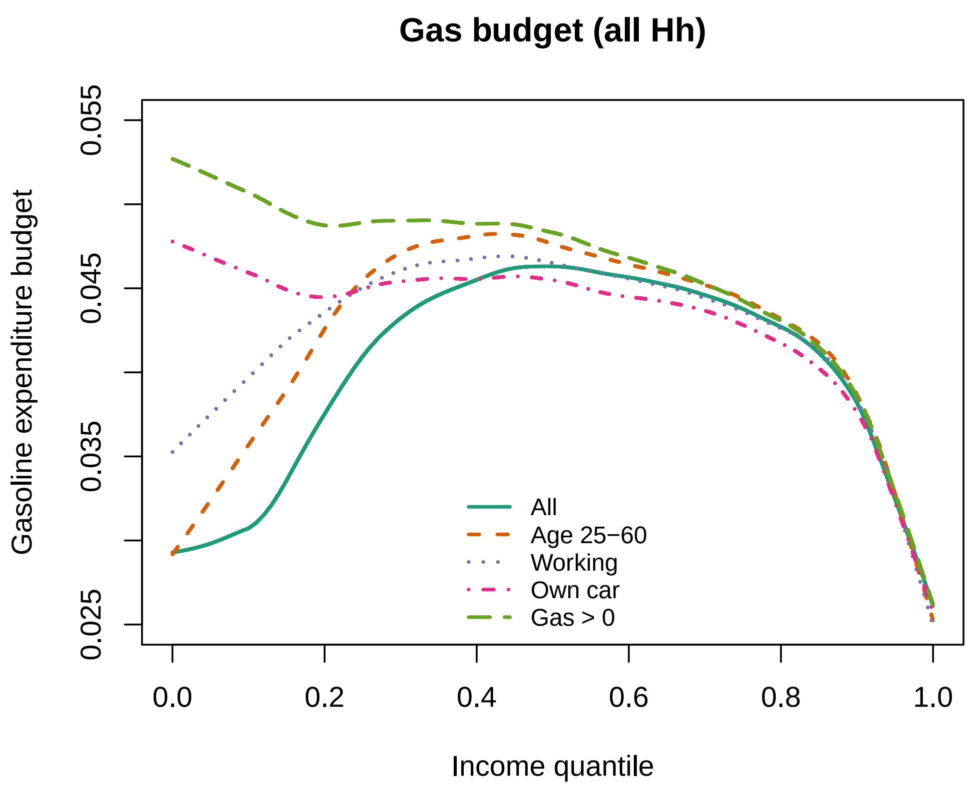
3.2.4 Regression

Focusing on the intensive margin of gasoline usage, I then estimate regression coefficients on the set of observations with strictly positive gasoline expenditures. I consider a number of specifications where income is proxied by income rank, annual income, or total quarterly expenditures, respectively, and where gasoline consumption is measured as the absolute expenditure budget, or its log transform. A number of demographic regressors and a time trend are also included.

Table 3.2 collects these OLS regression results. The controls not shown are a quarterly time dummy, a region dummy (Midwest, Northeast, South, West), a categorical variable for size of the metropolitan area, gender, race, age and education of the reference person, as well as marital status and family size.

For all measures of income and gas budget the relationship after controlling for the demographic characteristics is negative. The first three columns express the effect on the gas budget in absolute terms: For example, as listed in column 1, moving up one decile in the income distribution decreases the gas budget by 0.25 percentage points. Columns (4) and (5) contain the gas

Figure 3.5: Gas budget as function of income / total expenditures



Nonparametric estimation of expected gas budget conditional on income quantile for several subsamples. “All”: full sample (as in figure 3.3, “Age 25-60”: reference person is in that age range, “Working”: reference person worked at least 1 week in the previous year, “Own car”: own at least 1 automobile, “Gas ¿ 0”: positive gas usage (as in figure 3.4).

budget's log transform as dependent variable, so that one decile increase in the income rank is associated with a 4.27% reduction in the gas budget.

3.3 Model

3.3.1 Households

Households purchase gasoline for two purposes: they buy a fixed amount $\bar{E}$ every period in order to commute to work, and a quantity E_{it}^{H} from which they derive utility, for example in the form of going on joyrides or driving a car with higher gas usage. A household i's total gas purchases (in gallons) in period t are therefore $\bar{E} + E_{it}^{H}$. The oil price q_t evolves exogenously according to an autoregressive process given by

$$\log q_t = (1 - \rho_q) \log \tilde{q} + \rho_q \log q_{t-1} + \varepsilon_t^q.$$

The parameter $\tilde{q}$ defines the steady-state level of the gasoline price.

Households are indexed by their type i. Each type i has a weight ω_i and the total mass is normalized to 1. Household types differ in their labor efficiency: One hour of labor supplied by household i yields e_i units of labor services to a firm. Denoting labor supply (in hours) by L_{it}^{H}, effective labor services provided by the household are $e_i L_{it}^{H}$. There is a common labor market for all households in which one unit of labor service is paid a wage w_t. Households have to pay a tax at rate τ^L on their labor income and receive profits π_t from firms. Since there is no saving, the flow budget constraint is

$$\left(1 - \tau^L\right) w_t e_{it} L_t^H + \pi_t = C_{it} + q_t E_{it}^H + q_t \bar{E} \tag{3.1}$$

Table 3.2: Regression results

	Dependent variable:				
	Expenditure Share			log(Expenditure Share)	
	(1)	(2)	(3)	(4)	(5)
Income Rank	-0.0253^{***}			-0.4271^{***}	
	(0.0003)			(0.0060)	
log(Income)		-0.0045^{***}			
		(0.0001)			
log(Expenditures)			-0.0244^{***}		-0.5161^{***}
			(0.0001)		(0.0019)
Constant	0.0681^{***}	0.1011^{***}	0.2638^{***}	-3.0483^{***}	1.1467^{***}
	(0.0019)	(0.0020)	(0.0018)	(0.0341)	(0.0316)
Observations	281,167	298,549	398,139	281,167	398,139
R^2	0.1722	0.1640	0.2308	0.1875	0.2872

Controls not shown included in all regressions: Quarterly time effects, region, metropolitan area, gender, race, age, education, marital status and family size.

where C_{it} is consumption of the output good which serves as the economy's numeraire.

Lifetime utility is given by

$$\max_{C_t, E_t, L_t} E\left[\sum_{t=0}^{\infty} \beta^t \left\{\log\left(\varphi C_t^{\frac{\zeta-1}{\zeta}} + (1-\varphi)\left(E_t^H\right)^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} - \psi \frac{\left(L_t^H\right)^{1+\nu}}{1+\nu}\right\}\right]$$

subject to the sequence of budget constraints (3.1). This utility function specifies that households have a constant elasticity of substitution between consumption of the output good and gasoline consumption. The first-order conditions are

$$\begin{aligned} E_{it}^H &= \left(\frac{1-\varphi}{\varphi}\frac{1}{q_t}\right)^{\zeta} C_{it} \\ C_{it} &= \frac{\left(1-\tau^L\right) w_t e_{it}}{\psi\left(1+\left(\frac{1-\varphi}{\varphi}\right)^{\zeta} q_t^{1-\zeta}\right)\left(L_{it}^H\right)^{\nu}}. \end{aligned}$$

3.3.2 Firms

The economy's production side is very simple: A representative firms produces output goods Y_t according to

$$Y_t = \left(A_t^L L_t\right)^{\alpha}$$

where A_t^L is exogenously evolving labor productivity (which like the oil price will be specified as an AR-1 process), and L_t is total labor services employed by the firm.

The firm maximizes

$$\max_{L_t} \left(A_t^L L_t\right)^{\alpha} - w_t L_t$$

such that the optimality condition requires $\alpha \left(A_t^L\right)^\alpha L_t^{\alpha-1} = w_t$.

3.3.3 Market clearing and equilibrium

There are only 2 markets in the economy: The market for labor and the market for final goods. The market for oil does not clear inside the model, instead there is an infinite supply of oil at the exogenous price q_t (an alternative way to think about this is that households "mine" oil at a constant cost of q_t in their backyards).

The market clearing for final goods is the economy's resource constraint

$$Y_t - q_t \left(\bar{E} + \sum_i \omega_i E_{it}^{HH}\right) = \sum_i \omega_i C_{it},$$

and labor market clearing requires that the household's labor supply in efficiency units corresponds to the firm's labor demand

$$L_t = \sum_i \omega_i L_{it}^H.$$

An equilibrium consists of plans for the endogenous quantities $\{C_{it}, E_{it}^H, L_{it}^s, L_t\}$, the market clearing real wage $\{w_t\}$, fixed processes for the exogenous state variables $\{A_t^L, q_t\}$ such that optimality and market clearing conditions are satisfied. This is summarized by the system of equations

$$\begin{aligned} w_t &= \alpha \left(A_t^L\right)^\alpha L_t^{\alpha-1} \\ q_t^\zeta &= \left(\frac{1-\varphi}{\varphi}\right)^\zeta \frac{C_{it}}{E_{it}^H} \\ \psi\left(1+\varphi^\zeta (1-\varphi)^\zeta q_t^{1-\zeta}\right) \left(L_{it}^H\right)^\nu C_{it} &= \left(1-\tau^L\right) w_t e_{it} \\ \left(1-\tau^L\right) w_t e_{it} L_{it}^H + \omega_i \left(\left(A_t^L L_t\right)^\alpha - w_t L_t\right) &= C_{it} + q_t E_{it}^H + q_t \bar{E} \\ L_t &= \sum_i e_{it}\omega_i L_{it}^H, \end{aligned}$$

and the exogenous block

$$\begin{aligned} \log A_t^L &= \rho_L \log A_{t-1}^L + \varepsilon_t^L \\ \log q_t &= (1-\rho_q)\log \tilde{q} + \rho_q \log q_{t-1} + \varepsilon_t^q. \end{aligned}$$

3.3.4 Calibration and results

Calibration

To analyze how welfare effects of oil price shocks vary with income I split the household sector in two groups: “rich” and “poor” households with equal mass $\omega_r = \omega_p = 0.5$. I calibrate their respective labor efficiencies to match the relative total consumption expenditures observed in the data and setting mean labor efficiency to 1. Since in the CEX households in the top half of the income distribution have mean quarterly expenditures that are a bit more than twice as high as the expenditures of households in the bottom half ($\$15,800$ vs. $\$7,300$) which implies values of approximately $e_r = 1.37$ and $e_p = 0.63$. Table 3.3 collects these and all other parameter values.

Several parameters are set to standard values. The elasticity of production with respect to labor is $\alpha = 2/3$. Households discount the future at a quarterly rate of $\beta = 0.99$ and have a Frisch elasticity of labor supply of $1/\nu = 2$. The households' utility coefficient on labor is normalized to $\phi = 1$. The labor efficiency process has a persistence of $\rho_L = 0.9$ and the standard deviation of its innovation $\varepsilon^L = 0.012$ is set to match the long-run variance of detrended US GDP of 1.7%.

The labor tax rate $\tau^L = 0.2$ roughly targets total receipts of income tax and payroll tax in the US as a share of total labor income. For the exogenous process describing the gas price, running an AR-1 estimation on quarterly retail gasoline prices from 1976Q1 to 2014Q4 yields a persistence parameter of $\rho_q = 0.95$ and a residual standard deviation of $\varepsilon_t^q = 0.086$.

This leaves the four parameters $\tilde{q}, \bar{E}, \zeta$ and φ governing the household's gasoline consumption. Of these, both the steady state gas price $\tilde{q}$ and the preference weight φ determine the steady-state level of the gas expenditure budget and are not separately identified. I therefore normalize the long-run gas price $\tilde{q}$ to 1. I rely on Kehrig and Ziebarth (2009) who find that consumption and oil usage enter as complements in household preferences to set the elasticity of substitution $\zeta = 0.73$. Finally I calibrate the two remaining parameters $\varphi = 0.961$ and $\bar{E} = 0.02$ to match the mean gas budget (as share of total expenditures) for the top and bottom half of the income distribution in the CEX, in which the observed expenditure budgets are 4.85% and 6.21%, respectively.

Table 3.3: Calibration

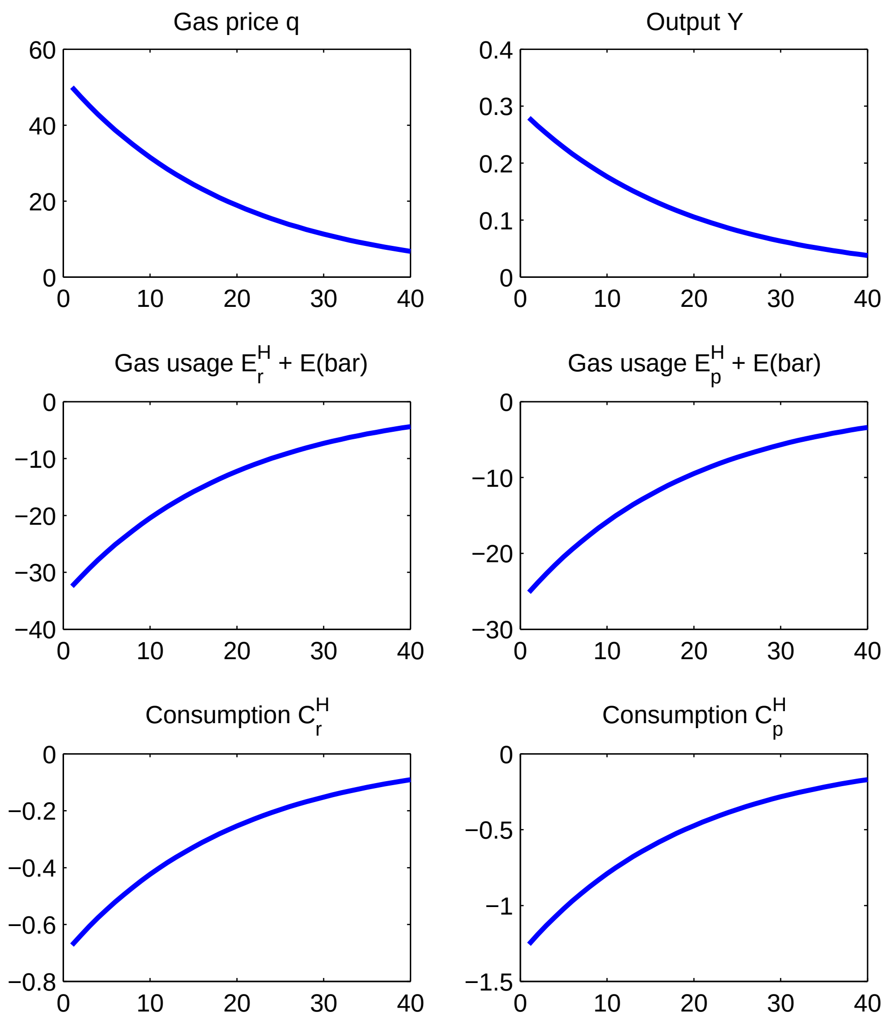
Parameter	Value	Description	Source
α	$\frac{2}{3}$	Production elasticity	Standard value
β	0.99	Hh discount factor	Standard value
ν	$\frac{1}{2}$	Inv. labor elasticity	Standard value
ϕ	1	Labor preference coefficient	Normalization
ζ	0.73	EOS gas / consumption	Kehrig and Ziebarth (2009)
$\tilde{q}$	1	Mean gas price	Normalization
φ	0.961	Preference weight on C_i	Mean gas expenditure share
$\bar{E}$	0.02	Minimum gas purchase	Gas expenditure heterogeneity
ρ_L	0.9	Persistence labor efficiency shock	Output data
$\mathrm{sd}(\varepsilon_t^L)$	0.012	S.d. labor efficiency shock	Output data
ρ_q	0.95	Persistence gas price shock	Gas price data
$\mathrm{sd}(\varepsilon_t^q)$	0.08	S.d. gas price shock	Gas price data
e_r, e_p	1 ± 0.37	Heterogeneity in labor efficiency	Mean consumption expenditures
τ^L	0.2	Labor income tax	Tax receipts / labor income

Result

I simulate the economy up to a first-order approximation around its non-stochastic steady state. Figure 3.6 displays the impulse response functions in percentage deviations from steady state after a significant gas price shock of 50%, which corresponds approximately to an increase of the gasoline price from \$2 to \$3 per gallon. Since in the model oil has no role in the production function, there is a small positive impact on output because households partially compensate the reduction in their budget set by working longer hours. Total gas usage decreases for both households, but more so in relative terms for the rich since the fixed part $\bar{E}$ constitutes a smaller share of their gasoline budget. Conversely, the relatively large share of $\bar{E}$ in the poor household's income means that the poor type has to reduce their consumption C_p to a relatively larger extent.

To evaluate the welfare loss for the households quantitatively one can run a counterfactual experiment in which, instead of the one-time oil price shock, steady-state labor taxes are raised by an amount that makes the household indifferent between the higher tax rate and the oil price shock. The rich type suffers a welfare loss equivalent to a steady-state tax increase of 0.01 percentage points with an oil price shock of 1%, while the poor type suffers a loss equivalent to a tax hike of 0.019 percentage points. In the example this means that, in order to avoid an increase in the gas price from \$2 to \$3, rich households would tolerate a tax increase from 20% to 20.5%, and poor households from 20% to 20.95%, even though households know that in the long run the gasoline price

Figure 3.6: IRF to gas price shock



One-time shock to ε_t^q in period 1. Y

Table 3.4: Permanent labor tax hike equivalent with one-time 50% oil price shock

Household type	Equivalent tax increase
High-income	0.5% (from 20% to 20.5%)
Low-income	0.95% (from 20% to 20.95%)

returns to its steady-state value.

3.4 Summary and future work

The model of household gasoline consumption outlined above mirrors the empirical finding that high-income households allocate a lower share of their budget to gasoline consumption. Calibrated to the CEX data on household income and expenditures, the model predicts that oil price shocks have direct effects on welfare that are almost twice as large for households in the lower half of the income distribution as for those in the upper half. For example, a one-time increase in the gas price from \$2 to \$3 would be equivalent with a permanent tax increase of 0.5% for high-income households but equivalent with a 0.95% tax hike for low-income households.

There are several possible ways for further research. First, this paper currently does not take advantage of the time dimension in the dataset. On an aggregate level one could exploit the fluctuation in the oil price over time, whereas on the household level one could make use of the (albeit short) panel dimension to better control for household heterogeneity. Second, while the model has focused on the direct effects of the oil price on household welfare through the gas price, one could follow the large literature that considers a role for oil in production, and include indirect effects through labor demand. These indirect effects could potentially have further significant distributional impacts.

Appendix

Appendix 1

1.1 Aggregator's demand function

The aggregator's problem is to

$$\max_{\{y_i\}_{i=0}^{1}} \left[\int b_i^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} + \lambda\left[I - \int p_i y_i di\right] + \int \mu_i \left[\bar{y} - y_i\right] di$$

such that the first-order necessary conditions with respect to y_i are given by

$$\left[\int b_i^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{1}{\sigma-1}} \left(\frac{b_i}{y_i}\right)^{\frac{1}{\sigma}} = \lambda p_i + \mu_i y_i \qquad \forall i.$$

For any given variety i either we have to consider two cases. If the aggregator is unconstrained in this variety, i.e. $\mu_i = 0$, then

$$\lambda = \left[\int b_i^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{1}{\sigma-1}} (b_j/y_j)^{\frac{1}{\sigma}},$$

whereas the aggregator is limited to purchasing $\bar{y}$ of variety i if

$$\mu_i = \left[\int b_i^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{1}{\sigma-1}} \left(\frac{b_i}{y_i}\right)^{\frac{1}{\sigma}} - \lambda p_i > 0$$

.

For any two varieties i, j with $\mu_i = \mu_j = 0$ then the relationship

$$\frac{y_i}{y_j} = \frac{b_i}{b_j}\left(\frac{p_j}{p_i}\right)^{\sigma}$$

holds. Integrating over all i one then has

$$I = \int_0^1 p_i y_i di = \left(\int_{i \in U} p_i^{1-\sigma} b_i di \right) \frac{y_j}{b_j} p_j^\sigma + \int_{i \in C} p_i \bar{y} di$$
$$= P_U^{1-\sigma} y_j \frac{p_j^\sigma}{b_j} + \int_{i \in C} p_i \bar{y},$$

where $U \equiv \{i : \mu_i = 0\}$ and $C \equiv \{i : \mu_i > 0\}$ are index sets over unconstrained and constrained varieties, respectively, and $P_U \equiv \left(\int_{i \in U} p_i^{1-\sigma} b_i di \right)^{\frac{1}{1-\sigma}}$ is a price index over unconstrained varities.

Demand for an unconstrained variety j is then given by

$$y_j = b_j \frac{\left(I - \int_{i \in C} p_i \bar{y} di \right) P_U^{\sigma-1}}{p_j^\sigma}$$
$$= b_j \frac{I_U P_U^{\sigma-1}}{p_j^\sigma},$$

where $I_U \equiv I - \int_{i \in C} p_i \bar{y} di$ are the aggregator's expenses over unconstrained varieties.

1.2 Equilibrium conditions

First-order conditions for p_i and k_i:

$$
\begin{aligned}
& E_S\left[\xi\left(\Pi_t^{\text{ppi}}-1\right)\Pi_t^{\text{ppi}}+y_t^s\overline{rp}_t\left(\sigma-1\right)\int_0^{\bar{b}_t}\frac{b}{\bar{b}_t}df\left(b\right)\right] \\
& = E_S\left[y_t^s\overline{rp}_t\left\{1-F\left(\bar{b}_t\right)+\sigma\int_0^{\bar{b}_t}\left(\frac{b}{\bar{b}_t}\right)^{\frac{\psi}{\alpha+\psi(1-\alpha)}}df\left(b\right)\right\}+\right. \\
& \qquad \left.\xi\left(\Pi_{t+1}^{\text{ppi}}-1\right)\Pi_{t+1}^{\text{ppi}}\right]
\end{aligned}
$$

$$
\begin{aligned}
R_t-(1-\delta) = E_S & \left[\frac{\alpha\left(\psi-1\right)}{\psi}\overline{rp}_t\frac{y_t^s}{k_t}\times\right. \\
& \left.\left\{\left[1-F\left(\bar{b}_t\right)\right]+\int_0^{\bar{b}_t}\left(\frac{b}{\bar{b}_t}\right)^{\frac{\psi}{\alpha+\psi(1-\alpha)}}df\left(b\right)\right\}\right]
\end{aligned}
$$

Firm supply y^s:

$$
y_t^s=\left(\frac{\alpha}{\chi}\right)^{\frac{1}{\psi-1}}\left(\frac{1-\alpha}{w_t}\right)^{\frac{\psi(1-\alpha)}{\alpha(\psi-1)}}\overline{rp}_t^{\frac{\alpha+\psi(1-\alpha)}{\alpha(\psi-1)}}k_t
$$

Aggregate supply and factor demands from firms:

$$
\begin{aligned}
Y_t &= \bar{b}_t^{\frac{1}{\sigma-1}}y_t^s\left\{\left[\int_0^{\bar{b}_t}\frac{b}{\bar{b}_t}df\left(b\right)+\int_{\bar{b}_t}^{\infty}\left(\frac{b}{\bar{b}_t}\right)^{\frac{1}{\sigma}}df\left(b\right)\right]\right\}^{\frac{\sigma}{\sigma-1}} \\
L_t^d &= \frac{1-\alpha}{w_t}\overline{rp}_t y_t^s\left(\int_0^{\bar{b}_t}\left(\frac{b}{\bar{b}_t}\right)^{\frac{\psi}{\alpha+\psi(1-\alpha)}}df\left(b\right)+\left[1-F\left(\bar{b}_t\right)\right]\right) \\
CU_t &= \frac{\alpha}{\psi}\overline{rp}_t y_t^s\left(\int_0^{\bar{b}_t}\left(\frac{b}{\bar{b}_t}\right)^{\frac{\psi}{\alpha+\psi(1-\alpha)}}df\left(b\right)+\left[1-F\left(\bar{b}_t\right)\right]\right)
\end{aligned}
$$

Household optimality conditions (Euler equation, no-arbitrage, labor supply):

$$\frac{1}{C_t} = \beta \mathcal{R}_t E\left[\frac{1}{C_{t+1}\Pi_{t+1}}\right]$$
$$\mathcal{R}_t E\left[\frac{1}{C_{t+1}\Pi_{t+1}}\right] = E\left[\frac{R_t}{C_{t+1}}\right]$$
$$w_t = \varphi_t L_t^{\varepsilon} C_t^{\tau}$$

Definition of producer price inflation:

$$\Pi_t^{\mathrm{ppi}} = \Pi_t \frac{\overline{rp}_t}{\overline{rp}_{t-1}}$$

Market clearing conditions:

$$k_t = K_t$$

$$L_t^d = L_t$$

Taylor rule:

$$\log\left(\mathcal{R}_t\right) = \log\left(1/\beta\right) + CB_{rf}\log\left(\Pi_t\right)$$

Aggregate resource constraint:

$$Y_t = C_t + CU_t + \frac{\xi}{2}\left(\Pi_t^{\mathrm{ppi}} - 1\right)^2 + \left[K_{t+1} - (1-\delta)\,K_t\right] + G_t$$

Aggregator's zero-profit condition $I_t = \mathcal{P}_t Y_t$:

$$Y_t = \overline{rp}_t y_t^s \left(\frac{\int_0^{\bar{b}_t} bdf(b)}{\bar{b}_t} + \left[1 - F\left(\bar{b}_t\right)\right]\right)$$

1.3 Variance of firm profitability

A firm's profitability is given as

$$\begin{aligned}
p_i SR_i &= \frac{p_i y_i}{k_i^{\alpha} l_i^{1-\alpha}} \\
&= p_i \left(\frac{\tilde{k}_i}{k}\right)^{\alpha} \\
&= \frac{p_i^2}{\mathcal{P}} \left(\frac{\min\left\{b_i, \bar{b}\right\}}{\bar{b}}\right)^{\frac{\alpha}{2-\alpha}} \left(\frac{\alpha}{2}\frac{1}{\chi}\right)^{\alpha} \left(\frac{1-\alpha}{w}\right)^{1-\alpha}.
\end{aligned}$$

Since all firms set the same price $p_i = p$, it follows for the variance of log profitability

$$\begin{aligned}
\mathrm{Var}\left(\log(p_i SR_i)\right) = \mathrm{Var}\Bigg(& \frac{\alpha}{2-\alpha}\log\left(\min\left\{b_i, \bar{b}\right\}\right) + \\
& \log\left[\left(\frac{1}{\bar{b}}\right)^{\frac{\alpha}{2-\alpha}} \frac{p^2}{\mathcal{P}} \left(\frac{\alpha}{2}\frac{1}{\chi}\right)^{\alpha} \left(\frac{1-\alpha}{w}\right)^{1-\alpha}\right]\Bigg) \\
= \left(\frac{\alpha}{2-\alpha}\right)^2 & \mathrm{Var}\left(\log\left(\min\left\{b_i, \bar{b}\right\}\right)\right).
\end{aligned}$$

Appendix 2

2.1 Derivation of the firm-specific bond price

As mentioned in section 2.2.4 firm revenue net of wages is given by

$$\pi(z,k,\Sigma) = \frac{1-\alpha+\alpha\sigma}{\sigma}\left\{\left[\frac{(\sigma-1)(1-\alpha)}{\sigma w}\right]^{(\sigma-1)(1-\alpha)}(Azk^{\alpha})^{\sigma-1}Y\right\}^{\frac{1}{1-\alpha+\alpha\sigma}}$$
$$C(w)\left[(Azk^{\alpha})^{\sigma-1}Y\right]^{\frac{1}{1-\alpha+\alpha\sigma}}.$$

Then the firm's assets after production in any period are $\pi(z,k,A)+(1-\delta)k$ whereas its liabilities consist of debt b carried over from last period (although b could theoretically be negative if the firm decided to save). Consequently net worth is $n=\pi(z,k,A)+(1-\delta)k-b$. The assumption behind the friction is that for exogenous institutional reasons there is a lower bound of net worth $\bar{n}$ which is not enforceable for repayment — so if the firm's net worth would fall below $\bar{n}$ it partially or fully defaults instead.

Given capital, debt and aggregate state this implies a cutoff $\bar{z}$ for the level of idiosyncratic productivity that triggers default, implied by $\bar{n} = (1-\delta)k' + C(w)\left[(A\bar{z}k^{\alpha})^{\sigma-1}Y\right]^{\frac{1}{1-\alpha+\alpha\sigma}} - b$. Solving for $\bar{z}$ one has

$$\bar{z} = \left(\frac{\bar{n}+b-(1-\delta)k'}{C(w)}\right)^{\frac{1-\alpha+\alpha\sigma}{\sigma-1}}(Ak^{\alpha})^{-1}Y^{-\frac{1}{\sigma-1}}.$$

The autoregressive nature of z in turn determines a cutoff value $\bar{\epsilon}$ for the lognormally distributed shock ϵ given as

$$\rho_z \log z_{-1} + \log \bar{\epsilon} \equiv \log \bar{z}$$
$$\bar{\epsilon} = e^{\log \bar{z} - \rho_z \log z_{-1}},$$

determining the likelihood of default.

The second piece of information needed to determine the risk premium is the fraction of the loan that is recoverable in case of default (i.e. the recovery rate). In default, the lender can claim all but the minimum level $\bar{n}$ of the borrower's assets so that the actual repayment $\bar{b}$ is defined as

$$\bar{b}(z,k,\Sigma) \equiv \max\left\{(1-\delta)k + C(w)\left[(Azk^{\alpha})^{\sigma-1}Y\right]^{\frac{1}{1-\alpha+\alpha\sigma}} - \bar{n}, 0\right\}.$$

Repayments are bounded below by zero. This matters only for the rare case of total default, i.e. a realization of z which is so low that $(1-\delta)k + \pi(z,k,A) < \bar{n}$. This inequality implies a second cutoff value defining the threshold for total default

$$\underline{\epsilon} = \left(\frac{\bar{n} - (1-\delta)k}{C(w)}\right)^{\frac{1-\alpha+\alpha\sigma}{\sigma-1}} \left[Az^{\rho_z}k^{\alpha}\right]^{-1} Y^{-\frac{1}{\sigma-1}}.$$

There is a default cost χb amounting to a fraction χ of the original loan. So the recovery rate $\tilde{R}(z,k,b,\Sigma)$ can be defined as

$$\tilde{R}(z,k,b,\Sigma) \equiv \frac{\bar{b}(z,k)}{b} - \chi.$$

The price of the bond, then, makes the lender indifferent between lending to a firm that chooses k' and b' and is in state z today, and lending at the risk-free interest rate R.

$$\begin{aligned}
q\left(z,k',b',\Sigma\right) =& \frac{1}{R}E_A\left[1+\int_{\epsilon'<\bar{\epsilon}}\tilde{R}\left(z'\left(\epsilon'\right),\left(1-\delta\right)k',b',\Sigma\right)-1\mathrm{d}F\left(\epsilon'\right)\right]\\
=& \frac{1}{R}E_A\Bigg[1-F\left(\bar{\epsilon}\right)\left(1+\chi\right)\\
&+\int_{\underline{\epsilon}<\epsilon'<\bar{\epsilon}}\frac{\left(1-\delta\right)k'+C\left(w'\right)\left[\left(A'z'k'^{\alpha}\right)^{\sigma-1}Y'\right]^{\frac{1}{1-\alpha+\alpha\sigma}}-\bar{n}}{b'}\mathrm{d}F\left(\epsilon'\right)\Bigg]\\
=& \frac{1}{R}E_A\Bigg[1-F\left(\bar{\epsilon}\right)\left(1+\chi\right)+\left[F\left(\bar{\epsilon}\right)-F\left(\underline{\epsilon}\right)\right]\left[\frac{\left(1-\delta\right)k-\bar{n}}{b'}\right]\\
&+z^{\frac{\rho_z\left(\sigma-1\right)}{1-\alpha+\alpha\sigma}}\frac{C\left(w'\right)\left[\left(A'k'^{\alpha}\right)^{\sigma-1}Y'\right]^{\frac{1}{1-\alpha+\alpha\sigma}}}{b'}\int_{\underline{\epsilon}<\epsilon'<\bar{\epsilon}}\epsilon'^{\frac{\sigma-1}{1-\alpha+\alpha\sigma}}\mathrm{d}F\left(\epsilon'\right)\Bigg],
\end{aligned}$$

from which equation (2.3) follows with $\nu \equiv \epsilon^{\frac{\sigma-1}{1-\alpha+\alpha\sigma}}$ and $\bar{\nu}$ and $\underline{\nu}$ defined correspondingly.

2.2 Outline of numerical model solution

Using Khan and Thomas (2008)'s approach of normalizing the price of output with the household's marginal utility of consumption define $P \equiv u'\left(C\right)$. With the household's discount factor given as $d(\Sigma',\Sigma) \equiv \beta u'\left(C'\right)/u'\left(C\right)$ equations (2.4)-(2.6) can be rewritten as

$$v\left(z,k,b,\Sigma\right) = \max_{\mathbb{1}_{\text{adj}}} \mathbb{1}_{\text{adj}}\left[v_a\left(z,-\phi+\tilde{n},\Sigma\right)\right]+\left(1-\mathbb{1}_{\text{adj}}\right)v_n\left(z,k,b,\Sigma\right), \tag{2.1}$$

$$v_a\left(z,\tilde{n},\Sigma\right) = \max_{k',b'} P\left[\tilde{n}+q\left(z,k',b',\Sigma\right)b'-k'-\phi\right]+\gamma\beta E_{\Sigma}\left[v\left(z',k',b',\Sigma'\right)\right], \tag{2.2}$$

$$v_n\left(z,k,b,\Sigma\right) = \max_{b'} P\left[\tilde{n}+q\left(z,k,b',\Sigma\right)b'-k\right]+\gamma\beta E_{\Sigma}\left[v\left(z',k,b',\Sigma'\right)\right]. \tag{2.3}$$

As outlined in section 2.4.1, the distribution μ is approximated using a grid over values for aggregate capital K. The algorithm then proceeds as follows:

1. Guess an initial set of log-linear functions $K'(A, K)$, $C(A, K)$, $Y(A, K)$ and $w(A, K)$ which can be represented by their coefficients. Agents use these functions to forecast aggregate variables given the aggregate state.

2. Given the approximating functions the remaining aggregate variables $P(A, K)$ and $R(A, K)$ can be computed as functions of the aggregate state.

3. Derive the firms' value functions (2.1)-(2.3) and associated policy functions by value function iteration.

4. Simulate the economy for a large number of periods using the firms' policy functions. To this purpose a sequence of aggregate TFP A is drawn at random for the first simulation and held constant throughout all following iterations. The discretized steady state distribution is simulated forward using the policy functions for next period's endogenous state variables k' and b' and the stochastic transition rule for z'. The first few hundred periods are being discarded, and the aggregate variables of the remaining simulated periods are stored.

5. Regress the stored values for K', C, Y and w from the simulation onto A and K to obtain new estimates for the coefficients of the log-linear

relationships from 1. If the new estimates and the previously used coefficients are close, stop. Otherwise update the coefficients by using a convex combination of the previous ones and the new estimates, and return to 2.

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