MODELLING ELECTRICITY LOAD SCHEDULING AND RETAILER DECISION-MAKING

by

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Summary

Electricity is critical to the economic and social development of humanity. Significant effort has been spent on the effective management thereof and with the growth of the renewable energy sector, traditionally regulated markets are no longer sufficient. This has resulted in privatization of the sector over the last three decades, and has largely been met with success internationally. South Africa however, continues to suffer rolling black-outs and rising energy costs. Many attribute this to the closed system under which the country operates.

In order for there to be sufficient buy-in for policy-change, key stakeholders such as the consumer and retailer must be made aware of the new reality under which they would operate, the factors that would affect their interests, and the extent to which they would be affected. Furthermore, the conflicting objectives of these parties must also be addressed through their simultaneous achievement, taken to be social welfare.

This dissertation satisfied these aims by creating an accurate depiction of the locally unique consumer and retailer's realities through the development of operations research models. The resident's problem was modelled as a load scheduling one which considered the vastly divergent socio-economic status of South Africans and how this affects their energy consumption patterns. The spot market dynamics that a retailer is confronted with was modelled as a three-regime Markov switching model. Because social welfare was the overwhelming interest of the study, a novel problem formulation was proposed to combine the resident's interests in reducing bill payments and inconvenience levels, and the utility's interest in increasing revenues.

The developed models and problem formulation were applied to South African scheduling data for residents operating under a fixed rate tariff. It was found that, under the guidelines of Eskom's pricing boundaries, the relationship of the consumer's price elasticity relative to the retailer was not a linear one. Social welfare was found to be a function of this relationship, and static tariffs that achieved optimal social welfare at varying degrees of relative price elasticity were identified. It was noted that insufficient

research has been conducted on validating the effect of the retail tariff on the resident and utility. Furthermore, this effect varies from one society to the next and is dependent on factors such as consumer attitudes and electricity profit margins.

Time-varying tariffs increased model complexity but are capable of achieving demand response which are believed to broaden the interests of the retailer and consumer. A trail-and-error algorithm was proposed as an appropriate tool for demonstrating the effects of demand responsiveness under a time-of-use (TOU) tariff. This was applied to the South African context with the inclusion of the novel problem formulation.

The novelty of this thesis is four-fold: firstly, a problem formulation that captures social welfare, which has previously not been considered in literature, is proposed. Secondly, the assumption of most works in this field that the effect of retail tariff changes on the consumer and retailer are the same, is disproved. In fact, this relative sensitivity is shown to be far greater for the resident than for the utility. Thirdly, a three-regime Markov switching model is successfully applied to the Australian market with no restrictions on the transition probability matrix. Finally, initial computations for this unique perspective on the problem are conducted with a trial-and-error algorithm and findings will certainly assist in guiding future research.

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LIST OF ACRONYMS

APX	: United Kingdom Power Exchange
AR	: Autoregressive model
BPSO	: Binary Particle Swarm Optimisation
BRICS	: Brazil, Russia, India, China, South Africa
CAES	: Compressed Air Energy System
DLC	: Direct Load Control
DM	: Demand Management
DR	: Demand Response
DSM	: Demand Side Management
EA	: Evolutionary Algorithm
EEX	: European Energy Exchange
EMC	: Energy Management Controller
EMS	: Energy Management System
EPEX	: European Power Exchange
EXAA	: Energy Exchange Austria
GA	: Genetic Algorithm
GCPSO	: Guaranteed Convergence Particle Swarm Optimisation
IEEE	: Institute of Electrical and Electronics Engineers
ISO	: Independent System Operator
MILP	: Mixed Integer Linear Programming Problem

MINLP : Mixed Integer Nonlinear Programming Problem

MIP : Mixed Integer Programming Problem

MOOP : Multi-Objective Optimisation Problem

MRS : Markov Regime Switching Model

NEM : Australian National Electricity Market

NERSA : National Energy Regulator of South Africa

NSW : New South Wales

PAR : Peak-to-Average Ratio

PHEV : Plug-in Hybrid Electric Vehicle

PPX : Polish Power Exchange

PSA : Present-State-of-Art

PSO : Particle Swarm Optimisation

RLC : Remote Load Control

RTP : Real-Time Pricing

SOC : State of Charge

TOU : Time-of-Use

UREM : University of Regina Energy Model

ZAR : South African Rand

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Since its modest beginnings in 600BC, electricity has grown to be an indispensable asset in modern society. It is the backbone for economic, social and infrastructural growth and success in every nation. Electrification has increased comfort and human activity which has resulted in paradigm-shifting inventions such as that of the refrigerator, telephone and computer. Every effort has thus been made to effectively manage this precious resource in order to derive maximum benefit and with this has come the evolution of the energy industry. With the advent of competitive electricity markets over the last thirty years and the gradually increasing contribution of renewable energy to power generation, this sector has grown to be a complex one with multiple stakeholders, conflicting objectives and persisting challenges. At the forefront of these challenges is providing a satisfactory and reliable service to consumers consistently, and many deregulated markets around the world have not only shown great success in achieving this, but have also enjoyed many other benefits in the process.

Open markets were initially established in the fields of aviation, telecommunications and natural gas during the 1970s and 1980s. Soon, they were a common phenomenon amongst advanced industrial economies. Their birth was prompted by the increasing inefficiencies of government regulation and a sense of distrust and risk that agencies under the old regime were manipulating policy for their own benefit at the detriment of the general public. A similar international trend has developed for the energy sector and in countries such as Australia, the United States, Brazil, Russia, and Denmark where the transition had been initiated for similar reasons, significant benefits have resulted. Energy providers have enjoyed the freedom to design their own tariff structures which has bred industry rivalry and competition, lower retail prices and bill payments for the energy consumer have been experienced as a result of this, and system reliability has been enhanced through the modernization of infrastructure and the introduction of smart grids and smart technology (NRG Expert, 2012, Pentland, 2013). One other interesting and slightly more complex benefit has been the growth of new renewable energy and its increasingly significant contribution to power supplies. Prior to deregulation it was well-known that electric power generation produced more pollution than any other single industry in the United States. This pollution was altering global climates and harming ecosystems for future generations. The eco-conscious consumer thus became increasingly interested in how their energy was produced, but had little option as far as choice was concerned. The market restructure has since given small pockets of solar and wind energy producers the platform to trade their product sustainably for economic gain. Furthermore, the transition has resulted in what is considered to be a cyclical effect in that, because renewable energy is also associated with near-negligible generation costs (Ziel et al, 2015), it is often associated with the lowest rate. In a market where consumers get to shop for their electricity amongst a wide range of options, green energy is an obvious choice. With these trends in mind, wind, solar and hydro-electricity have collectively seen a market share of 19% as of 2012 (Renewable Energy Global Status Report, 2014) and this is expected to rise in the coming years for most industrialised countries. However, it makes for a stochastic and intermittent energy source (Zugno et al, 2013), and needs the infrastructural support and management of a competitive market to make a significant and reliable contribution to meeting global needs. The relationship between open markets and green energy is thus clearly symbiotic, and society cannot move in the direction of one without the other.

It has been made clear thus far that deregulation has been adopted by many countries and global trends indicate that it is both necessary and beneficial for the future of energy. The South African electricity industry differs significantly from its international counterparts however primarily because of its unique history. Marquard (2006) identifies the development of the local electricity system to have unfolded in two main phases. In the first phase, electricity was initially generated and distributed at a regional level and based in local authorities with the exception of a private initiative that served the emerging gold-mining sector. Escom, a state utility, was created to integrate regional systems into a national grid that serviced local authorities, industry and mining. In the second phase, a distinct division between the generation, transmission and distribution activities of the industry was observed, with Escom controlling the generation and transmission rights. What enhanced the clout of the utility however was its dominating role in supplying energy to the mining sector and other intensive users, both of which drove the national economy at the time. Distribution was in the hands of local authorities and was thus fundamentally affected by apartheid so that white households were almost entirely electrified whilst black households were left energy-poor. With the transition into democracy, Eskom naturally assumed the right of supply to these black households which ultimately meant the vast majority of the country, solidifying their status as a central player in the local industry. This has however meant a radical increase in demand for the utility, one which many argue they were, and still are, ill-prepared for. The energy crisis of 2008 for example, which entailed a shortage of electricity, led to rolling blackouts, economic losses of R200 million per day (Crowley, 2014a) and numerous retrenchments due to the dwindling profits of large, medium and small enterprises. Furthermore, had the contributions of renewable energy and private generators to the national grid not been capped to 9% by 2030 (which has already been exceeded by other BRICS nations) due to government policy, the deficit of supply would not have been felt so devastatingly, the crisis would have been more manageable and perhaps, may not even have occurred at all (Montmasson-Clair and Ryan, 2014). Indeed, South Africa has a promising future in green energy, ranked third amongst thirty-five nations in its potential to attract capital for low-carbon energy resources, but the necessary tools are currently not in place to nurture its growth or feed its demand (Department of Energy, 2015). Since then, the National Energy Regulator of South Africa (NERSA) has approved tariff increases of between 8-20% per year (Crowley, 2014b). This has been higher than consumer inflation rates in an already struggling economy, with the promise that load-shedding and blackouts would be a thing of the past and that rising energy demands at the time would be met. In this regulated market however, Eskom, the parastatal responsible for serving 95% of South Africa's energy needs, find themselves even worse off than they were six years ago.

Solutions to the current local energy crisis can clearly be sourced from overseas. Although the success of deregulation depends heavily on the country in question, empirical evidence supports its successful implementation in both progressing and developed nations. Furthermore, South Africa proves itself to be a viable and promising candidate due to the failure of the current regulated system, rising energy costs and its impressive potential for green energy. Proposing a radical change such as a shift to deregulation seems both necessary and inevitable for South Africa's energy future, but also requires, firstly, significant buy-in from key stakeholders, and secondly, knowledge of the unchartered. This study thus aims to further this proposition by satisfying these two objectives. Before defining exactly how this will be accomplished however, a more thorough discussion of some key concepts is required. This is presented in Section 1.2. Section 1.3 provides clarity on research objectives and how they fulfil the aims highlighted above, and Section 1.4 outlines the contributions. Section 1.5 concludes the first chapter with a brief outline of the rest of this dissertation.

1.2 KEY CONCEPTS

Energy as a commodity has unique characteristics that must be addressed in its management. For example because it cannot be sustainably stored, its supply and demand must continuously be balanced in real-time. Trends such as seasonality, time-varying volatility, mean reversion and jumps and spikes are also exhibited and must be accounted for. These factors amongst others have prompted the development of different market structures, stakeholders, tariff structures and decision-making horizons. Each of these aspects play a fundamental role in shaping the dynamic of the industry as will become evident throughout the rest of this section.

1.2.1 Market structure

There are two types of market structures that exist for every energy industry: vertically integrated (regulated) and open market (deregulated) systems. In a regulated market the government sets rules that define how an industry and operators in that industry may behave (Abhyankar and Khaparde, 2013). Historically all markets were regulated as it offered a risk-free way to finance the creation of utilities without the added burden of competition and there existed an obligation to serve the people. South Africa still operates under a regulated system where electricity supply is dominated by a state-owned enterprise and a structure of the local industry is given in Figure 1.1. In a deregulated system rules and economic incentives set up by government to control an industry are restructured. This ultimately means that the pricing, generating, distribution and transmission activities of the industry are left unregulated. Like any retail product, electricity is then purchased at a given rate by end-users, but service providers have a number of procurement options at their disposal. Futures or contract trading, in which purchases are made in bulk in advance at an agreed-upon price, reduces settlement risk and the possibility of insufficient supply. Nevertheless, deviations between supply and demand in real-time are guaranteed to occur, usually as a result of fluctuating demand, generation or transmission failures, and to ensure that a balance is maintained, trading is enabled in spot markets in which retailers may bid to purchase or sell electricity back to the grid at some regulating price. Because of the reduced time to delivery in spot markets and the resulting reduced capability to circumvent problems as they arise, significant attention is paid to spot market price predictions (Weron, 2007). The United States, Russia and several Nordic countries shifted toward this structure in recent decades due to technological innovations improving efficiency of supply on a smaller scale, the historically better performance of other privatized industries and the resulting drop in cost when competition is introduced. Figure 1.2 represents the generic structure of an open market system. It is clear that any proposition of this structure must be accompanied by an appropriate representation of trading in these respective markets so that its effect on the retailer may be quantified and analysed.

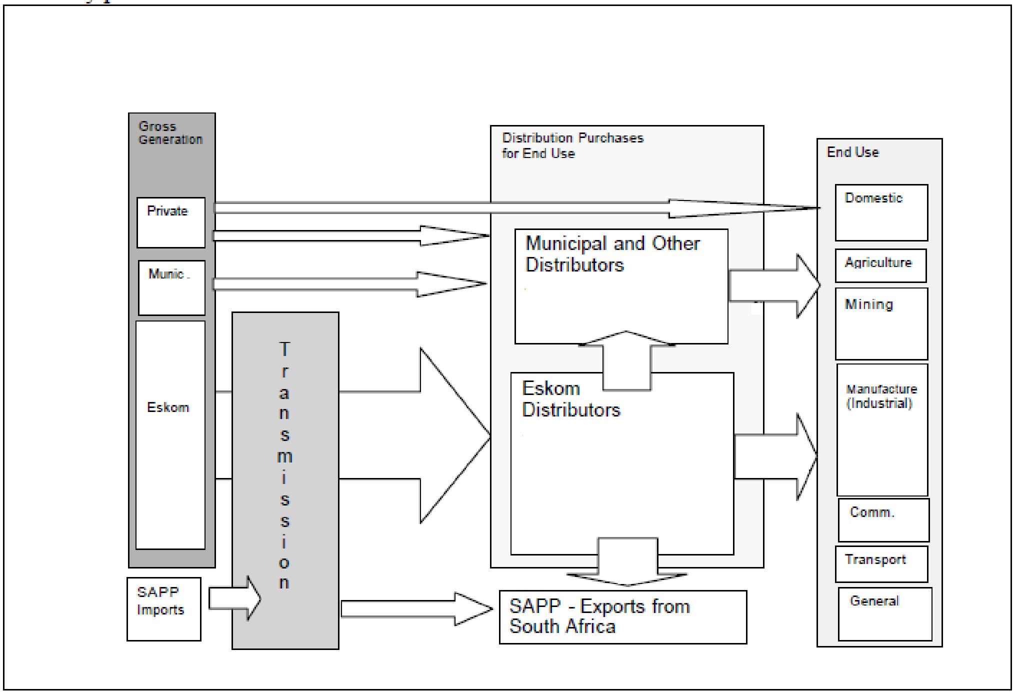
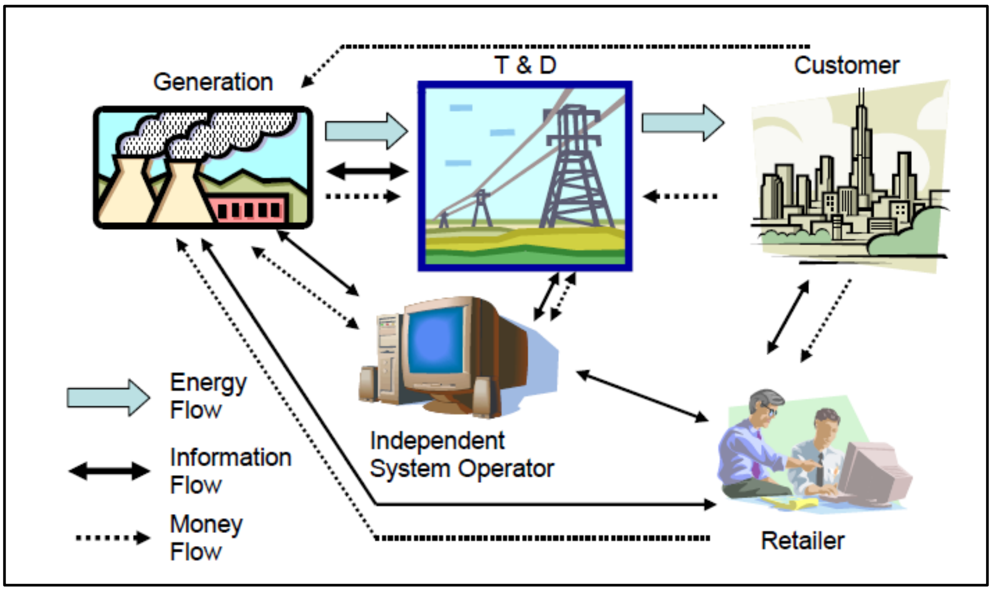


Figure 1.1: The South African Electricity Supply Industry (Department of Minerals and Energy, 2008)



*T&D refers to transmission and distribution activities

Figure 1.2: Deregulated electricity system (Abhyankar and Khaparde, 2013)

1.2.2 Stakeholders

As can be seen in Figure 1.2 several stakeholders exist in the deregulated electricity market, each with their own expectations. The degree to which these objectives are addressed dominates literature surrounding the energy industry. Some of these key objectives have been summarised in Table 1.1. As can be seen, the outcomes of one party often times conflict with those of others in the energy value chain, and this is regardless of the market structure adopted. For example, the recovery of revenue for the generator cannot be maximised without the expense of the retailer, and consumers cannot receive an affordable and predictable tariff without the retailer suffering some procurement risk and profit losses. The resolution of the conflict between the residential user and the retailer requires buy-in and herein lies the focus of this study. Whilst parties accept that a certain loss must be incurred due to industry dynamics, little work has been done in the way of identifying this so-called tipping point, or the space in which two stakeholder's conflicting objectives are satisfied without the excessive deterioration of the other. If this can be found, key stakeholders will be motivated to support change with the knowledge that their interests will not be neglected. This concept will be referred to as social welfare for the remainder of this dissertation and is taken to be the overwhelming interest of two parties whose other objectives are otherwise independently conflicting. Currently, no problem formulation exists in literature that prioritises this over individual stakeholder's interests, and it is of the belief that herein lies the solution to sustainable change.

Table 1.1: Stakeholder objectives

Stakeholder	**Objective**	**Description**
Generator	Revenue recovery	Revenue from sales to the retailer should reflect the full cost to supply electricity and ensure that the industry is economically viable and fundable in the short, medium and long-term
	High generation capacity	Capacity should match consumption needs to ensure greater potential for

		revenue
	Low cost of implementation	Operating costs should be minimised to ensure higher profit margins
Transmission and distribution utility	Low cost of maintenance	Because these utilities are paid for ownership and use of their lines (but not operation), management, engineering and maintenance costs should be minimised to ensure higher profit margins
Independent System Operator (ISO)	Reliable supply	As an independent regulating authority, the ISO is responsible for ensuring the balance of the system and secure supply
Retailer	Revenue recovery	Revenue from sales to the consumer should reflect the full cost to purchase and transfer electricity from the generator to the consumer, and ensure that the industry is economically viable and fundable in the short, medium and long-term
	Low procurement risk	Power supply options (see decision-making horizons) should cumulatively meet consumption requirements in real-time
Consumer/ Resident	Affordable tariff	Price levels should assume an efficient and prudent utility with sufficient thought to consumer welfare and in the case of dynamic schemes, elasticity
	Predictable and stable tariff	Prevent price shocks and keep consumers informed about future trends

1.2.3 Tariff structures

Broadly a tariff structure can be defined as a set of charges levied to a party for supply of a resource. Electricity has been paid by the consumer in the past using a variety of structures that can be classified as fixed, static and dynamic. These classifications have become increasingly important due to its inextricable link to Demand Response (DR), a topic which has garnered significant attention in the energy field of research. DR can be defined as "changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments" (United States Department of Energy, 2006). In a fixed structure a flat-rate is charged to the customer without consideration of variations in supply or demand, seasonality or time-variance. This protects the consumer from price volatility, is simple to manage and implement for the retailer, simpler to respond to for the consumer, and is often times applied in pilot markets not yet mature enough to respond to more advanced schemes. However, it also inhibits the end-user from playing a more active role in DR and responding to price signals accordingly. A subsequent improvement of this structure has been static time-varying pricing methods. These prices are known months in advance and ‘communicate’ with the consumer based on fluctuations in supply or demand. For example, in a time-of-use (TOU) tariff, prices vary based on block periods of the day that represent high (higher tariffs) and low (lower tariffs) periods of demand. Another example is the inclining-block rate tariff which resembles a unit step function. Here, energy consumed above certain quantities is charged at increasing premiums. Currently, Eskom charges TOU and inclining block rate tariffs to its business and residential customers respectively. The primary criticisms of static tariffs have been its inadequacy in capturing variations within block periods and the infrequency of price adjustments. Real-time pricing (RTP) was developed to address these shortcomings and has since been applied extensively in deregulated industries. These tariff structures have no pre-set components but apply retail prices, regulated by an ISO, that translate wholesale prices to the consumer to achieve effective levels of demand response. An example is day-ahead pricing in which rates for a 24-hour period are published a day in advance and reflect prices hourly up to five-minute periods. Any imbalances in supply and demand are then met in the spot market. It is clear that this tariff structure is the most complex to manage and respond to for the consumer, and has thus only been implemented in established markets despite its obvious benefits.

The implications of not achieving DR in a deregulated market can be catastrophic, as was seen in the California energy crisis. In June 2000, the state, due to a mere shortage of 300MW in a grid of 50 000MW, experienced an 800% increase in wholesale prices which filtered down to retailers and consumers. This forced the institution of multiple large-scale blackouts, saw a drastic cut to profit margins in the industry, and resulted in the closure and near-closure of two of the largest energy companies. Clearly, the lack of regulation had a cascading and devastating effect on all parties in the market despite customers receiving price signals which served as warnings of the impending crisis. A post-analysis by Triki and Violi (2009) indicated that price elasticity, defined as "a measure of the effect of a price change or a change in the quantity supplied" (Investopedia, January 2015), was insufficient to trigger changes in consumption patterns for the consumer, resulting in a shortage of supply. Elasticity demonstrates the sensitivity of users to price changes. A higher elasticity means a greater sensitivity and likelihood of changes in behaviour, thereby circumventing the California crisis. According to the findings of Kirschen (2003), Aalami et al (2008) and Corradi et al (2013), effective demand response can only be achieved when price signals reflect a level of change that consumers are sensitive to. For example, if an increase of R0.1/kWh was incurred, it is of interest as to whether this would result in a reduction of residential users' loads so as to save on bill payments. Only then can a deregulated market operate successfully and achieve DR, thereby deriving significant benefits for the consumer and retailer. The sensitivity of consumers must therefore be captured so that the selected tariff is capable of eliciting appropriate changes. Furthermore in the way of achieving social welfare, the price elasticity of consumers relative to the retailer is of critical interest, and has not yet been addressed in literature. It is hypothesised that the effect of a change such as, for example R1/kWh, has vastly different impacts on each stakeholder's outcomes. This must first be tested then quantified by an appropriate problem formulation that represents social welfare instead of favouring one party over another.

1.2.4 Decision-making horizon

The sequence of decisions and availability of information for generators, retailers and customers in making purchasing and selling decisions is considered in the decision-making horizon. Figure 1.3 depicts the sequence of decisions made by a retailer in procuring and subsequently selling electricity to consumers. Note that in the event of demand exceeding the quantity purchased, the market is said to be up-regulated and relationships between the day-ahead market and this regulating price are often dependent on the country in question. Similarly, in the event of demand being lower than the quantity purchased, the market is said to be down-regulated. Figure 1.4 extends these concepts to a decision framework spanning one month.

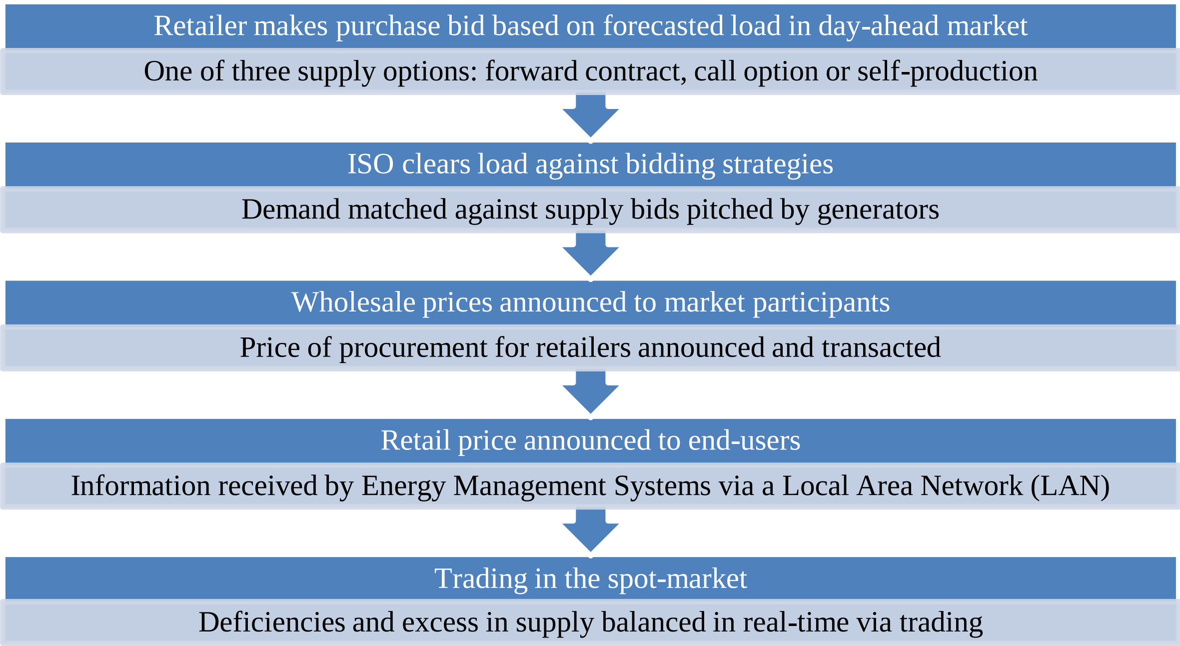


Figure 1.3: Retailer's decision-making framework

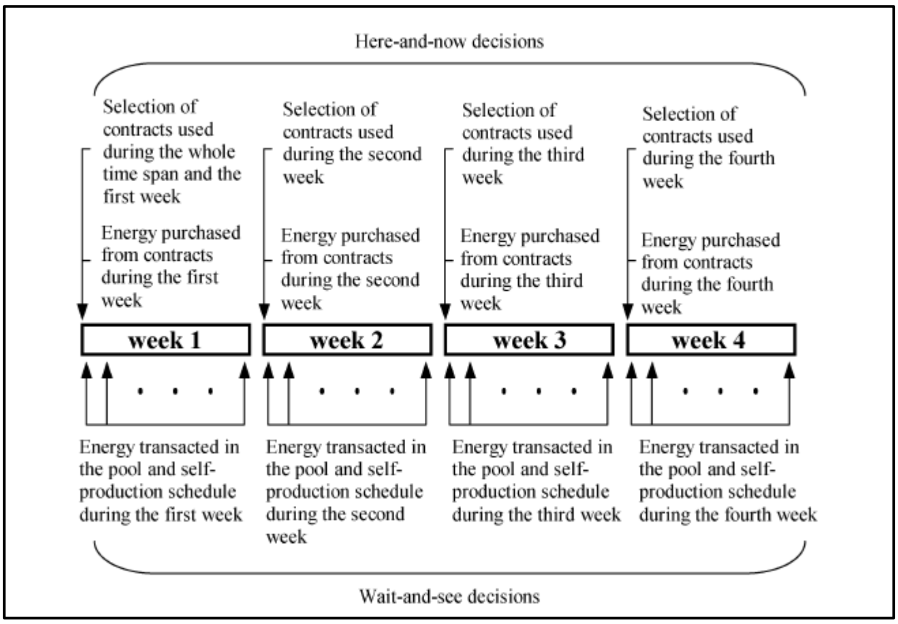


Figure 1.4: Decision framework spanning one month (Carrion et al, 2007)

1.2.5 Summary

From the above discussion, several conclusions can be drawn:

- Before the local energy sector can progress to a new era, the retailer and consumer must be provided with sufficient buy-in as well as an accurate depiction of their locally unique new reality in order to motivate change.
- Achieving social welfare is integral to creating buy-in, and this occurs when consumers neither incur exorbitant charges, nor do retailer revenues suffer excessively.
- No problem formulation that simultaneously addresses both stakeholders' objectives and measures their relative performance is available in literature.
- One of the primary benefits of deregulation in electricity markets is that of demand response which is in turn only achieved through the effective capture of consumers' price elasticity for the country in question.
- A representation of the consumer and retailer's problems must encompass demand response tools so that its benefits can be quantified and the impact of more advanced pricing schemes can be analysed.
- No problem formulation that seeks to achieve social welfare is available in literature, and one that does so must capture the price sensitivity of the consumer relative to the retailer.

1.3 OBJECTIVES

The South African electricity industry has thus far, failed to consistently deliver a reliable service to its public. This, coupled with global trends towards deregulation and green energy, makes South Africa a natural candidate for change. As a country "for the people" even when in a competitive environment, in order to create buy-in as well as preserve the interests of independent parties, social welfare must be the prevailing objective. In an open system, the consumer and retailer will also be confronted with uncertainty due to new market dynamics and its effect on their outcomes must be quantified. As with all forecasting and predictive efforts employed in industry, this will improve the opportunity for success of any fledgling deregulated energy market operating in South Africa. With these areas of focus in mind, the following sub-objectives have been defined:

- To contextualise the South African resident's load scheduling problem with regards to existing literature so that a realistic yet tractable model that accurately depicts the unique local reality is formulated.
- To contextualise the retailer problem with regards to existing literature so that the dynamics of a deregulated industry, specifically that of spot market and futures contract trading, is appropriately formulated in a realistic yet tractable model.
- To identify and encompass the infrastructural, technological and social requirements of demand response so that the deregulated market may operate successfully.
- To develop a novel problem formulation that is capable of capturing the relative price elasticity of retailers and consumers, identified as the key to achieving social welfare
- To demonstrate the principle theories of the derived model and problem formulation through its application to a South African case study operating under a fixed rate tariff.
- To identify, justify and propose an appropriate solution algorithm that extends the model's application to include more advanced TOU and dynamic pricing schemes for a mature deregulated market operating in South Africa.

1.4 CONTRIBUTIONS

The main contributions of this thesis can be discussed in a similar fashion and include:

- The first attempt at modelling a residential load scheduling problem with keen interests in the unique challenges of the South African environment, which addresses direct load control, battery storage and scheduling inconvenience

- The development of a model that simulates the dynamics of an energy retailer operating in South Africa
- The first application of a Markov regime-switching model with a non-restricted transition probability matrix to the Australian spot market, which is identified as a balancing market similar to one which would operate in the South African context, and which acts as an appropriate substitute
- The proposal of a novel approach to formulating the consumer and retailer problem that, when in this format, seeks to maximise social welfare instead of a single party's interests by appropriately capturing the relative price sensitivity of both stakeholders
- The demonstration of this problem formulation and its effectiveness in measuring social welfare, as well as demand responsiveness, when applied to a South African case study operating under a fixed tariff, as well as the identification of other factors requiring further investigation to ensure its success
- The proposal of a trial-and-error algorithm as a means of demonstrating the effects and impacts of demand responsiveness and battery usage on stakeholder objectives when operating under a TOU pricing scheme, which is markedly more computationally expensive due to the inherent nature of scheduling problems with non-convex search spaces.

1.5 SCOPE

Chapter 1 is introductory in nature and provides some context as to the energy industry, the problems South Africa is confronted with, and how this thesis aims to improve the outlook for the application of deregulation so as to curb the current crisis.

In Chapter 2 presents existing literature to develop an understanding of the energy environment with particular interest to the interactions between the consumer and retailer. Attention is paid to key features such as load management, battery storage and scheduling inconvenience for the resident, and procurement strategies and trading under uncertainty in the spot market for the energy provider.

The developed retailer and consumer models are presented in Chapter 3. Included is the three-regime Markov switching model that was identified as the strategy of choice for capturing and forecasting spot prices under uncertainty. A novel problem formulation that captures social welfare, achieved through hypothesising the relative price elasticity of retailers and consumers, is also presented.

In chapter 4 the results of the three-regime Markov switching model are presented and analysed. Data for a South African case study which is used as the basis for numerical work is also provided, followed by a demonstration of the proposed problem formulation to this case study when a fixed rate tariff structure is employed.

Consideration is given to the application of time-varying tariffs in Chapter 5. To do this, a brief overview and analysis of existing solution strategies which may be used to solve the electricity market's social welfare problem is presented. The trial-and-error algorithm is identified as an appropriate strategy for demonstrating the effects of demand response and battery storage when operating under a TOU tariff (or dynamic pricing scheme in future research). Preliminary results when the algorithm is applied to the case study under a TOU tariff is also discussed with regards to challenges experienced and recommendations for improvement.

Chapter 6 concludes the thesis with a brief description of major findings and avenues for future research. Limitations of the study that were identified during its completion as well as assumptions are also presented.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

As has already been established, electric power markets are complex environments within which a number of stakeholders and decision-making horizons are considered. In recent years the activities of governments, utilities and distributors have focused on changing the quantity and demand profile of energy consumption. This can be attributed to developments in smart grid technology enabling bidirectional communication between utilities and consumers. Holistically, this field of research has been referred to as Demand Side Management (DSM) since the 1980s, and now comprises activities such as Load Management and Demand Response. The reported benefits of effective DR are several-fold. Firstly, a more reliable electricity system is achieved as a result of smaller differences between peak and low load periods (Saele and Grande, 2011). This reduced volatility is observed due to improved consumer-retailer interactions which ultimately protects the user from risk, intermittent supplies and higher probabilities of transmission or distribution failures from overloads. Secondly, the consumer is able to incur lower bill payments when their load schedules are adjusted accordingly in response to price signals that they are sufficiently sensitive to, and herein is buy-in created not only for DR, but for deregulation as well. Finally, for the retailer a more uniform and predictable load profile is achieved, measured as the peak-to-average ratio (PAR) of demand. This means that generators are able to avoid the construction of expensive power plants to satisfy only several hours of peak demand per year (Mohsenian-Rad et al, 2010), and these heightened fixed costs are not filtered down to the service provider.

Based on the above discussion, a model that is developed for the consumer and retailer must therefore be an accurate depiction of their realities under a deregulated regime, but must also be equipped to realise the benefits of DR, and demonstrate and measure its effects on stakeholder and social welfare interests. To do this, a thorough understanding of the consumer-retailer interaction as well as their respective behaviours within the energy context is required, which in turn calls for a review of available literature. Section 2.2 gives an overview of the consumer problem and discusses some key features in detail. Section 2.3 presents the retailer problem in a similar fashion to Section 2.2 with emphasis placed on the prediction of spot prices, a volatile market that has a significant impact on retailer revenues. Section 2.4 summarises this chapter.

2.2 THE CONSUMER PROBLEM

Table 1.1 identified cost minimisation and a reliable power supply as the primary concerns of the residential user. Both outcomes were found to be inextricably linked to the load scheduling problem as it is referred to by Mohsenian-Rad and Leon-Garcia (2010), Chen et al (2012a) and Wang et al (2013), and which can be considered a subsection of DR. The purpose of load scheduling is to create an optimal power consumption schedule under a given tariff structure so that the user is able to incur the lowest electricity charges. An added benefit to this method is a reported reduction in PAR, the benefits of which have already been discussed. A review of literature revealed that in the consumer problem there are several key aspects present. They include formulating the interaction between the consumer and retailer, discussing the role of smart technologies in DR, addressing customer behaviour, managing load, addressing consumer inconvenience and evaluating the effect of energy storage facilities on consumer and retailer outcomes. Each of these are discussed in further detail in Section 2.2.1 to Section 2.2.6 respectively. Publications in this field provide a unique mix of investigation into each of these aspects to varying degrees, and many have added their own improvements. These valuable contributions are noted in the "Present-State-of-Art" that follows the subsequent discussions.

The PSA has been compiled for a period of 11 years, from 2004 to 2015. Table 2.1 is divided into five headings: year, author/s, "method used", "key features" and "contributions". The "method used" identifies the primary modelling technique or methodology of the publication and "key features" highlights which salient feature/s formed the focus of the study. Finally, the "contributions" outlines the significance of the results obtained and forms a valuable basis for comparison in the analysis phase. It is of the opinion that this approach gives an accurate overview of the current analytical modelling techniques applicable to the consumer problem.

2.2.1 Formulating the interaction between the consumer and retailer

The relationship between the consumer and retailer is a mutually dependent but conflicting one. This is because the consumer relies on the energy provider to provide a service at a reasonable retail rate so as to minimise their electricity payments, whilst it is in the best interest of the retailer to charge a higher tariff so as to achieve higher profit margins. Furthermore, the consumer may have other interests apart from a reduced bill such as reducing their inconvenience for shifting appliance usage. The same holds true for retailers. Two primary setups have been reported in literature to describe the interaction between these two parties, as can be seen in the Present-State-of-Art provided in Table 2.1. These are Stackelberg games and multi-objective optimisation problems (MOOP).

The more popular approach is that of a Stackelberg game in which the service provider is the leader who sets a tariff and residential users are the followers responding to this tariff. Mohsenian-Rad et al (2010), the University of British Columbia (2011) study, Chen et al (2011) and Chen et al (2012a) apply this technique to depict the power balance between these stakeholders. For Chen et al (2011) and Chen et al (2012a), a conventional approach is followed as the utility aims to maximise revenue and the follower is some automated decision-maker aiming to minimise payments thereafter. Mohsenian-Rad et al (2010) focus on the minimisation of PAR, an indicator of aggregated load profile, instead of on individual user's utility. The University of British Columbia (2011) study focus their attentions on the pricing scheme rendered to consumers and adopt a two-fold structure comprising mismatch and usage pricing. Although not stated explicitly, Mohsenian-Rad and Leon-Garcia (2010) also seem to adopt a Stackelberg formulation but operate with a pricing structure that combines real-time pricing and inclining block rate tariffs. The authors find that RTP creates confusion amongst users not familiar with dynamic pricing and contributes to load synchronization, but coupled with inclining block rate tariffs, these effects are avoided. The primary benefit of a Stackelberg strategy is that a unique and optimal solution known as the Stackelberg or Nash Equilibrium can be found, as is reported by Mohsenian-Rad et al (2010), Chen et al (2011) and Chen et al

(2012a). These solutions are typically derived using backward induction and in one case, a distributed algorithm (Mohsenian-Rad et al, 2010). For the University of British Columbia (2011) study, no experimental or numerical results are provided and this poses a topic for future research as the price formulation presented is promising. This problem formulation is also effective in representing the leader-follower relationship between the service provider and user. However, three criticisms of this model setup exist. Firstly, only a sequential solution that is scenario-based is permitted, which proves difficult when applying such solutions to general cases. Secondly, because the retailer is consistently favoured as the party responsible for setting a price signal and the consumer is at its mercy, social welfare is not guaranteed. Finally, under this problem formulation both parties objectives are studied and satisfied independently. Their relative effect on one another is therefore not adequately captured and price elasticity, which is key to achieving DR benefits for both parties, is ignored or at best, assumed to be linear.

Multi-objective optimisation problem setups have been employed frequently in the energy industry, and particularly in the direct load control (DLC) context, because of their desirable trait of simultaneously considering numerous outcomes. Gomes et al (2007) and Pedrasa et al (2009) are two such authors that have done so. In Gomes et al (2007), no fewer than seven objectives that consider both the customer and retailer are included. In Pedrasa et al (2009) a non-convex, non-continuous objective function is derived due to the scheduling of interruptible loads and the presence of binary variables, which both significantly increase model complexity. Thus, the authors opt for simplifying this to a single aggregate function. Both of these studies make use of metaheuristics for sub-optimal but sufficient model solutions due to their inherent complexity, and both emphasise the importance of setting well-validated parameter values due to their critical impact on results. Therefore, MOOPs increase the required number of parameter estimates, as is the case with Gomes et al (2007) whose solution is dependent on predetermined aspiration and reservation levels for the consumer, and for which no guidelines are offered. This further means that if parameters are not appropriately justified and selected, they call into question the integrity of a study’s fundamental findings. Lastly, the simplification of MOOPs to aggregate functions also assumes a relative price elasticity between selected parties that is linear, and this has not been substantiated in literature.

Based on the above discussion, it is clear that a completely novel problem formulation must be proposed and tested that captures the power balance inherent to the relationship between the retailer and consumer, enables the simultaneous consideration of stakeholder objectives so that social welfare is maintained, and addresses the relative price elasticity of the retailer and consumer so as to appropriately capture this social welfare in a single function. For this study the consumer and retailer models will be developed separately but will be inextricably linked by the decision of an appropriate tariff. The objective is then to determine the relationship between the retailer and consumer as a function of the price charged so that the tariff at which optimal social welfare is established can be defined. In response to this price, end-users may then respond with an optimal load schedule that reduces their bill payments and incurred inconvenience, and retailers may enjoy sufficient revenue recovery.

2.2.2 The role of smart technologies in Demand Response

Effective demand response cannot be achieved without smart grids that are equipped with smart technology. These grids are key infrastructural assets that enables two-way communication and in turn facilitates the interaction between consumers and their service provider. The installation of devices such as Energy Management Controllers (EMCs) and Local Home Area Networks (LHANs) that enable DR is a voluntary effort on the part of consumers and this means that sufficient incentive in terms of cost savings must be realised (Conejo et al, 2010). On a macro level, these technologies then work in coordination with Energy Management Systems (EMS) such as UREM (Cai et al, 2009) and CAES (O'Neill et al, 2010) to make planning, generating and capacity decisions (the definition of UREM and CAES can be found in the List of Acronyms). Therefore, the employment of smart technologies is not only critical to meeting stakeholder objectives, but it offers significant benefits for consumers, retailers, distributors and generators. Sou et al (2011), like Chen et al (2011), highlight the importance of smart grid technologies for enabling information flow between service providers and consumers, and handling the large data volumes and velocities typical of DR. Rastegar et al (2012) give a satisfactory description of a variety of smart technologies and EMSs with focus on the system data transfer structure. In a context where devices enabling bi-directional communication are not available, perhaps for financial and infrastructural reasons or due to technological immaturity, Corradi et al (2013) propose a one-way price signal to control electricity consumption. This is however not ideal as data gathered on load profiles is aggregated and thus not specific to customer behaviour, limiting its effectiveness in achieving DR. Finally, Saele and Grande (2011) comment that the use of manual practices, such as the ‘El Button’ in their case, to remind customers to avoid usage of energy-intensive appliances is also highly recommended.

What is clear from these works is that the best demand response is achieved when homes are equipped with technologies that enable the communication of load and pricing data between the retailer and consumer. Furthermore, manual reminders are also a tool for affecting change in user consumption patterns that has seen positive results. The identification of these requirements speaks to one of this study’s aims of establishing and encompassing infrastructural, technological and social parameters that achieve DR in the proposed model so that the deregulated market may enhance its success. For this study, it will thus be assumed that all residences are equipped with EMCs that hold the data for user preferences and which automatically adjust load in response to price signals. This will eliminate the need for manual reminders, and the model developed can easily be integrated with current EMSs (Conejo et al, 2010). For a review of the functional and constructional requirements of smart grid technologies, the reader is referred to Schweppe et al (1989).

2.2.3 Addressing consumer behaviour

Customer behaviour is fundamentally linked to price elasticity and thus demand response. This is evident from the number of publications that have attempted to classify their load patterns and predict their attitudes, as can be seen in the Present-State-of-Art (see Table 2.1). Criteria such as rated electrical values, activity-type parameters or even the socio-economic status of a user have a significant impact on their demand profiles

(Chicco et al, 2004). In Gomes et al (2007) loads are classified into 20 groups, each with their own distinct profile, but little substantiation is given as to how these groups are derived and such a large segmentation could prove tedious. Other factors such as the type of pricing scheme, the presence or absence of incentives to participate in DR programmes and manual reminders to reduce consumption have also proved to be significant (Mohsenian-rad and Leon-Garcia, 2010, the University of British Columbia study, 2011, Saele and Grande, 2011). This is due to the varying degrees of flexibility and attitudes of users towards energy consumption. For example, an indigent resident would be far more willing to incur inconvenience for the sake of a lower bill payment in comparison to a wealthy user. According to Baboli et al (2012), incentive-based programmes have proved more successful than price-based programmes in achieving higher levels of DR due to its foundation as a reward rather than a punishment-based system. However, no methods are presented to assist in defining the price of incentives or tariff structures, unlike in Aalami et al (2008) where a trial-and-error algorithm is used to find the value of incentives required to achieve a reduced peak load equal to a pre-selected base load line based on price elasticity. Setlhaolo et al (2014) also used the guidelines of Wood and Newborough (2003) to define an incentive value. The focus of this study however is on the selection of a tariff that delivers optimal social welfare, and to this end, price-based studies such as those by Mohsenian-Rad and Leon-Garcia (2010) and Saele and Grande (2011) have reported success in achieving better demand response as well as benefits for the consumer and retailer.

Chicco et al (2004), Aalami et al (2008) and Baboli et al (2012) all take different perspectives on analysing consumer behaviour. Chicco et al (2004) compare two approaches for classifying users by load profile and their technique can be used complementarily with tariff structure design to maximise service provider revenue whilst meeting consumer demand. In Baboli et al (2012) the psychological element and consumer habit formation is emphasised. Importantly, in this work the authors begin to explore the effect of customer segmentation and behaviour on energy consumption patterns. However, little qualitative or quantitative analysis in the form of questionnaires, historical data or socio-economic studies is presented, and no justified basis for classification is provided. Thus, assumptions regarding the customer segmentation weighted coefficient (see Table 2.1) and baseline energy consumption levels are made, both of which heavily impact the results of the study. Of the four categories by which customers are segmented, Baboli et al (2012) neglect to analyse which has the greatest impact on energy consumption patterns. Finally, Mohsenian-Rad et al (2010) indicate that users might be tempted to act strategically and lie about their load schedules so as to maximise personal cost savings, especially when retail prices are based on aggregated profiles for a specific region. However, the study proves that this is in fact not in their best interests as minimised aggregated energy costs also result in reduced individual electricity charges.

Based on the above findings, it is clear that the diverging attitudes of residents to DR, especially in a socio-economically diverse country such as South Africa, must be considered in model development so as to depict an accurate representation of reality. To this end, price elasticity is acknowledged as key to affecting consumer behaviour, but how this should be quantified, managed and related to the service provider so as to achieve social welfare has not yet been addressed in literature. Lastly, it is noted that all residents considered in this study do not act strategically and that information relayed via EMCs is transparent and accurate.

2.2.4 Load management

Direct Load Control (DLC) and Remote Load Control (RLC), especially for residential consumers, has received significant attention in literature. In fact the bulk of research in DR has focused on this strategy, as is indicated in the Present-State-of-Art. This is because this sector is more flexible and thus capable of adjusting their demand profiles in comparison to industrial users, and because they, together with the commercial sector, are the largest contributors to system peaks (Ramanathan and Vittal, 2008). Load management is by far the most integral aspect of this study’s model development as it dominates the formulation of the consumer’s problem and satisfies one of the key aims as stated in Section 1.3. In fact, all other research pertaining to the user (specifically Section 2.2) simply complements and enhances the realism of the load scheduling problem, the bulk of which is borne of this section. It can be seen from literature that the load scheduling problem has four aspects to it, namely the number of loads considered and size of time slots, the types of loads considered, the technical and operational appliance constraints, and load uncertainty. Each of these now requires some discussion.

Consideration of the number of loads and time-discretization in a scheduling problem has a significant impact on the computational time and thus the feasibility of a model. Specifically, increasing the number of appliances in a scheduling problem causes an increase in the size of binary decision variables which are notoriously difficult to solve and in this context, cannot be relaxed with Lagrangian techniques. In the energy market especially, this is an important factor to consider as prices are released on a daily basis. Excellent sensitivity analyses are conducted by Sou et al (2011) which give insight into the significant effect of the number of appliances and size of time slots on computational time. It is however found that whilst the size of time slots greatly increases solving time, it has minimal effect on objectives. According to the authors, their study is limited to a single household with less than five appliances due to the computational burden that a larger problem would cause. This is likely due to their strict time constraints (their model is solved in a handful of minutes) as well as the discretization of time into 5-minute intervals. In contrast, Pedrasa et al (2009), Mohsenian-Rad and Leon-Garcia (2010) and Setlhaolo et al (2014), all of whom model with larger time slots, are able to include between 10-40 loads with comparative results.

End-use devices can be characterised by distinct load profiles, user attitudes towards their consumption and from an operations research perspective, their effect on model tractability. The classification of appliances by Schweppe et al (1989) seems to be appropriately defined based on their scheduling flexibility and consumption patterns. For example, authors who focus their attention on thermostatic loads such as electric water heaters or air-conditioning systems do not also consider other load types in their study due to the limited user discomfort incurred with these devices in contrast to others (Ramanathan and Vittal, 2008, Wang et al, 2013). On the other hand, reschedulable, discretionary activity and non-reschedulable appliances are often modelled together as can be seen in Chen et al (2011), Sou et al (2011) and Setlhaolo et al (2014). Reschedulable devices refer to dishwashers, washing machines and dryers, and are named such because of the typically flexible attitude that customers have towards the times of their use. The key parameter to consider for these appliances according to Schweppe et al (1989) is kW/usage because they vary so greatly from one model to the next and are highly dependent on the selected cycle (for example, the wash only or wash and dry functions in a dishwasher). These and discretionary activity devices, which are broadly categorised as those used for cooking, hobbies and chores, differ primarily because the energy consumed per usage in the latter is highly variable and time-dependent. Finally, the authors state that the modelling of non-reschedulable appliances such as lights and televisions is not meaningful, but that if done, the necessary parameters be taken from published data as opposed to direct measurement. Other load classifications have also been found in literature. Gomes et al (2007) classify loads into 20 groups, each with its own distinct profile and aggregate power consumption. However, little justification is given as to the need for this number of groups or how it would vary from one context to the next. In Mohsenian-Rad et al (2010) for example, a load is classified as shiftable or non-shiftable but the EMC then has no impact on scheduling for a non-shiftable appliance, and its inclusion is merely to capture total energy consumption data. For Pedrasa et al (2009) the focus is on interruptible loads. These are devices that need not complete their cycle continuously, and the objective is then to minimise disruptions. The authors report a non-smooth feasible region and a non-convex, non-continuous objective function which significantly increases model complexity, and from which only a limited number of appliances would benefit.

The technical and operational constraints for appliance modelling are extremely important as they create the feasible region within which optimal results are located. Because the objective of this study is also to create a model that accurately reflects reality, the permitted and non-permitted behaviours of end-use devices must be appropriately defined. The various constraints enforced on appliance behaviour can be found in the works of Mohsenian-Rad et al (2010), Mohsenian-Rad and Leon-Garcia (2010), Conejo et al (2010), Sou et al (2011) and Setlhaolo et al (2014). Commonly enforced constraints include the appliance-specific minimum duration of operation, sequential operation of appliances, maximum daily or hourly consumption and valid hours of operation. Several differences exist between these works however. Setlhaolo et al (2014) include an incentive in their model which enables them to draw conclusions regarding its effect on cost savings. Rastegar et al (2012) and Setlhaolo et al (2014) differ from Sou et al (2011) in that their inclusion of valid hours of operation for each appliance enables the difference between some pre-specified baseline and optimal schedule to be modelled as a binary matrix from which scheduling inconvenience can be calculated (this is discussed further in Section 2.2.5). Chen et al (2011), Tarasak (2011) and Chen et al (2012a) address appliance scheduling to a lesser extent than Sou et al (2011) and Setlhaolo et al (2014). Chen et al (2011) focus their attention on the hours of operation for each appliance, whilst Chen et al (2012a) bound the power consumption for each time slot. The validity of this assumption remains untested, firstly because consumers are typically self-serving and would not limit their consumption unless incentivised (Chicco et al, 2004), and secondly because several end-use devices may be non-reschedulable but energy-intensive, causing them to exceed consumption boundaries (Schweppe et al, 1989). Tarasak (2011) provide an improvement on minimum and maximum time boundaries by stipulating that they correspond to the power usage of nonstop and all appliances respectively. Still, this approach poses challenges in scalability. Setlhaolo et al (2014) include a maximum cost that the consumer is willing to incur over a 24-hour period, but it is recommended that further analysis be conducted for this value. This is because in South Africa particularly where the socio-economic demographic of the country is extremely divergent, users may have radically different attitudes and behaviours to cost savings, and such a cost ceiling should accurately reflect this. Adjustable power ratings and minimum and maximum power standby power levels are also modelled by Mohsenian-Rad et al (2010), Mohsenian-Rad and Leon-Garcia (2010) and Sou et al (2011). This approach assumes greater flexibility of residential appliances than Setlhaolo et al (2014), and perhaps reflects a more realistic residential load scheduling problem. However, end-use devices that offer such flexibility only include reschedulable and energy-intensive appliances such as dishwashers and washing machines. In general, whilst these methods are effective in capturing the load scheduling problem and should certainly be studied during model development, they all offer little to no consideration or analysis for the design of tariff structures. In fact, these are largely assumed to be input parameters in the above studies. A work that combines both the perspective of the retailer in setting such a tariff, and then studies the resultant effect on the consumer would be valuable indeed.

Load uncertainty offers important practical value to demand side problems within the energy market. Whilst forecasting and statistical modelling of load demand can certainly assist generators and service providers in planning and decision-making, unavoidable sources of uncertainty can stem from measurement errors and discrepancies in customers' expected and actual demand. Its effect on utility revenue as well as the retail price received by consumers must therefore be considered. Schweppe et al (1989) identify several ways for addressing uncertainty, the most promising of which is mini-max control due to its ease of implementation and computational inexpensiveness. Tarasak (2011) and Chen et al (2012a) both consider load uncertainty to be some random variable attached to the aggregated expected demand with a given mean and variance. Chen et al (2012a) assume a constant value for variance whilst Tarasak (2011) perform a comparison of several methods, one of which is variance with a bounded magnitude as described by Schweppe et al (1989) as the mini-max approach. Both Tarasak (2011) and Chen et al (2012a) found that load uncertainty increased the optimal price and thus retailer revenue and Tarasak (2011) indicated that a variance with unknown distribution produced a higher optimal price. The finding of Tarasak (2011) that unknown distribution of the variance has different effects on consumption depending on the time of day hints that higher uncertainty may be associated with peak periods and lower uncertainty with off-peak periods, but further analysis to test this hypothesis is required. It is also recommended that some comparative study be conducted to quantify the effects of uncertainty on retailer revenue. Chen et al (2012a) are the only authors to present the adverse effect of load uncertainty on user payoff, due to the fact that it drives up the retail price per unit of energy. Finally, Conejo et al (2010) make use of robust optimisation techniques in combination with an ARIMA model to develop certainty bounds for hourly loads. Whilst this was found to increase consumer utility, little guidance is given as to selecting the robustness parameter value, which has a significant effect on results.

2.2.5 Addressing scheduling inconvenience

Much like the financial, oil and natural gas markets, electricity is first and foremost a commodity. According to microeconomic theory, this means that there exists a degree of price elasticity in which consumers will adjust their demand up to the point where the benefit they believe they derive from accessing a resource is equal to the price they pay (Kirschen, 2003). For electricity, this derived benefit is subject to the inconvenience suffered by a user in adjusting their ideal load pattern. This in turn affects their elasticity to prices which must be reflected in their relationship with the retailer through the proposed social welfare function. For example, it may be more cost-effective for a resident to use the stove during an off-peak hour such as 3AM, but this would be highly inconvenient and impractical, and as a result the price signal must be sufficiently loud (and large) in order to not be ignored. Too high a retail rate however would detriment social welfare as the consumer would be left disgruntled. Studies by Ramanathan and Vittal (2008), O'Neill et al (2010), Mohsenian-Rad and Leon-Garcia (2010), Chen et al (2011), Rastegar et al (2012), Wang et al (2013) and Setlhaolo et al (2014) seek to appropriately address and quantify this inconvenience and its effect on user behaviour and pricing policies. Details on these publications can be found in Table 2.1.

O'Neill et al (2010) address scheduling inconvenience by assuming that users prefer devices to be operated sooner rather than later. The authors thus define an *average pending workload* that is always positive and increasing but must be minimised in conjunction with financial costs. It is modelled as an exponentially smoothed, strictly convex, parameterized dis-utility function. A diagonal matrix is used to control consumer concern for the average delay of a device completing service The primary limitations of this method are that developing a utility function that is more effective in capturing customer dissatisfaction would require a non-diagonal matrix and the development of similar functions for each device owned by the resident, making the technique cumbersome. Each individual device's user constraints could be modelled and this offers the opportunity to better reflect statistical relationships between appliances but this would largely differ from one user to the next, once again making the technique tedious. As a result, expanding the study to include a more realistic number of residences would threaten model scalability and increase parameter requirements. Like O'Neill et al (2010), Mohsenian-Rad and Leon-Garcia (2010), Rastegar et al (2012) and Setlhaolo et al (2014) also make use of a trade-off parameter to control the importance of consumer inconvenience relative to cost savings. For Rastegar et al (2012), this knob is given as a cost incurred per watt-hour so that inconvenience and electricity payments are measured in the same unit. This coefficient represents the sensitivity of users to DLC shifting and the authors provide guidelines for the threshold value used for comparative purposes. Interestingly, the study by Setlhaolo et al (2014) revealed that after a trade-off parameter value of 25, overall costs incurred by the resident plateaued, indicating an exponential flattening effect. Whilst this value may be relative as it depends on factors such as the appliances considered and tariff rates, it highlights the strong impact of the waiting parameter on the overall consumer objective.

Mohsenian-Rad and Leon-Garcia (2010) also seek to minimise the waiting time of devices but follow a slightly different strategy to O'Neill et al (2010). The authors define a 'valid' schedule with beginning and ending time intervals representing the hours that a user is amicable to appliance usage (thus preventing a stove from being used at 3AM). A similar approach is also adopted by Rastegar et al (2012) and Setlhaolo et al (2014). Significant benefits of this technique are that it reduces the number of scheduling decision variables and is also flexible enough to be implemented in a number of residences. In practice, each user would specify what they consider to be valid hours of operation for each appliance as input parameters to their EMC. A second control parameter is also introduced for each appliance i, enabling a user to, for example, have a more flexible attitude towards shifting the operation of a kettle over a geyser. This technique is far more representative of reality than the uniform flexibility that O'Neill et al (2010) assume of all appliances. The exponential function used by the authors to demonstrate the customer's attitude to increases in waiting time is also a truer representation of reality. That is, a user would become progressively more disgruntled as they were forced to wait longer for an appliance, their dissatisfaction resembling that of an exponential curve rather than a linear one. The study also provides a set of guidelines for selecting appliance flexibility parameters δ_i, where appliance $i \in I$, and discusses their implications for monthly payments (Mohsenian-Rad and Leon-Garcia, 2010):

1. $\delta_i = 1$. This implies a strict cost reduction and the operational flexibility associated with this appliance is high, that is, the cost of waiting has no impact on the model selecting an optimal schedule.
2. $\delta_i > 1$. This implies a medium cost reduction and the operational flexibility associated with this appliance is fair. The study found that an increase in δ_i from 1 to 1.01 caused a 19.58% increase in electricity payments whilst the waiting time decreased by 75.7% of the valid schedule. This signifies the strong impact that flexibility parameters have on the model objective.
3. $\delta_i >> 1$. This implies no cost reduction and flexibility associated with operating this appliance is negligible.

The primary criticism of this technique is that only the postponement of appliance operation from its ideal usage is considered to be an inconvenience, and not the advancement. The beginning time interval of the valid schedule for each appliance is thus considered to be the optimal period of usage which is not necessarily the case in reality: a resident may prefer to use the kettle at 7AM, but is flexible to using it between 6AM and 8AM. Setlhaolo et al (2014) are the only authors who assume that any deviation from the preferred schedule incurs inconvenience for the user. Chen et al (2011) opt not to use waiting parameters but rather assume that each time slot representing a delay in device operation incurs a cost in dollars, and that this cost may vary from one appliance to the next, but remains fixed regardless of the period in the day. A maximum allowable delay for each appliance is also specified.

It was established in Section 2.2.4 that thermostatic loads such as heating and cooling devices have been treated differently in literature in comparison to conventional household appliances. This too is the case when it comes to quantifying consumer inconvenience. In general, such devices are modelled for research purposes, numerical and pilot studies due to minimal end-user discomfort incurred and the ability of the load to be disrupted. According to Ramanathan and Vittal (2008), inconvenience is incurred with such devices when there is a deviation between the internal temperature and the thermostat set points. Wang et al (2013) opt for scheduling electric water heaters and are slightly more flexible in their approach to modelling inconvenience. They allow upper and lower limits of temperature set points to be specified by each user, and this forms a comfort band. The objective is then to keep water temperature in this band with the least electricity cost. This technique is far more representative of reality as the attitude of residents towards comfortable temperatures is unlikely to be as restrictive as Ramanathan and Vittal (2008) propose, and also offers the model more flexibility in generating an optimal solution.

After evaluating the key techniques employed in literature to address consumer scheduling inconvenience, it is noted that the work of Mohsenian-Rad and Leon-Garcia (2010) best addresses the areas of importance. The user’s appliance-specific attitude towards scheduling flexibility, the exponential-like curve resembling customer dissatisfaction and the guidelines provided for parameter selection are promising features of the study. It is however recommended that in order to more appropriately reflect reality, advancing the use of an appliance to incur lower financial cost (and not only postponing usage) should also be considered an inconvenience to the consumer, which is not the case in the original authors’ work.

2.2.6 The role of storage facilities in Demand Response

Energy production and the subsequent storage thereof are two topics that have grabbed the attention of governments, policy-makers and businesses worldwide since the 21st century. Energy production techniques are primarily the concern of generators and thus fall beyond the scope of this study. Storage options however are now becoming more decentralized and offer promising benefits to the resident. Because of this, their potential to reduce consumer’s payments and this resulting effect on social welfare achievement must be addressed.

Rising energy demands, increasing consumer payments, the development of PHEVs, and renewable energy production have been the biggest catalysts for investigating the viability of electrical energy storage. The primary driver for rising energy demands is an ever-increasing population and according to Francois L'homme of Schneider Electric, a multinational corporation specialising in energy management, a 200% increase in demand is expected over the next thirty-five years (Mining IQ, 2012). Storing electricity will enable a more economical production of the higher required output so that fewer facility start-up costs are incurred. Residential storage in particular has seen vast growth with companies such as Solar City pioneering viable solar storage and lithium-ion battery models, with the biggest take-up to date being in California. Pilot studies performed in several countries by Chen et al (2009) indicated that energy storage is extremely effective in achieving peak-demand shaving. For the consumer, this means reduced payments as they are able to store electricity (or charge their battery) during off-peak, low-cost periods of the day and utilize this stored energy during peak-periods without drawing from the grid and incurring a higher bill. For the generator, a more uniform and predictable demand profile is created, enabling them to better manage capacity and reduce or eliminate excessive facility start-up costs required for peak periods. PHEVs have also grown in popularity over the past decade due to their ability to allow sustainable personal travel without harming the environment. The use of a battery and electric motor (instead of a gasoline tank and internal combustion engine) means that a resident's vehicle may be treated as any other household appliance to be optimally scheduled for recharge (Mohsenian-Rad and Leon Garcia, 2010). PHEVs reached a market share of 0.72% in 2014 in the United States and its limited energy requirements (roughly 16KWh for a 65km distance) coupled with future developments in the industry make it a significant motivator for residential energy storage. Indeed, works such as Rastegar et al (2012) not only attempt to model the effects of PHEV ownership on load scheduling, but also highlight that it is often feasible and profitable to return their stored energy back to the grid. In South Africa however, PHEVs have not achieved significant market share or even publicity and including it in the proposed model would be somewhat unrealistic and not reflective of the typical local resident in the medium future.

Finally, renewable energy is an environmentally-friendly alternative to fossil fuels and crucial to meeting future energy needs. Although unpredictable, these energy sources offer great potential in terms of flexibility in global supply chains but this will only be possible if effective storage strategies are developed to harness them. This, coupled with distribution grid level constraints such as disturbances, transmission limits and voltage drops, means that the demand for decentralised energy storage is ever-increasing.

Typically, battery storage systems comprise a rechargeable battery, bi-directional inverter/charger and controller, as can be seen in Figure 2.1 below. According to Leadbetter and Swan (2012), when modelling for energy storage the capacity, power and cycle life of the selected battery are of key importance. In their study, the authors assume the use of lithium-ion batteries due to its extensive application in households, its high power and energy density characteristics, long cycle life and limited maintenance costs. So as to extend life, the storage system only operates between a 15-85% state of charge (SOC), and the battery is assumed to always exist in one of three states: charging, discharging or standby. A five-minute time interval is used to sufficiently capture peaks and eliminate the averaging effect (Saldanha and Beausoleil-Morrison, 2012), and the battery is scheduled to recharge during a five-hour nightly period when tariffs are at their lowest. Results indicated that a single optimal battery storage system in terms of size, cost and capacity did not exist, but was rather dependent on the household under consideration and its respective demand profile. Specifically, it was found that a 4kWh system reduced maximum peaks by 40% whilst a battery double the size only achieved a 51% reduction. Overall, systems ranging from 5kWh for a low-use home to 22kWh for an intensive-use home were deemed sufficient. It is believed that this model is prescriptive however in that it does not enable the optimised scheduling of the battery system similar to any other appliance, thus compromising its ability to reduce costs for the user. This is highlighted by results indicating the infrequent usage of the battery, and only to one-fifth of its capacity. Furthermore, it is assumed that if the battery is unable to meet excess demand, this goes unmet and yet no penalty is imposed for dissatisfying the customer. It should also be noted that the choice of time step significantly increases the number of considered variables and computational time of the model. It is of the opinion that only three works consider battery storage models in conjunction with load scheduling, namely Chen et al (2012a), Chen et al (2012b) and Rastegar et al (2012). The studies are similar in their constraint modelling and reported significant cost savings as is expected. In the case of Rastegar et al (2012), results also indicated a reduction in average load and peak load increment but an increase in PAR. Chen et al (2012b) however are the only authors to consider the charging and discharging efficiency of a battery. This relates to the inherent loss of energy during these processes to the environment in the form of heat. This factor has a significant impact on the amount of energy available to appliances and must thus be accounted for. Typically, values range between 10-30% and are affected by a number of factors such as rate of charge/discharge and energy state of the battery. For the purposes of this study a constant rate will be assumed.

It is clear that the trend towards energy storage is growing, whether for financial, environmental or technological reasons, and any effective load scheduling tool must consider its effects. From the literature investigated it appears that batteries are the most popular and cost-effective means of storage. Guidelines as to model development can be found in Chen et al (2012a) and Rastegar et al (2012).

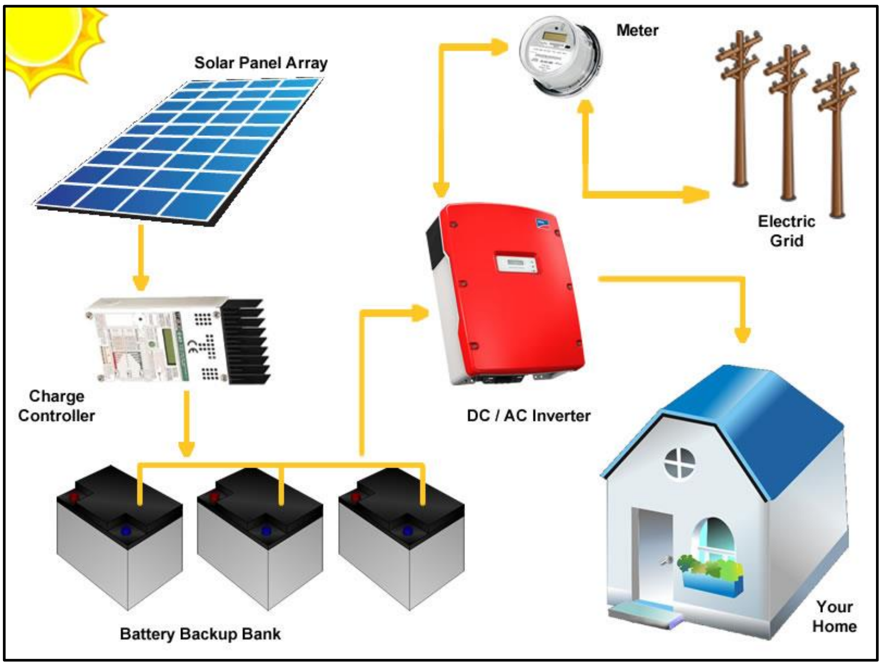


Figure 2.1: Generic battery storage system

Table 2.1: Present State-of-Art: Consumer Problem

Year	**Author/s**	**Method used**	**Key features**	**Contributions**
1989	Schweppe et al	• Spot price based algorithms presented for use in smart grid technologies to enable residential load control	• Describes the four main functions of smart grid technologies • Defines the basic control actions for end-use devices • Categorises end-use devices as thermal storage, water heating, periodic use requirement, reschedulable appliances, discretionary activity devices and non-reschedulable appliances	• Parameter values for each of the end-use device categories are discussed • Identifies three ways to manage uncertainty, namely open loop feedback control, stochastic control and mini-max control • Identifies customer attitudes and characteristics, utility characteristics and communication media as deciding factors in selecting an appropriate algorithm
2004	Chicco et al	• Customer classification by load profile	• Load patterns classified by normalising the peak value in a time interval • Follow-the-leader algorithm and self-organizing maps used for customer classification • Distance threshold (distance between two representative load profiles) used by algorithm to calculate number of clusters	• Numerical study conducted for 234 customers (industrial, service, small-business) over a 3-week period in the Romanian market • Trial-and-error approach used to calculate distance threshold • Results were the number of customer classes and class composition • Follow-the-leader algorithm is superior

			• Approaches compared using two adequacy indicators: mean index adequacy and clustering dispersion indicator	according to both indicators for the number of classes ranging between 10 and 30 • Self-organising map has better visualization of results
2007	Gomes et al	• Multi-objective optimisation problem to identify and select direct load control actions in a distribution network	• 7 objectives considered: customer discomfort, peak power demand and loss factor is minimised, profits maximised • Monte Carlo simulation used to evaluate changes in demand provoked by DLC actions • Aspiration (optimistic) and reservation (unacceptable) levels for each objective set by decision-maker (DM) • Loads classified into 20 groups, each with its own distinct load profile and power aggregation level	• Interactive evolutionary algorithm applied to accommodate progressive articulation of DM's preferences for aspiration and reservation levels • Highlights the importance of parameter selection when applying evolutionary algorithms • Aspiration and reservation levels have a critical impact on evaluated non-dominated solutions • Each generated solution is only for a particular scenario of aspiration and reservation levels
2008	Aalami et al	• DR model with single-period and multi-period elasticity with the	• Emergency Demand Response and TOU programmes evaluated (individually and simultaneously) for their effect on demand reaction	• Value of incentive taken to be average price of electricity over a given time period • Higher elasticity results in greater effectiveness of TOU in achieving demand response

		aim of maximising consumer benefit	• Price elasticity depicted as change in customer demand based on value of incentives • Numerical study applied to Iranian Power Grid	• Higher values of incentives result in greater peak reduction • Reported the occurrence of peak-shifting • TOU programme reduced range of the load curve (and thus peak-to-average (PAR) ratio) by 6500MW as well as peak magnitude
2008	Ramanathan and Vittal	• Framework to develop DLC programme that minimises user discomfort under stochasticity	• The effect of different load parameters, ambient parameters and artificial constraints on DLC is studied • Focus is on thermostatically-driven loads, specifically air-conditioning • Load scheduling: target load levels, minimum on and maximum off durations, maximum internal temperature specified • User discomfort is measured as the difference between internal temperature and thermostat set-points	• Monte Carlo-based simulation-cum-optimisation framework developed to analyse parameter effects • Dynamic programming applied • Stringent on/off constraints result in better load distribution but higher discomfort • Smaller differences between min-on and max-off times result in a better distribution of control effects • More diverse loads have a positive effect on control
2009	Pedrasa et al	• MOO scheduling problem to minimise	• MOOP simplified to single aggregate objective function and penalties assigned for violation of	• Builds on the work of Huang et al (2004) who solved the same problem using fuzzy dynamic

		total payments and frequency of interruptions for interruptible loads	constraints • Appliance scheduling: interruptible loads considered • Binary PSO applied when decision variables are discrete-valued	programming, but without the consideration of social welfare • 19 interruptible loads considered over a 16-hour horizon • BPSO achieved near-optimal solutions in manageable computational time-frames • Multiple sub-swarms significantly increased the probability of arriving at a valid solution • The probability of generating a high quality solution without interruptible load violations decreases as demand increases
2010	Conejo et al	• Linear scheduling problem to maximise commercial consumer utility under RTP-based scheme	• Appliance scheduling: minimum daily consumption, minimum and maximum hourly load levels, ramping limits on load levels considered • Price uncertainty addressed through robust optimisation techniques • ARIMA-based model used to define certainty intervals for prices	• Model solved using CPLEX 11.2.1 under GAMS on a Linux-based server with four processors clocking at 2.6GHz and 32GB of RAM • A lower robustness parameter value (45% of unknown prices) achieved maximised consumer utility with the smart grid than without (75-100% of unknown prices)

			• Rolling window model used to optimise power consumption on an hourly basis • Two scenarios considered: with smart grid (hourly adjustments to consumption allowed) and without smart grid (only price bounds are known and schedule for the whole day must be developed) • Numerical study applied to Iberian Pensinsula	• Maximised utility was 5.86% higher with a smart grid than without • The availability of real-time prices is more important than price bound updates to maximise consumer utility • Robust optimisation results in a 16.22% increase in utility than forecasting
2010	Mohsenian-Rad et al	• Load scheduling model to reduce PAR and minimise energy costs	• Stackelberg game formulation • DR should have the objective of minimising aggregated properties such as PAR instead of individual daily charges • Appliance scheduling: predetermined total daily energy consumption, valid hours of appliance operation, minimum and maximum standby power levels for each appliance, non-shiftable and shiftable devices considered for each user • Quadratic cost model for generating or	• 1-hour intervals, 10 users, 20-40 appliances considered • Distributed algorithm used to generate optimal schedule and power level of appliances for each user • Algorithm converged after 22 iterations, that is, roughly 2 iterations per user • Reduced PAR (by 17%), total energy costs (by 18%) and individual charges achieved at the Nash Equilibrium • Proposes that RTP schemes create confusion

			distributing electricity for each hour developed	amongst users and cause load synchronization
2010	Mohsenian-Rad and Leon-Garcia	• Load scheduling model to minimise energy costs and waiting time	• Tariff structure combines RTP and inclining block rate • Prices predicted by user with a weighted average filter developed from real-time data that estimates coefficients for each day • Appliance scheduling: predetermined energy consumption for each device, minimum standby and maximum power levels, shiftable and non-shiftable devices, valid hours of operation considered • Scheduling inconvenience is the amount of time the consumer waits after their preferred usage • Various scenarios proposed for future research: discrete consumption levels, interruptible and uninterruptible loads, multiple retail sources, load reduction requests, electricity storage	• 1-hour time slots, 10-20 appliances, 1 user considered • Interior-point method used for model solution in polynomial computational time • Price prediction capabilities only necessary for the user when exposed to extreme forms of dynamic pricing • Reduced PAR (by 38%) and user payments (by 25%) achieved with price predictor and energy scheduler • Inclining block rate tariff enables the avoidance of load synchronization • Increasing the number of users further balances the aggregated load, and more flexible users benefit more in general
2011	University of British	• Power scheduling	• Stackelberg game formulation	• Pricing scheme seeks to simultaneously

	Columbia study	model for multiple users under two-fold and uniform pricing schemes	• Appliance scheduling: elastic and inelastic appliances considered, where elastic loads are bounded • Interaction amongst end-users also is evaluated • Pricing scheme is a combination of mismatch and usage pricing • Mismatch pricing is used to encourage users to adjust elastic load • Mismatch and usage pricing are optimised both independently (two-fold) and simultaneously (uniform)	maximise profits and match supply and demand • No numerical results presented
2011	Chen et al	• Real-Time-Pricing (RTP)-based power scheduling model for residents	• Stackelberg game formulation • Schedule formed on appliance-basis and not on hourly aggregate consumption • Appliance scheduling: expected duration of operation, operating power stipulated • Inconvenience modelled as incurred cost of mismatch between planned and supply load, with maximum allowable delay permitted	• 80 customers, 3 appliances and 10 minute time intervals considered • Peak periods modelled as appliances being requested with higher probability • Backward induction used for model solution • Cost savings of approximately 10% (versus 6% for day-ahead trading), peak reduction of approximately 30%, reduced variation between

			• Retail price modelled as the sum of wholesale price and price gap • Lower price gap is the result of a larger difference between planned and real-time load	planned and actual supply resulting in stability for the service provider reported • Peak shifting avoided through sequential decision-making
2011	Saele and Grande	• Pilot study for demand response of electrical water heaters in Norway	• Water heaters selected for the limited inconvenience placed on residents • Focus was on identifying the times of day that load shifting is needed and how to provide price signals to customers that reflects power situation • Time-of-day network tariff considered to be a sum of user-specific costs, network losses and variable energy costs linked to DR • Retail tariff structure designed to be predictable for consumers but dynamic enough to reflect market fluctuations	• Study considered 40 customers over a period of 1 year • DR was more effective in comparison to previous studies due to the "El-button" which served as a reminder to customers to avoid using energy-intensive appliances • Predictable price signals, smart technology, remote load control and reminders were key to achieving good DR results
2011	Sou et al	• MILP scheduling problem to minimise	• Appliance scheduling: expected duration of operation, adjustable peak power consumption,	• Premature termination (for realism purposes) produced excellent suboptimal results with

		cost of operating smart home appliances, subject to consumer preferences	uninterruptible and sequential operation, maximum operating power, idle power maximum execution time stipulated • Spot prices and tariff structure given • Perfect supply assumed, that is, demand is always less than supply for every time period • Numerical study applied to New York City and Sweden • Solving scenarios such as optimality, first feasible solution, appliance operation ASAP are evaluated	relative errors less than 1% • To measure model performance objective was maximised and difference in results compared • Large tariff fluctuations are required to cause changes in consumption patterns • Length of time slots has a significant impact on computational time but minimal effect on objective • Number of appliances considered has a significant impact on computational time
2011	Tarasak	• Extends on Samadi et al (2010) who propose a utility framework and distributed algorithm under RTP scheme to include load uncertainty	• Load uncertainty captures measurement error from non-ideal transmission of demand information • Load uncertainty defined as the sum of expected energy consumption and some random variable • Three uncertainty models considered: bounded uncertainty, Gaussian and unknown distribution	• One hour time intervals and ten customers considered • Model is solved for optimal power consumption and optimal energy generation for the user and service provider respectively • In general, load uncertainty was found to increase the optimal price due to increased generation requirements

			• Assumed that customers do not act strategically, that is, each acts independently • Appliance scheduling: power consumption for each time slot is bounded but varied for hours of the day • Utility assumed to generate energy within predetermined boundaries for every time slot • Retail price is updated with the gradient projection method	• Bounded uncertainty causes power consumption to be consistently lower than generating capacity • Load uncertainty with unknown distribution has a higher optimal price than the Gaussian model • During peak hours, the unknown distribution model cannot account for the minimum power requirement and will result in outages
2012	Baboli et al	• Customer behaviour-based DR model with single-period (self-elastic) and multi-period (cross-elastic) loads	• Incentive (reward) and price (punishment) – based programs evaluated for their effect on consumer habit formation • Weighted coefficient defined as a non-linear function of customer size, sector, income level and social/cultural level defined to differentiate between the two programmes • 24-bus IEEE Reliability Test System used for validation in a numerical study	• Reward-based programmes lead to significant and sustainable improvements in habit formation in contrast to punishment-based programmes • Education and publicity results in higher demand response, even in the face of lower incentives • Exhaustive socio-economic and psychological studies required to more accurately estimate weighted coefficient • Incentive-programmes have long-run benefits

				for the consumer but reduce utility revenue • Price-based programmes increase retailer revenue as well as consumer payments in peak periods
2012	Chen et al	• RTP-based power scheduling model for residents with focus on load uncertainty	• Stackelberg game formulation • Appliance scheduling: power consumption for each time slot is bounded • Load uncertainty defined as the sum of planned power supply and some random variable • Consumer satisfaction modelled as a weighting parameter that varies with user type and time of day • Power consumption modelled as a piece-wise equality constraint subject to scenarios representing supply from the utility	• 10 users and 2 hour time slots considered • Load uncertainty defined with mean of zero and variance equal to 0.005 (bounded magnitudes subject to sensitivity analysis) • Backward induction used to determine the Stackelberg Equilibrium • Load uncertainty decreases each user's payoff but increases retailer revenue • In comparison to Tarasak (2011), load uncertainty does not incur lower revenue • Service provider revenue increases with higher bounded magnitudes placed on load uncertainty
2012	Rastegar et al	• MIP appliance scheduling framework for	• Batteries, responsive appliances and plug-in hybrid electric vehicles (PHEVs) considered • Customer inconvenience modelled as a factor	• Model outputs are appliance operation periods, battery charge/discharge cycle, energy purchasing and selling schedule

		minimising residential payment with incentives	restricting complete direct load control • Incentives dedicated to customers consenting to participate in DLC • TOU tariff structure assumed to be given (taken from Baltimore gas and electric) • Appliance scheduling: elastic and cross-elastic devices, valid hours of appliance operation, continuous operation considered • Several cases involving PHEV and storage system status are considered • Energy transfer capped for every time period to prevent distribution congestion and PAR increase • 10-minute time interval over a six day period, 11 non-responsive and 3 responsive devices considered	• Load control reduces payments but not necessarily peak load or PAR • Multiple optimal solutions possible under TOU tariff • Non-identical valid schedules amongst households results in a more uniform consumption distribution over 24 hours • The battery storage system was more effective in reducing costs than PHEVs, but not in reducing PAR • PHEVs and a storage system were most effective in reducing cost and PAR • Energy transfer limits achieved lowest PAR as appliances, batteries and PHEVs could not be active simultaneously • Level of DLC execution, number of periods in which devices could be switched off and consequential inconvenience integral to decision of participation

2013	Corradi et al	• Data-driven approach to forecasting price-elasticity dynamics with the objective of maintaining constant consumption	• Model for price-responsivity of users must evolve to reflect demographic development and so forth • To reduce communication infrastructure needs, consumption is measured at an aggregated (grid) level • Three models evaluated: effect of external variables on price-consumption assumed to be linear, non-linear, or autoregressive • Applied to heating systems	• Price-responsiveness reached saturation for price changes higher than 1 and lower than -1 (using a standardized price) • Responsiveness is dependent on the time-of-day, reaching maximum levels during periods of high heating demand • Peak consumption reduced by nearly 5% and 11% of mean daily consumption was shifted • FIR model (linear effect of external variables) sufficiently describes price response of aggregation of households • Response duration is between 5-6 hours
2013	Wang et al	• MINLP load scheduling problem with consideration of payment and comfort level	• Novelty of the proposed algorithm is due to combination of original algorithm operation and characteristics of the specific appliance • Appliance scheduling: electrical water heater, upper/lower limits and temperature set point considered • Real-time prices and hot water use assumed to	• Novel inferior Traversal-and-Pruning algorithm applied due to trade-off between speed and optimality • The approach can be combined with other general algorithms • 5-minute intervals considered • 20% more cost savings with the same comfort

			be obtained from historical data	levels than with the original algorithm
2014	Setlhaolo et al	• MINLP scheduling problem to maximise social welfare for the consumer under TOU tariff	• Social welfare is defined as the sum of costs, subject to earned incentives, and consumer inconvenience • Incentives earned by switching off appliances during peak periods • Appliance scheduling: minimum appliance operation, continuous operation, sequential operation, valid hours of appliance operation considered • Maximum cost that consumers are willing to incur over a 24-hour period is defined • Case study performed for a South African household, with TOU structure adopted from Eskom's Homeflex tariff structure	• 10-minute time interval and 10 appliances considered • Model solved with Aimms Outer Approximation Algorithm • Cost savings of more than 25% reported • Consumer inconvenience modelled as the difference between a user's preferred schedule and the optimal schedule with the aid of a weighting factor • Sensitivity analysis performed for various weighting factors which represents the extent to which consumers are willing to be inconvenienced

2.3 THE RETAILER PROBLEM

Retailers in the electricity industry have a number of complex decisions to make due to their central position in the energy supply chain. Upstream, there are a number of suppliers and procurement options from which to select and downstream, there exists a large set of end-users to service. Their ultimate intention is thus to manage these contracts so that revenue from end-users covers the cost of goods paid to generators. Municipalities, which fulfil the same role as retailer in a regulated market, are confronted with the same challenge but do not have the same breadth of procurement options neither are they exposed to the level of risk and uncertainty as would be the case when operating in an open market system. Similar to literature for the consumer's problem, there are also a multitude of objectives that they must address, such as that of minimised settlement risk (Gabriel et al, 2004), optimal procurement strategies (Carrion et al, 2007), optimal selling price (Hatami et al, 2009) and minimised network losses (Saele and Grande, 2011). Because the focus of this study is on consumer-retailer interaction, attention will only be paid to factors pertaining to the end-user that affect retailer decisions. One such factor is that of the balance between supply and demand.

As was seen in Figure 1.3, retailers are required to submit bids in advance for the purchase of electricity. This form of procurement, known as a forward option (or the wholesale market) because trading occurs in advance of delivery of the commodity, is often associated with less risk and financial expense than other trading options (Gabriel et al, 2004). Significant effort is thus spent on appropriately forecasting expected demand to take advantage of this option, but the uncertainty associated with supply such as transmission failures or unexpected user demand patterns means that deviations often occur. In these scenarios, retailers must resort to the spot market to balance supply and demand in real-time. When demand is over-estimated, that is, the predicted demand is more than the observed demand, the retailer has the opportunity to sell excess electricity back to the national grid and when it is under-estimated, they may procure electricity to meet consumer load. In works such as Sou et al (2011) and Tarasak (2011) where the balancing market for a deregulated industry is not considered, perfect supply is assumed.

This means that the predicted and observed demands are always equal to one another. Naturally, this is not an accurate depiction of reality because it is an integral part of open market dynamics and therefore fails to satisfy objectives stated in Section 1.3. Furthermore, the energy provider is interested in maximising net profits and not simply revenues, and this spot market dynamic, specifically the prices at which trading occurs, has a significant impact on their outcomes.

According to Gabriel et al (2004), there are six cases that must be analysed when considering spot market prices, supplier prices and retail prices in relation to one another and these are shown in Table 2.2. The occurrence of these scenarios is however stochastic due to uncertainty in load profiles market dynamics. Because these outcomes affect delivered profits for the retailer, they in turn affect their price elasticity and the identification of a tariff at which social welfare is optimised. These factors are critical to the problem at hand and as such, sufficient attention must be paid to selecting an appropriate technique that will predict spot market prices.

The literature available on electricity price modelling is very rich. Two challenges persist however. The first is that the South African energy market operates under a vertically integrated utility and does not have the historical data necessary for selecting an appropriate model to spot prices. In fact, according to Serati et al (2007) and Aiube et al (2013), there is no one specific model that has been supported by empirical evidence, but rather the suitability of models depends largely on the nature of a market and the decision-maker’s problem. The second challenge is that most recent literature in this field of study is focused on renewable energy generation, specifically wind and solar power, the modified marginal cost structure it introduces, and its reportedly significant effect on the dynamics of energy markets (Woo et al, 2011, Wurzberg et al, 2013 and Ziel et al, 2015). Because the contributions of the local renewable energy industry are capped to a meagre 9% by 2030, it would not play a determining role in spot price signals unlike other markets in Europe. Indeed, the South African energy market is thus still very much operating in a time that is almost a decade behind its global counterparts.

To overcome the challenges stated above, a model must be identified that balances mathematical convenience but also creates the realism of a South African dynamic energy market. In order to achieve this, a brief discussion on the characteristics that make electricity distinctive from other commodities is given in Section 2.3.1. In Section 2.3.2 an overview of the pricing models that have successfully addressed these features and been applied to existing markets is presented. Finally, Section 2.3.3 presents the justification for the selected technique applied to modelling spot prices under uncertainty.

Table 2.2: Various outcomes for over- and under-estimating load (Gabriel et al, 2004)

Prices	**Load estimate**	**Result**
$Price_{supplier} < Price_{retail} < Price_{spot}$	Over	Profit
$Price_{supplier} < Price_{retail} < Price_{spot}$	Under	Loss
$Price_{supplier} < Price_{spot} < Price_{retail}$	Over	Profit
$Price_{supplier} < Price_{spot} < Price_{retail}$	Under	Profit
$Price_{spot} < Price_{supplier} < Price_{retail}$	Over	Profit
$Price_{spot} < Price_{supplier} < Price_{retail}$	Under	Loss

2.3.1 Characteristics of electricity prices

The increased availability of supply and demand data since the shift towards deregulated industries has enabled the relationships between spot prices and their underpinning drivers to be better understood and analysed. Serati et al (2007), Hardle and Truck (2010) and Carmona and Coulon (2013) note the following characteristics:

- Mean-reversion
  Lucia and Schwartz (2002) and Knittel and Roberts (2005) were among the first to demonstrate mean-reversion as a classical behaviour of electricity spot market prices. In dynamic markets, this is an important stabilising property for reducing prices during jumps and allowing them to oscillate around long run averages during normal periods (Mari, 2006 and Koopman et al, 2007).

- Non-storability

Electricity cannot be stored or transported economically in large quantities (Weron, 2007). Since it is often traded hours or at most, days in advance of when it is to be consumed, prices are largely determined by the overall costs of production, the bulk of which can be attributed to the costs of fossil fuels such as coal or natural gas. In the case of substantial renewable energy generation where production costs are approximately zero (Ziel et al, 2015), as in the cases of the Nord Pool and the European Power Exchange (EPEX), an interesting phenomenon of negative pricing has arisen (discussed later in this section). Hydro-electric storage in the Nord Pool has also introduced price stability that is not necessarily present in other markets. Thus, price formation, as in most commodity markets, is strongly driven by the balance of supply and demand, as is shown by Carmona and Coulon (2013) with the use of equilibrium pricing.

- Seasonality

  Regular and predictable changes which occur periodically are said to be seasonal. In electricity, three types of seasonality are reflected in time series data: annual, weekly and intra-day cycles. Market prices are notoriously vulnerable to annual seasonality due to the effects of fluctuating temperatures (Jablonska-Sabuka et al, 2011), sometimes even resembling mirror behaviour; weekly seasonality is attributed to different business and social patterns on weekends versus weekdays that results in higher demand during the latter (Huisman et al, 2007); and intra-day periodicity results in hours of peak and off-peak consumption that largely correlate with hours of the day and night respectively (Higgs and Worthington, 2005).

- Extreme volatility

  Daily electricity price volatility is excessively high and is known to frequently reach levels of 30%-200% compared to 3%-5% for other commodities such as oil and natural gas in the same period (Serati et al, 2007 and Carmona and Coulon, 2010). According to Hardle and Truck (2010), spot prices may increase tenfold in a single hour before reverting to their mean in a very short period of time.

- Price jumps and spikes

  Spikes are the result of shocks in both the supply and demand of electricity. For example, extreme load fluctuations caused by strong seasonality, generation outages and transmission failures all culminate in imbalanced supply-demand curves. Low marginal production costs but high start-up costs incurred by generation facilities during periods of high demand can also often be attributed to positive price shocks. This is further exacerbated by the non-storability of electricity, network capacity constraints and the inflexibility of electricity markets, all of which vary from one market to the next.

- Negative and zero prices

  Negative prices occur in geographic clusters during low-peak months of the year and for short periods during the early mornings or late evenings when demand is especially low. They occur more frequently in markets with inflexible or renewable generation methods because facilities are too costly to shut down temporarily or production costs are negligible, resulting in imbalances of supply and demand. Other causes identified include errors in load predictions due to high temperature volatilities and network congestion that results in an oversupply in one area and an undersupply in another. Negative pricing proves problematic during modelling as certain transformations such as log transformations are unable to handle these values. Schneider (2011) suggests removing these observations from time series data, shifting prices or using transformations that are equipped to handle non-positive prices.

- Inverse leverage effect

  Spot prices have a tendency of responding asymmetrically to positive and negative shocks, that is, volatility is more intense in response to positive price shocks than negative (Karakatsani and Bunn, 2004 and Knittel and Roberts, 2005). Thomas and Mitchell (2005) reported the effects of inverse leverage in the Australian market to be more predominant for intra-day seasonality. Because this particular feature is still establishing itself as an inherent descriptor of electricity prices, its effects on modelling and model selection are still unknown.

2.3.2 Modelling of uncertainty: existing techniques

Modelling the dynamics of electricity prices in the spot and regulating markets means addressing each of the characteristics just discussed. One of the first works presented in this field was the classic Ornstein-Uhlenbeck process by Lucia and Schwartz (2002). This technique was selected to address the mean-reverting characteristics and predictable seasonality typical of electricity prices. Here, prices are modelled as the sum of a deterministic and stochastic component, and this methodology has been largely adopted as the 'industry standard' by most researchers in the field (Mari, 2006, Huisman et al, 2007, Higgs and Worthington, 2008, and Auibe et al, 2013), with techniques varying on how the deterministic and stochastic components are found with respect to each market. Attention has primarily been paid in the existing literature to modelling the stochastic component and Serati et al (2007) propose a classification of the methodologies into three broad categories, namely autoregressive models, volatility models and jump diffusion and regime-switching models. Section 2.3.2.1 to Section 2.3.2.3 which follows provides further discussion on each of these models and Table 2.3 summarizes some representative papers.

2.3.2.1 Continuous stochastic and autoregressive models

Geometric Brownian motion, otherwise known as the Wiener process, has its roots in the modelling of stock market prices with the Black-Scholes model (Black and Scholes, 1973). It was first extended to electricity markets by Lucia and Schwartz (2002) to model stochasticity in the Nord Pool. Since then, the technique has been overlooked in favour of other continuous stochastic processes capable of generating spikes and heavy tailed random variables, such as the Poisson process (Mari, 2006, Jablonska-Sabuka et al, 2011 and Mayer et al, 2015) Nevertheless, Brownian motion is still being used in literature to address the normal equilibrium behaviour of prices (Mayer et al, 2015).

Autoregressive (AR) models were a natural candidate for first generation electricity models due to their ability to capture seasonality and significant lags in other commodity markets. Early comparative works such as Karakatsani and Bunn (2004) and Misiorek et al (2006) soon revealed the limitations of AR models in capturing fluctuating volatility, price spikes and other unique characteristics of electricity such as the inverse leverage effect. To overcome this, AR features have since been combined with other volatility, mean-reverting and regime switching models to report improved goodness-of-fit results and forecasting errors, such as in the cases of Aiube et al (2013) and Ziel et al (2015). The AR features prove especially effective in investigating the effects of lags and model orders for increasing model accuracy. The finding of Aiube et al (2013) that increasing model order and lags improves fit bodes poorly for modellers and researchers seeking mathematical convenience.

2.3.2.2 Volatility models

Volatility can vary dramatically over time, especially in commodity markets. The most popular measurement of volatility, variance, can be classified as homoscedastic (constant) or heteroscedastic (non-constant). Homoscedastic models such as ARMA models were initially used to model financial markets, but were found to be severely lacking due to the fluctuating volatility of data they were unable to capture. This is even more so the case for electricity prices (Carmona and Coulon, 2010). Techniques aimed at capturing conditional variance and heteroscedasticity, such as GARCH and GARCH-extension models, have thus garnered much attention in this field of research, as can be seen in Table 2.3.

Karakatsani and Bunn (2004), Misiorek et al (2006) and Aiube et al (2013) indicate improved performance of GARCH models when combined with autoregressive and seasonality components respectively. This is likely due to the extended models' ability to better anticipate abnormalities in price levels. Of the GARCH-derivative models, EGARCH and TARCH were reported to effectively capture price volatility (Mayer et al, 2015) and conditional heteroscedasticity (Ziel et al, 2015) in the British and German markets, whilst EGARCH and PARCH reported superior results in detecting the inverse leverage effect and negative pricing (Thomas and Mitchell, 2005). In Hatami et al (2009), little analysis is provided as to the ability of the models in fitting or forecasting prices and the GARCH-jump model remains invalidated. The SARMA approach of Aiube et al (2013) with lags can be seen as an alternative to panel frameworks (Huisman et al, 2007 and Pena, 2012), but is less cumbersome whilst still revealing the effects of intra-day seasonality. Overall, the GARCH model has earned its formidable reputation in financial markets, which are significantly less volatile than electricity prices. A critical flaw of this category of models is thus that whilst they are effective in modelling volatility of electricity prices, they prove inadequate in capturing short-lived spikes without significantly high parameter values to quickly force prices back to their equilibrium state.

2.3.2.3 Mean-reverting jump diffusion and regime switching models

Inarguably the biggest shortcoming of Lucia and Schwartz (2002) was the assumption of constant mean reversion. Not only did this directly contradict market observations (Christensen et al, 2009), but the proposed method was unable to capture the volatility and short-lived nature of spikes because they did not follow distinct patterns or have stable rates of mean-reversion. To overcome this, Jablonska-Sabuka et al (2011) modelled electricity dynamics as three separate states defined as ‘regular, ‘spike’ and ‘after-spike’ periods, each with their own mean-reversion rates and volatility parameters. Across literature, classifying behaviour by these three states in multiple mean-reversion, jump diffusion or regime-switching models has become common practice, as can been seen in Karakatsani and Bunn (2004), Mari (2006), Higgs and Worthington (2008) and Janczura and Weron (2012). Whilst the authors met their original goal of capturing spike behaviour, their inattention to addressing intra-day seasonality could be attributed to the 24-hour reversion time interval used that ignored the cross-sectional correlation identified by Huisman et al (2007) accounting for hourly dynamics in a time series. As

Huisman and Mahieu (2003) reflect, jump diffusion models do not distinguish between mean-reversion and jump-reversal which can cause mis-specified volatility. Furthermore, these models require large parameters to create short-lived spikes and are better suited when jumps are sustained. For a general specification of mean-reversion jump diffusion models that accounts for many of the features of electricity prices, the reader is referred to Weron (2007).

Huisman et al (2007) focus their study on the effects of intra-day and inter-day seasonality. Their work found that prices in peak hours are highly correlated and occur in block structured patterns (the same is true for off-peak hours). Pena (2012) also applied a panel model to each hourly series but adopted an autoregressive periodic component that yielded better results than a standard mean-reverting one. The attention paid to intra-day patterns is fairly rare for this category of models due to the parameter estimates required for each of the 24 models developed for each hour of the day. More popularly, intra-day seasonality has most typically been documented by means of GARCH models such as in Aiube et al (2013) and Higgs and Worthington (2005). Naturally, this brings into question the mathematical convenience and cumbersome nature of the required numerical work.

Markov Regime Switching (MRS) models extend on the concept of mean-reversion and reportedly exceed the performance of its autoregressive, volatility and mean-reverting jump diffusion counterparts (Karakatsani and Bunn, 2004, Misiorek et al, 2006 and Janczura and Weron, 2010). In applying this approach, separate states are defined to model the dynamics of electricity, each characterised by a mean-reversion rate and volatility parameter. MRS models are thus able to deliver the short-lived spikes, mean reversion and high volatility typical of electricity prices whilst also avoiding the high parameter estimate values typical of mean-reversion and diffusion models. According to Mari (2006), two different price movements are generally observed: normal periods defined by prices fluctuating around a long-run equilibrium, and periods of turbulence characterised by jumps and short-lived spikes. Similarities exist between Mari (2006), Higgs and Worthington (2008) and Jablonska-Sabuka et al (2011), and all studies disprove the constant mean-reversion assumption of Lucia and Schwartz (2002). In Mari (2006) and Higgs and Worthington (2008), regime-switching is controlled by a Markovian probability matrix. In contrast, Karakatsani and Bunn (2004) define regimes by probabilistic inference, but this is due to their aim of investigating the effects of market structures on price volatility. Unlike Mari (2006), Higgs and Worthington (2008) model seasonality both weekly and annually but do not account for multiple consecutive jumps or intra-day behaviour. In one of few comparative studies in the field of Markov Regime Switching, Janczura and Weron (2012) found that threshold type regime switching models such as TAR, STAR and SETAR increased modelling risk because the threshold variable would need to be specified in advance. Although latent variable models were a greater challenge to calibrate, they did not pose the same disadvantage. Of the models investigated, a three-regime model with time-varying transition probability matrix, heteroscedastic diffusion type base regime dynamics and shifted spike regime distributions offered the best results in addressing seasonal spikes, spike intensity and the inverse leverage effect. Their goodness-of-fit hypothesis results and descriptive statistics also proved superior to other known models. Due to the complexity of the proposed model however, it is argued that this be a topic of future research as it falls beyond the scope of this study.

2.3.3 Modelling of uncertainty: selecting an appropriate technique

In Section 2.3 it was established that model selection depends heavily on the market to which it is applied. Because South Africa cannot rely on its own market dynamics or historical data for model selection, an appropriate substitute market must be selected. Table 2.3 provides an indication of the most popular markets for which spot prices have been modelled, namely the cross-continental European exchanges such as Nord Pool, EEX and EPEX, Australia and less frequently, the United States. The lack of research conducted for emerging markets can be attributed to fewer countries adopting deregulation and a resulting lack of data availability. Karakatsani and Bunn (2004) use fuel prices as a proxy for seasonal spot price behaviour, but Eydeland (2003) provides evidence to suggest that this may not be sufficient to account for actual marginal costs in generation and the resulting supply curve. Furthermore, supply stack structures may be complex in markets where there is more than one major fuel source driving dynamics. In studies by Mari (2006), Higgs and Worthington (2008) and Jablonska-Sabuka et al (2011), it was revealed that volatility was consistently different between cool and warm seasons and that this particular aspect of seasonality estimation was crucial to accurate price predictions. This entrenched the view that a strong correlation exists between temperature and electricity pricing. Along this vein, a comparison of the most popular markets for spot price estimation, namely the Nord Pool (representing European market dynamics) and Australia, is done in order to determine appropriateness of fit to the South African environment.

Nord Pool currently operates in Norway, Denmark, Sweden, Finland, Estonia, Latvia, Lithuania, Germany and the United Kingdom. It is the largest, most mature and successful electricity trading market in Europe, with good collaboration efforts amongst the parties that enables some of the lowest cost structures in the world. Temperature and daylight trends indicate higher demand for lighting and heating facilities in winter, and cooling and ventilation facilities in summer, especially in Scandinavian climates where temperatures fluctuate annually between -12ºC and 26ºC. Nord Pool are the pioneers in renewable energy and hydro-storage, both of which introduce price stability and low production costs that are unique to the market. As such, selecting it to simulate the effects of a South African environment would create supply, demand and cost curves that are poor representations. The work performed with relation to the Australian market is indeed promising in comparison to other literature due to similarities with the South African market. Seasonally, Australia experiences hot summers and cool winters with temperatures averaging 25ºC and 7ºC respectively; similar to the local climate. Daylight hours are also comparable to South Africa as both countries fall within the Southern Hemisphere, meaning that energy requirements would follow similar seasonal and demand patterns. Lastly, both countries rely heavily on coal-sourced energy with little renewable fuels capacity, implying that their market resilience, ability to store electricity and its effect on supply curves and equilibrium pricing are comparable. For these reasons, the Australian market is selected as a proxy for the South African deregulated market.

Attention must now be paid to model selection. It is proposed that the three-regime switching model presented in the work of Higgs and Worthington (2008) be applied due, firstly, to its reported success in the Australian market and, secondly, to the superiority of regime switching models over other categories of models for the reasons discussed in Section 2.2. The primary limitations of the study however were its inattention to intra-day seasonality and its restrictive assumptions regarding spike behaviour. Their model is thus amended to include dummy variables accounting for intra-day seasonality and a constant transition probability matrix that allows for multiple consecutive jumps and spikes similar to the work of Mari (2006). Such a model applied to the Australian market accounts for the stylized features of electricity, specifically mean-reversion, non-storability, seasonality, volatility and spikes. Although it neglects to address the inverse leverage effect and negative pricing, neither of these features have been indicated by past literature to be significant for the Australian market. It is further assumed that strategic bidding is not allowed which substantially reduces the frequency of negative pricing (Ziel et al, 2015) and postulated that the presence of the inverse leverage effect will be detected during parameter estimation and analysis.

Table 2.3: Representative works: modelling under uncertainty

Year	Author/s	Model/s	Main features	Market	Contributions
2004	Karakatsani and Bunn	• Regression model with GLE, GARCH, time-varying parameter, regime-switching	• The model is implemented for each load period (half hourly) • Regime-switching is defined by probabilistic inference • Seasonality approximated with a sinusoidal function as a proxy for annual fuel price pattern	• United Kingdom	• Regression-GARCH outperformed GARCH, GLE and time-varying parameter extensions • Regime-switching was most effective in capturing stochastic behaviour
2005	Thomas and Mitchell	• GARCH • TARCH • EGARCH • PARCH	• Seasonality and price spikes are filtered to study only the underlying volatility process • The selected models are used to address the presence and significance of the inverse leverage effect and negative prices	• Australia	• Significant ARCH and GARCH effects were present in the data • PARCH and EGARCH were most successful in detecting the inverse leverage effect and addressing negative pricing • AR effects were significant at specific lags and location-dependent
2006	Mari	• 2-regime switching model	• States modelled as mean-reverting processes with unique mean-reversion and volatility rates	• APX • EEX • Nord Pool	• Extends on Huisman-Mahieu method (2003) to include multiple jumps and consecutive spikes

		• 3-regime switching model	• Jumps modelled as a Poisson process • Constant Markov transitions control regime-switching • Seasonality is deterministic with dummy variables to account for weekly behaviour	• Powernext • EXAA	• 3-regime model with multiple jumps was most effective in capturing fluctuations for all markets except APX, where 2-regime model performed better • Extreme jump behaviour was observed in EEX and APX
2006	Misiorek et al	• AR/ARX • AR-GARCH • TAR (non-linear, threshold regime switching) • Regime-switching model	• The observed period coincided with the Californian market crash • Relatively simply time series models (AR) were compared with typical volatility (GARCH) and regime-switching models	• California	• The Markov model outperformed its counterparts • TAR models outperformed linear AR models, which in turn produced higher point forecasting efficiency than GARCH approaches • The non-linear regime switching model systematically underestimated next-day prices
2007	Huisman et al	• Panel model with 24 cross-sectional hours	• Mean reversion rates modelled for each hour of the day • Seasonality is deterministic with dummy variables to account for weekly	• APX • EEX • PPX	• Hourly prices in a day behave cross-sectionally and hourly dynamics over days behave as a time series • Weekend mean price levels were

			behaviour		significantly lower than average means • Peak week hours exhibited slower mean-reversion and higher volatility
2008	Higgs and Worthington	• Mean-reverting model • Stochastic model • 3-regime switching model	• States modelled as mean-reverting processes with unique mean-reversion and volatility rates • Constant Markov transitions control regime-switching • Seasonality is deterministic with dummy variables to account for weekly and yearly behaviour	• Australia	• Regime-switching was most effective in price predictions • Prices exhibited stronger mean-reversion after price spikes • Volatility was higher in spike periods and varied from one state to another • Higher prices were associated with weekdays and peak winter and summer months
2009	Hatami et al	• GARCH model • GARCH-jump model	• GARCH models retailer load • GARCH-jump models spot market prices	--	• Joint set of realizations for retailer load and spot market prices are generated • 100 scenarios were found to be sufficiently large • Highest retailer profits and lowest risk yielded from purchasing in spot market, forward contracts, call-options and self-

					production
2011	Jablonska-Sabuka et al	• Multiple mean reversion jump diffusion model	• Parameters modelled for each hour of the day • Jumps modelled as a Poisson process • Constant Markov transitions control mean-reversion switching • Mean-reversion switches are independent of one another but related to price level • Seasonality is included in mean hourly prices as a fitted linear regression of price versus temperature	• Nord Pool	• Builds on classical Ornstein-Uhlenbeck process that assumes constant rate of mean-reversion • Effectively modelled spikes and extreme behaviour of spot prices • Out-of-sample simulations indicated weaknesses in driving intra-day seasonality and price stability introduced by hydro-storage
2012	Janczura and Weron	• 3-regime switching model	• Focus is on the calibration of mean-reverting regime switching models • Parameter-switching regimes, independent mean-reverting processes for regimes and a combination of the two are compared	• EEX • Australia	• Australian prices did not exhibit significant price drops, unlike EEX prices • High speeds of mean-reversion were typically associated with parameter estimates of 0.20 to 0.44 • Staying in the same regime resulted in parameter estimates of approximately 0.6390

					(spike regime) and 0.9539 (for the base regime) • The proposed estimation procedure calculated estimates to within a 95% confidence of true values for samples of over 1000 observations • Lower errors were presented in comparison to that of Huisman and de Jong (2002), and allowed for 100 to over 1000 times faster calibration
2013	Aiube et al	• SARMA-GARCH	• Model presented to forecast hourly spot prices for a week • SARMA method used to capture seasonality through autoregressive means • Comparisons between varying lags and model orders are presented • GARCH method used to capture volatility	• Austria • Spain	• SARMA-GARCH model fitted data better than the SARMA model but presented with higher forecast errors for both countries • Static forecasts outperformed dynamic forecasts, especially in the presence of higher volatility • SARMA (7,7) model effectively captured weekly seasonality • In general, increasing order and lags

					improves model fit
2015	Mayer et al	• Levy process with constant volatility • Levy process with EGARCH volatility	• Annual and weekly seasonality is addressed deterministically • 'Normal' and 'extreme' price behaviour is modelled as two Levy processes with Brownian and Poisson jump motions respectively • Mean-reversion is modelled as an Ornstein-Uhlenbeck process	• France • Germany • Scandinavia • Great Britain	• Extends on the work of Benth et al (2003) to additionally address stochastic volatility, negative prices and self-contained parameter estimation • The proposed models performed better than the Ornstein-Uhlenbeck and jump diffusion model of Cartea and Figueroa (2005) in all markets • EGARCH model performed equally well to the constant volatility model in the Nordic market, but better in the German and British
2015	Ziel et al	• Periodic VAR-TARCH model	• The effects of load, wind and solar power on prices are investigated • Focus is paid to the inverse leverage effect and negative pricing in particular • Iteratively reweighted lasso approach is used for parameter estimation • Over 1000 lags are considered	• EPEX	• The model was tailor-made for hourly EPEX data and cannot be generalized • TARCH model was effective in capturing conditional heteroscedasticity • Load, wind and solar power had strong and significant impacts on price • Lags indicated that prices in the previous

					two hours and prices one week ago were significant

2.4 CONCLUSION

In this chapter the focus of the reviewed literature was on developing an accurate depiction of reality for the South African resident and energy provider operating in an open market. Furthermore, the infrastructural, technological and social requirements in order to achieve demand response needed to be highlighted so that its benefits could be realised by the proposed model and quantified for its effect on social welfare. To this end, six key factors for the consumer problem were discussed extensively, and this will form the basis for the formulated model which is presented in Chapter 3. These elements were also identified as the conditions under which effective demand response can be achieved. For the retailer, spot market trading and its inherent uncertainty was seen to have a significant impact on profits and thus social welfare. As such, an approach to predict its effect on the retailer, above and beyond other market dynamics, was required and reviewed. It was found that the application of a three-regime Markov model to the Australian market is an appropriate representation of the conditions under which a local retailer may operate, thereby satisfying the objective outlined in Section 1.3. Finally, it was shown that published works on problem formulations found in literature were insufficient in capturing the price elasticity of the consumer relative to the retailer, and as a result, social welfare. A novel approach is therefore required that lends itself to identifying the tariff at which optimal social welfare is achieved, and residents and service providers objectives are realised. This chapter thus provides the building blocks to construct the proposed model which is presented in the following chapter.

CHAPTER 3

MATHEMATICAL MODELS OF CONSUMER SCHEDULING AND RETAILER PROBLEMS

3.1 INTRODUCTION

It was established in Chapter 2 that there are several important aspects that must be considered when developing consumer and retailer models that reflect the South African reality under a deregulated market structure. The thread linking the satisfaction of both these stakeholders' aims lies in the achievement of social welfare, and to this end, the relative price sensitivity of consumers to their retailer is of integral importance. Neither social welfare nor the price-dependent relationship between the user and service provider has been appropriately captured or analysed in literature, and herein lies the primary contribution of this study. This is a novel approach to viewing the retail tariff as not simply a decision to be made by the retailer *to* the consumer, but rather as a tool for creating harmony amongst both parties, as long as the effect of price changes on each stakeholder can be quantified. Previous studies have either neglected this concept entirely, yet acknowledged its importance (Kirschen, 2003), or assumed this effect to be linear (Aalami et al, 2008). In order for social welfare to be addressed, its effect on the setting of a tariff to be quantified and hypotheses for relative price elasticity to be tested, operations research models for the consumer and retailer must first be proposed. From this, valuable analyses can be made that contribute to the existing body of knowledge in this field. The formulated models must therefore be sufficiently flexible to lend itself to parameter and input adjustments as similar expectations will exist of a model that is integrated with an Energy Management System for implementation. This chapter presents the numerical model that captures the salient features just discussed.

The remainder of this chapter is structured as follows: Section 3.2 provides the notation necessary for the rest of this chapter. In Section 3.3 the consumer problem is formulated. Section 3.4 presents the retailer problem with specific focus paid to the three-regime Markov switching model from which results will serve as an input parameter. The relationship between the independently developed consumer and retailer models to achieve the objective of maximised social welfare is presented in a novel approach in Section 3.5. Finally, Section 3.6 concludes this chapter.

3.2 NOTATION

$\alpha_{i,t}^{j}$: the inconvenience factor associated with user j using appliance i at time t

ρ_t : the price signal delivered to the consumer by the retailer at time t, in R/kWh

δ_i : the degree of flexibility associated with the user postponing or advancing the use of appliance i from the preferred schedule

λ_j : the importance of the waiting cost for user j (ZAR)

ω_c : the charging efficiency of a user's battery

ω_d : the discharging efficiency of a user's battery

μ_q : average wholesale price of electricity purchased by the retailer, obtained from NEM operating in NSW over the period January 2015, converted to R/kWh

$\sigma_{q,t}^2$: price variance in the day-ahead market for time t

$\sigma_{C,t}^2$: variance of aggregated load experienced in observed user demand for time t

b_t^j : $\begin{cases} 1 \text{ if user } j \text{ charges their battery at time } t \\ 0 \text{ otherwise} \end{cases}$

d_t^j : $\begin{cases} 1 \text{ if user } j \text{ discharges their battery at time } t \\ 0 \text{ otherwise} \end{cases}$

e_i : the initial time segment for the valid schedule in which appliance i may be used by the user

f_i : the final time segment for the valid schedule in which appliance i may be used by the user

h_i : the initial time segment for which appliance i is used in the preferred schedule by the user

q_t : price of electricity purchased (R/kWh) by the retailer in the day-ahead market for time t

s_t : the price at which electricity is traded in the spot market at time t

$u_{i,t}^j$: $\begin{cases} 1 \text{ if user } j \text{ uses appliance } i \text{ at time } t \\ 0 \text{ otherwise} \end{cases}$

$ubl_{i,t}^j$: $\begin{cases} 1 \text{ if user } j\text{'s preference is to use appliance } i \text{ at time } t \\ 0 \text{ otherwise} \end{cases}$

x_j : the maximum allowable energy to be charged/discharged to user j's battery, in kWh

$z_{i,t}^j$: $\begin{cases} 1 \text{ if user } j \text{ uses appliance } i \text{ at time } t, \text{ where } t \text{ is the first in consecutive segments} \\ 0 \text{ otherwise} \end{cases}$

B_t^j : the power charged to user j's battery at time t, in kW

C_t^* : aggregated power drawn from the grid by all users at time t, in kW

C_t^j : the power drawn from the grid by user j at time t, in kW

D_i : the minimum duration for using appliance i, in hours

E_t^j : the energy state of user j's battery at time t, in kWh

$Ecap_j$:the assigned capacity of user j's battery, in kWh

E_{max}^j: the maximum capacity of user j's battery, in kWh

E_{min}^j : the minimum capacity of user j's battery, in kWh

G_t^j : the power discharged to user j's battery at time t, in kW

N_t : the quantity of electricity sold by the retailer at time t in the spot market

P_i : the power rating of appliance i, in kW

Q_t : the quantity of electricity purchased by the retailer for time t in the day ahead market

S_t : the quantity of electricity purchased by the retailer at time t in the spot market

3.3 CONSUMER PROBLEM

In previous works, aspects of the load scheduling problem have been well addressed by the likes of Mohsenian-Rad and Leon-Garcia (2010), Chen et al (2012b) and Setlhaolo et al (2014). Specifically, each of these authors focused on appliance scheduling and consumer inconvenience, battery storage modelling, and incentives respectively. However, none of these works sought to identify the tariff structures at which social welfare is optimal, neither did they capture the relative sensitivity of retailers and consumers to changes in price. This study aims to do exactly this, with attention paid to the incorporation of tools that enable demand response in order to derive and quantify its effect on identified outcomes. To achieve this, each stakeholder's model must be well-segmented so that their individual and holistic impact on social welfare may be easily investigated. On a macro level the resident must firstly further their interests both financially and from the perspective of ensuring convenience. Secondly, their appliance and battery usage must be modelled realistically and with sufficient flexibility to enable optimisation. The unique and diverging socio-economic climate in South Africa which has a fundamental impact on whether homes are considered to be energy intensive or poor, must also be accounted for. Finally, the fundamental concept of physics, that is the Law of Conservation of Energy, must be obeyed so that heat and transfer losses to the environment are realistically accounted for. Only then will the proposed model meet the objectives outlined in Section 1.3, that is the development of a contextualised consumer model for the local environment that is equipped to achieve demand response.

For the consumer, it has been established that the load scheduling problem must encompass their interests which are two-fold, namely to minimise electricity payments and scheduling inconvenience. Their utilitymust then be the sum of these two components:

$$U = \sum_{t=1}^{T} \rho_t \sum_{j=1}^{J} C_t^j + \sum_{j=1}^{J} \lambda_j \sum_{t=1}^{T} \sum_{i=1}^{I} \alpha_{i,t}^j \left| ubl_{i,t}^j - u_{i,t}^j \right| \tag{3.1}$$

The first term represents the aggregated cost incurred by all residents for consuming electricity charged at retail price ρ_t. The presence of this nonlinear term in the user objective serves to significantly increase model intractability, as is discussed further below. The second term quantifies the inconvenience imposed on the consumer for shifting their load from the preferred schedule $ubl_{i,t}^{j}$ to the optimal schedule $u_{i,t}^{j}$. The waiting parameter λ_j and inconvenience factor $\alpha_{i,t}^{j}$ are used similarly to Mohsenian-Rad and Leon-Garcia (2010).

The net power drawn from the grid by each user is given in equation (3.2) as the sum of power drawn from appliances during usage and the battery during charging, less the power supplied by the battery to assist with peak shaving. It is noted that unlike Rastegar et al (2010) and Chen et al (2012a), a discharging efficiency is considered. From this expression it is clear that model complexity significantly increases and the reasons for this are two-fold. Firstly, the introduction of the three-dimensional binary variable $u_{i,t}^{j}$, and two-dimensional continuous variables B_t^j and G_t^j significantly increases the scale of the optimization problem, resulting in a potentially enormous parameter space that often is far too computationally expensive to effectively search. In fact, it is well-established in literature that many algorithms which have proven to be high-performing with small-scale problems are less so with large-scale ones, considered to constitute at least one thousand variables (Hager et al, 1994, Benson et al, 2003). Secondly, the expression in (3.2), when combined with that of ρ_t in the consumer utilityfunction, creates a nonlinear, non-convex search space when advanced schemes such as TOU and dynamic tariffs are employed. This causes the model to become increasingly more difficult and computationally expensive to solve as the number of discrete variables increases (Murray and Ng, 2010). Whilst in similar formulations Lagrangian relaxation techniques and Karush-Kuhn-Tucker conditions may be applied to overcome some of these challenges, this is rarely possible for scheduling problems due to their inherent binary nature (Grobler, 2008). At such points in research, attention is then turned to the application and feasibility of heuristics and metaheuristics, and this is studied further in Chapter 5. Under a fixed rate tariff however, which is far more suitable for a fledgling deregulated market operating in South Africa, a substitution approach that delivers global solutions is demonstrated in Chapter 4, and this is sufficient to demonstrate the principal theory and novelty of the social welfare function proposed in Section 3.5.

$$C_t^j = \sum_{i=1}^{I} P_i u_{i,t}^j + B_t^j - \omega_d G_t^j \tag{3.2}$$

Section 2.2.4 highlighted several technical and operational appliance scheduling constraints that must be enforced in order to depict a realistic residential load profile:

$$\sum_{t=e_i}^{f_i} u_{i,t}^j \geq D_i \tag{3.3}$$

$$\sum_{t=e_i}^{f_i - D_i + 1} z_{i,t}^j = 1 \tag{3.4}$$

$$u_{i,t}^j + \ldots u_{i,t+D_i-1}^j \geq D_i \, z_{i,t}^j, \qquad t \in [e_i, f_i - D_i + 1] \tag{3.5}$$

$$z_{i,t}^j = 0, \qquad t \notin [e_i, f_i - D_i + 1] \tag{3.6}$$

$$u_{i,t}^j = 0, \qquad t \notin [e_i, f_i] \tag{3.7}$$

The above constraints follow the work of Conejo et al (2010) and Setlhaolo et al (2014) among others by accounting for the typical behaviour of residential loads. Constraint (3.3) ensures that each appliance is used for its minimum duration within a valid time schedule. (3.4) and (3.5) require that each appliance is operated continuously for its minimum duration of operation, that is, an electric water heater cannot operate for two hours in the morning and one hour in the evening if its minimum duration of operation is three hours. It is noted that continuous operation is modelled slightly differently here in comparison to Setlhaolo et al (2014): the introduction of $z_{i,t}^j$ enables the constraint to be kept linear, unlike the authors' work who introduce nonlinearity and thus additional complexity to the model. Finally, constraints (3.6) and (3.7) ensure that appliances are not scheduled for operation during non-valid hours of the day.

Scheduling inconvenience is dependent on the flexibility of the end-user to altering a particular appliance's usage:

$$\alpha_{i,t}^{j} = \frac{\delta_i^{|t-h_i|}}{P_i D_i} \quad , \qquad t \in [e_i, f_i] \tag{3.8}$$

$$\alpha_{i,t}^{j} = 0, \qquad t \notin [e_i, f_i] \tag{3.9}$$

A resident is considered to be inconvenienced if they delay or accelerate the use of an appliance for the sake of a lower electricity bill. (3.8) defines this inconvenience to be exponentially increasing the longer a consumer waits or shifts usage forward. Constraint (3.9) ensures that no inconvenience is suffered for hours falling outside of the valid schedule or hours for which the optimal and preferred schedules coincide.

The last concept addressed in the consumer's problem is the inclusion of the battery storage system. These are capable of supplying power during peak periods so that the consumer incurs minimal inconvenience and cost, and drawing from the grid during off-peak hours to incur the lowest charges. For the purposes of this study, asset-associated costs are not considered. An area for future research is to compare electricity cost savings with the capital and maintenance costs of various residential storage facilities over their lifespan to evaluate the true financial effect on the consumer. Battery operation is modelled as follows:

$$b_t^j + d_t^j \leq 1 \tag{3.10}$$

$$E_{\min}^j = 0.15 Ecap_j \tag{3.11}$$

$$E_{\max}^j = 0.85 Ecap_j \tag{3.12}$$

$$x_j = E_{\max}^j - E_{\min}^j \tag{3.13}$$

$$B_t^j \leq b_t^j x_j \tag{3.14}$$

$$G_t^j \leq d_t^j x_j \tag{3.15}$$

$$E_t^j = E_{t-1}^j + B_t^j \omega_c - G_t^j \tag{3.16}$$

$$E_{min}^j \leq E_t^j \leq E_{\max}^j \tag{3.17}$$

$$B_t^j, G_t^j \geq 0 \tag{3.18}$$

(3.10) ensures that a battery is not charged and discharged simultaneously. (3.11) and (3.12) create boundaries for the energy state of the battery relative to its rated capacity. (3.13), (3.14) and (3.15) ensure that the charging and discharging rates of the battery are always less than the maximum allowable energy state. Constraint (3.16) indicates the energy state of the battery at time *t* to be the sum of stored energy at time *t-1* and the power charged, subject to charging efficiency, less the power discharged at time *t*. Finally, the energy state of the battery is constrained in (3.17) is bounded to its minimum and maximum capacities at all times so as to ensure low maintenance costs and a long storage life. (3.18) accounts for non-negativity.

3.4 RETAILER PROBLEM

Retailers are an integral part of the energy value chain. This is because it is their duty to procure electricity and supply it to a set of users at a predetermined fee. When social welfare is preserved as the overwhelming stakeholder interest, retailers can still be granted the freedoms that come with deregulation and consumers may still have the opportunity to reduce bill payments. For the retailer problem, creating this new market dynamic as described in Table 2.2 is the dominating element for developing a model that meets the objective stated in Section 1.3. For this study, only procurement in the form of a forward option and trading in the spot market will be considered, and all other strategies and their associated risks and/or rewards will be overlooked. The retailer must then make decisions regarding bids for the wholesale market and spot market, as well as the tariff structure. Their utility is thus the sum of four terms, as in Zugno et al (2013):

$$R = \sum_{t=1}^{T} \rho_t C_t^* - \sum_{t=1}^{T} q_t Q_t - \sum_{t=1}^{T} s_t S_t + \sum_{t=1}^{T} s_t N_t \tag{3.19}$$

The first component represents revenues collected by the retailer for aggregated power consumption in real-time. The second term accounts for purchasing electricity in the day-ahead market. The last two terms in (3.19) represent the losses (gains) of purchasing (selling) electricity in the spot market at the regulation price s_t. The inclusion of the last two terms implies that any of the outcomes highlighted in Table 2.2 may occur, which is an accurate reflection of the uncertainty and risk that the retailer is exposed to.

Similar to spot market prices, no historical data is available to predict the wholesale price q_t in the day-ahead market. It is thus assumed, with motivations already described in Section 2.3.3, that prices are taken to be the average wholesale price in the NSW market, subject to variance $\sigma^2_{q,t}$. Thus

$$q_t \sim N(\mu_q, \sigma^2_{q,t}) \tag{3.20}$$

$$Q_t = \sum_{j=1}^{J} C_t^j \tag{3.21}$$

In order to prevent the retailer from acting strategically, (3.21) ensures that only the predicted demand is purchased from the forward option. A similar assumption is made by Zugno et al (2013). Such strategic practices are often times prohibited by regulating bodies in a market, but the reader is referred to Philpott and Pettersen (2006) as an example in which strategic demand bidding is investigated.

It was previously hypothesised that demand uncertainty increases with load. Thus, load uncertainty is given as

$$C_t^* \sim \begin{cases} \sum_{j=1}^{J} C_t^j + N(0,1), & \text{if } \sum_{j=1}^{J}\sum_{i=1}^{I} ubl_{i,t}^j = 0 \\ \sum_{j=1}^{J} C_t^j + N(0, \sigma^2_{C,t} \sum_{j=1}^{J}\sum_{i=1}^{I} ubl_{i,t}^j), & \text{if } \sum_{j=1}^{J}\sum_{i=1}^{I} ubl_{i,t}^j \neq 0 \end{cases} \tag{3.22}$$

(3.22) ensures that total realised load is subject to some uncertainty at every time *t*. This volatility, which can only be met in the spot market because of (3.21), increases with the magnitude of aggregated load for time *t* of users' preferred schedules.

The quantity of electricity to be purchased and sold in the spot market is given by (3.23) and (3.24) respectively. These piecewise definitions ensure that demand left unsatisfied or any excess of supply after transactions in the day-ahead market can be resolved. Constraint (3.25) is enforced to ensure that all realised demand is satisfied between the day-ahead and spot markets, and (3.26) accounts for non-negativity.

$$S_t = \begin{cases} C_t^* - Q_t, & \text{if } C_t^* - Q_t \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.23}$$

$$N_t = \begin{cases} Q_t - C_t^*, & \text{if } C_t^* - Q_t \leq 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.24}$$

$$C_t^* \leq Q_t + S_t - N_t \tag{3.25}$$

$$Q_t, N_t, S_t \geq 0 \tag{3.26}$$

Prices in the spot market, s_t serve as an input parameter to the retailer problem and are predicted with a three-regime Markov switching model, presented next.

3.4.1 Three-regime Markov switching model

In Section 2.3 the theoretical requirements for an effective three-regime Markov switching model that captures the stylized features of electricity were developed. It was found that because of the significant impact that trading in the spot market has on retailer profits, their price elasticity and thus social welfare, effort must be made to accurately predicting these regulating rates. This section presents the numerical model to satisfy these requirements. It should be noted that the natural logarithm of hourly spot prices, n_t is applied due to its stabilising effect on data values and its ease of interpretation. Then n_t is defined as the sum of a deterministic and stochastic component such that

$$n_t = r_t + X_t \tag{3.27}$$

The deterministic component, r_t, accounts for seasonality and reflects a constant mean-reverting process. This price formation is an improvement on the work of Higgs and

Worthington (2008) in that all three aspects of seasonality identified in Section 2.3.1, namely annual, weekly and intra-day cycles, are captured in equation (3.28). Furthermore, this is achieved without the development of separate models as in Huisman et al (2007), due likely to developments in software that are capable of computing a larger volume of parameters. One such tool is MS Regress (Perlin, 2014), a MATLAB-compatible package for Markov Regime Switching models that was used to solve this model. Let seasonality be:

$$r_t = \mu_0 + \beta_1 W_t + \sum_{l=2}^{12} \beta_l M_{lt} + \sum_{y=1}^{23} \tau_y H_{yt} \tag{3.28}$$

where

μ_0 = long-run equilibrium mean spot price

W_t = dummy variable representing weekday/weekend seasonality (weekdays are the reference category)

M_{lt} = dummy variable representing monthly seasonality (January is the reference category)

H_{yt} = dummy variable representing hourly seasonality (Hour 1 is the reference category)

β_l for $l=1,2,\ldots 12$ and τ_y for $y=1,2,\ldots 23$ are parameter coefficients to be estimated by maximum likelihood

The stochastic component X_t is then given as

$$X_t = X_{t-1} + dX_t \tag{3.29}$$

Changes in the stochastic component, dX_t, are governed by a three-regime Markov switching model that follows the work of Huisman and Mahieu (2003). The framework assumes that the spot price demonstrates one of three tendencies: normal price dynamics (regime 0), sudden peaks or troughs (regime +1) and reversion to normal price dynamics (regime -1). It is defined as

$$dX_t = \begin{cases} -\omega_0 X_{t-1} + \sigma_0 \zeta_t & \text{in regime 0} \\ \mu_1 + \sigma_1 \zeta_t & \text{in regime +1} \\ -\omega_{-1} X_{t-1} + \sigma_{-1} \zeta_t & \text{in regime -1} \end{cases} \tag{3.30}$$

where

ω_0 = rate of mean-reversion in regime 0

σ_0 = volatility of changes in regime 0

μ_1 = mean price level in regime +1

σ_1 = volatility of changes in regime +1

ω_{-1} = rate of mean-reversion in regime -1

σ_{-1} = volatility of changes in regime -1

$\zeta_t \sim N(0,1)$

The shift from one regime to another is controlled by a constant Markovian transition matrix in which probabilities are estimated by maximum likelihood:

$$\begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 1-\gamma_{11}-\gamma_{21} & 1-\gamma_{12}-\gamma_{22} & 1-\gamma_{13}-\gamma_{23} \end{bmatrix}$$

Unlike Higgs and Worthington (2008), no constraints are placed on the jump behaviour of spot prices, and this implies that shifts may occur bi-directionally from any one regime to another.

3.5 SOCIAL WELFARE ACHIEVEMENT

The consumer and retailer utilities have been presented in Section 3.3 and Section 3.4 respectively. It is now required that an objective function be defined that measures the simultaneous achievement of both stakeholder's utilities represented by social welfare. To do this, the sensitivity of the consumer to changes in price relative to the retailer must be quantified and this is done by expressing social welfare $F(\rho_t)$ as an objective to be minimised. It is defined as the ratio of consumer utility to retailer profits, both expressed as functions of the retail price, such that

$$min\ F = \varepsilon \frac{U(\rho_t, \bar{x})}{R(\rho_t, \bar{y})} \tag{3.31}$$

$$U(\rho_t, \bar{x}) \geq 0 \tag{3.32}$$

$$R(\rho_t, \bar{y}) \geq 0 \tag{3.33}$$

where $\bar{x} = \left\{ u_{i,t}^j, b_t^j, d_t^j, z_{i,t}^j, B_t^j, G_t^j \right\}$ are the consumer's set of decision variables, $\bar{y} = \left\{ Q_t, N_t, S_t \right\}$ are the retailer's and ε is a weighting factor controlling the importance of each stakeholder's respective utility.

The objective function above is defined in this way so that optimal social welfare is achieved when residents' utility is at its lowest and a service provider's profits are at their highest. It should be noted that this entirely novel approach to expressing consumer and retailer interests will be key to establishing their relative effects on one another under various price elastic conditions. This will be a valuable contribution to the body of knowledge in this field as it will assist regulating bodies and other stakeholders in determining the effect of relative price elasticity on consumer and retailer outcomes, desirable levels of relative price elasticity, and inversely, it may be used to define tariff floor and ceiling constraints for varying levels of price sensitivity.

3.6 CONCLUSION

In this chapter the objective was to formulate the consumer and retailer models with specific focus on spot market price formulation in the latter. This was so that the objectives of a contextualised model for the user and service provider could be met, as well as that of the first proposal of a problem formulation that represents their respective outcomes in a social welfare function dependent on relative price elasticity. To do this, various factors of the energy environment were incorporated, namely load scheduling, tariff design and market dynamics. The consumer problem addresses the user's cost minimisation and scheduling convenience interests. The retailer problem incorporates the revenues and costs associated with servicing end-users and trading to balance supply and demand. The Markov-model analyses spot market prices and provides a sound means for predictions and forecasts. Lastly, the formation of the social welfare objective ensures policy buy-in from users and energy providers so that both party's outcomes can be met.

CHAPTER 4

DECISION-MAKING FOR THE CONSUMER-RETAILER PROBLEM

4.1 INTRODUCTION

Numerical evaluations form the basis upon which hypotheses are proved or disproved. It also reveals the conditions under which these hypotheses are valid which may not have been as easily discernible during theoretical justifications. As a result, this section applies the consumer-retailer model as well the proposed social welfare function to a South African case study of residents operating in an open market system. The aim is then to demonstrate the ability of the three-regime Markov switching model to predict spot prices and satisfy the objective of it being an appropriate tool for contextualising the retailer problem. The tools that were incorporated into the consumer problem to achieve demand response must also be validated, and most importantly, the ability of the novel problem formulation to capture relative price elasticity and study the effect of various tariffs on retailer and consumer outcomes must be demonstrated. To achieve this, Section 4.2 presents the Markov switching model whose results serve as input to the retailer problem. Section 4.3 presents data relevant to the case study and Section 4.4 provides the results of applying the model when a fixed rate tariff is used. Section 4.5 summarises.

4.2 THREE-REGIME MARKOV SWITCHING MODEL

In order for a model to be an accurate predictor of any real-time data under study, it must first adequately capture the key features of the commodity. In Section 2.3.3 it was established that a three-regime Markov switching model would be an effective tool for capturing the salient features of electricity spot market prices. Specifically, seasonality, mean-reversion, volatility and short-lived spike characteristics as prices fluctuate between their 'normal', 'spike' and 'after-spike' states were identified as key descriptors (Jablonska-Sabuka et al, 2011). The aim of this section is thus to validate the presence of these features for the selected sample of data and demonstrate the ability of the model to capture and forecast these values for an out-of-sample period. In order to achieve this, Section 4.2.1 provides a statistical description of in-sample data. Section 4.2.2 presents the results and subsequent analysis of applying the regime-switching model. Finally, Section 4.2.3 summarises the contribution of this section to the greater retailer-consumer problem.

4.2.1 Data and descriptive statistics

The data employed is that of daily spot prices from the Australian National Electricity Market (NEM) for the arbitrarily selected region of New South Wales (NSW). Because the NEM publishes data on a half-hourly basis but this study uses an hourly sample time, hourly arithmetic means have been calculated from the trading interval data. A sample period from 1 January 2013 to 31 December 2014 is selected for the purposes of parameter estimation. To perform an out-of-sample forecast, the hourly log spot prices for the period 1 January 2015 to 31 July 2015 are computed and compared against actual observations. As was hypothesized in Section 2.3.3, no negative or zero values were present in the data, but four extreme values were excluded from the sample set to prevent distortion of results. In order to estimate the hourly spot market price s_t which serves as an input parameter to the retailer problem, the forecasted values for an arbitrarily chosen weekday in January are then converted to South African Rands (ZAR) based on the arithmetic average exchange rate for the selected period.

Table 4.1 provides a summary of descriptive statistics for the in-sample data. Based on these statistics it is clear that the hourly spot prices of the NEM do indeed demonstrate the typical characteristics of electricity identified above. The data is extremely widely spread across a range of A$4012.86/MWh with a relatively high standard deviation of A$35.89/MWh. This corroborates the findings of Carmona and Coulon (2013) that price volatility is excessively high in electricity markets. The coefficient of variation expresses the standard deviation as a portion of the mean and confirms the large extent (almost 75%) to which stochasticity contributes to price formation. The kurtosis far exceeds three, indicating a leptokurtic distribution with heavier tails and values concentrated around the mean. This is supported by the findings of Knittel and Roberts (2005) and Higgs and Worthington (2008). The Jarque-Bera (J-B) test statistic and *p*-value together reject the null hypothesis of distributional normality at a significance level of 0.01. Finally, the Augmented Dickey-Fuller test, an effective unit root test for measuring whether time series data is stationary over time, concurs with Lucia and Schwartz (2002) in that the null hypothesis is rejected at a 0.01 level of significance, indicating that hourly spot market prices are indeed stationary. Additionally, Figure 4.1 provides a visual representation of the selected in-sample data from which the characteristics of mean-reversion and short-lived spikes are apparent. It is thus clear that the NEM is an appropriate representation of spot market prices in that the identified key features typical of the electricity market are present.

Table 4.1: Descriptive statistics of hourly spot prices (A$/MWh) for 1 January 2013-31 December 2014

Statistic	**Price**	**ln Price**
Number of observations	17520	17520
Mean	48.340	3.841
Minimum	4.280	1.454
Maximum	4017.135	8.298
Standard deviation	35.887	0.256
Coefficient of variation	0.742	0.067
Tenth percentile	31.989	3.465
Fiftieth percentile	49.860	3.909
Ninetieth percentile	56.670	4.037
Skewness	86.251	-0.366
Kurtosis	8960.222	11.867
J-B statistic	5.86E+10	1.03E+05
J-B *p*-value	0	0
ADF *t*-statistic	-19.974	--
ADF *p*-value	0.01	--

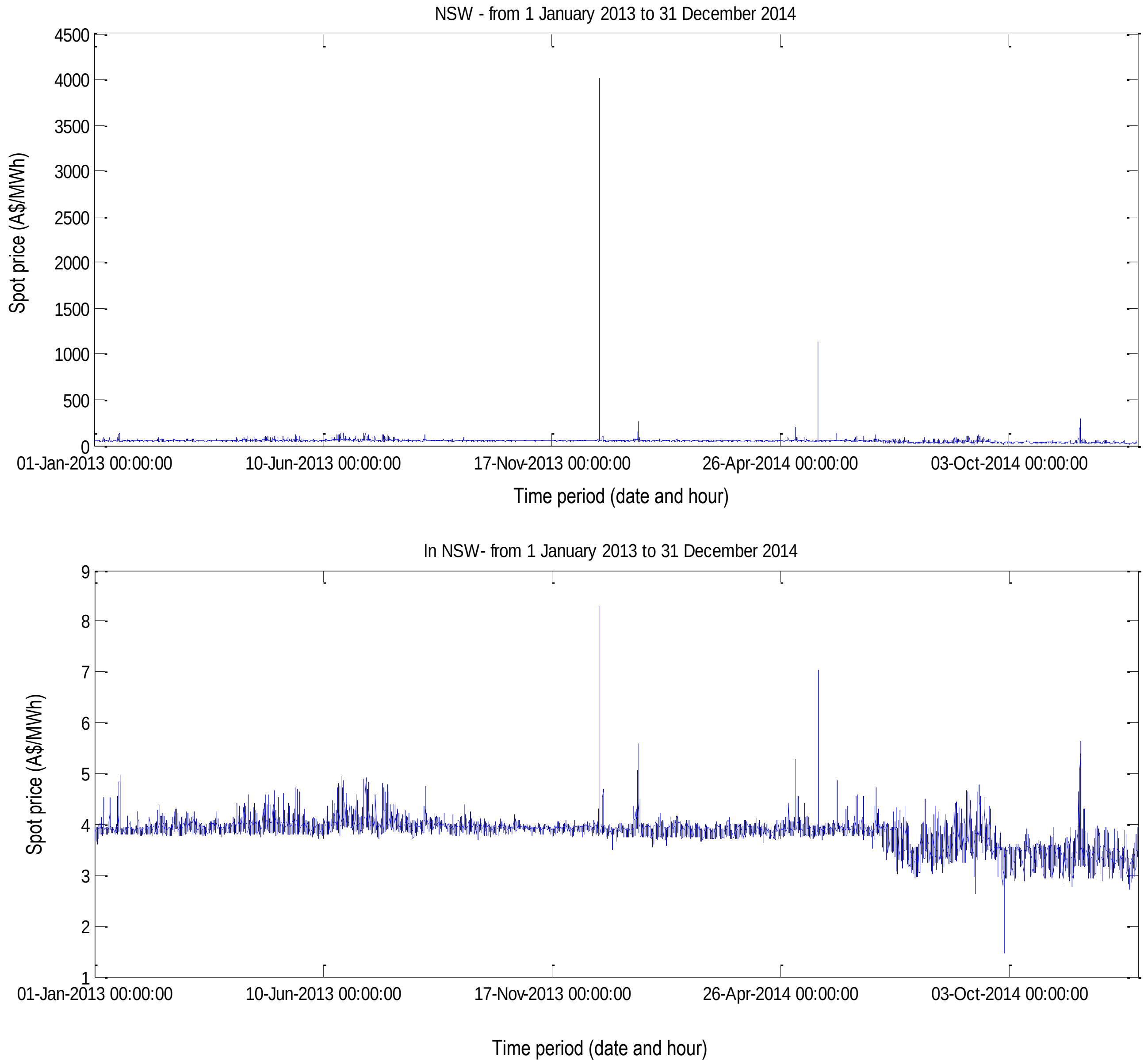


Figure 4.1: Hourly spot prices (A$/MWh) and natural logarithms of spot prices for the period 1 January 2013 to 31 December 2014 in the NSW region

4.2.2 Results and analysis

This section firstly presents and discusses the estimated coefficients and standard errors obtained via the model. Secondly, it applies the developed model to a set of out-of-sample data so as to validate its ability to predict hourly spot market prices. The three-regime Markov switching model was coded in MATLAB, version 2013a, on a standard personal computer with the aid of a user-friendly, open source software package called MS Regress (Perlin, 2014). The tool is capable of estimating, forecasting and simulating Markov Switching regression models. Table 4.2 presents the empirical results of applying the model to the selected in-sample data.

Table 4.2: Estimation results for the three-regime Markov switching model

Parameter	**Coefficient**	**Standard error**
Deterministic component		
μ_0	3.9359***	0.0018
β_1	-0.0715***	0.0005
β_2	0.0039**	0.0011
β_3	-0.0074***	0.0011
β_4	0.0039**	0.0011
β_5	0.0198***	0.0012
β_6	0.0296***	0.0012
β_7	-0.0751***	0.0014
β_8	-0.1569***	0.0014
β_9	-0.1253***	0.0013
β_{10}	-0.2404***	0.0013
β_{11}	-0.2361***	0.0013
β_{12}	-0.2545***	0.0013
τ_1	-0.0325***	0.0021
τ_2	-0.0974***	0.0020
τ_3	-0.1369***	0.0020
τ_4	-0.1398***	0.0020

τ_5	-0.0990***	0.0021
τ_6	-0.0499***	0.0021
τ_7	0.0072**	0.0022
τ_8	0.0642***	0.0021
τ_9	0.0799***	0.0021
τ_{10}	0.0652***	0.0020
τ_{11}	0.0403***	0.0020
τ_{12}	0.0340***	0.0020
τ_{13}	0.0275***	0.0020
τ_{14}	0.0317***	0.0020
τ_{15}	0.0342***	0.0020
τ_{16}	0.0458***	0.0020
τ_{17}	0.0679***	0.0022
τ_{18}	0.0866***	0.0023
τ_{19}	0.1376***	0.0023
τ_{20}	0.0807***	0.0021
τ_{21}	0.0466***	0.0021
τ_{22}	0.0148***	0.0020
τ_{23}	0.0177***	0.0021
Stochastic component		
ω_0	0.0115***	0.0025
σ_0	0.0006***	7.3435e-06
μ_1	-0.0735*	0.0385
σ_1	0.5869***	0.0126
ω_{-1}	0.0762***	0.0063
σ_{-1}	0.0110***	0.0002
Transition matrix		

γ_{11}	0.9222***	0.0058
γ_{21}	0.0041**	0.0016
γ_{12}	0.1507**	0.0551
γ_{22}	0.4060***	0.0439
γ_{13}	0.1679***	0.0094
γ_{23}	0.0349***	0.0045
LnL	27312.6108	
AIC	-54523.2215	
BIC	-54126.9101	

*** indicates significance at a 0.001 level
** indicates significance at a 0.01 level
* indicates significance at a 0.05 level
LnL, AIC and BIC are Log Likelihood, Akaike Information Criterion and Bayesian Information Criterion respectively

Overall, estimates proved to be significant at least at a 95% confidence level. The long-run equilibrium price μ_0 is only slightly higher (2.47%) than the mean value presented in Table 4.1 and translates to A$51.21/MWh. This indicates a strong similarity between the modelled and raw data as the equilibrium price falls within one standard deviation. The effect of whether a spot price falls on a working day or holiday is indicated by β_1 and is significant and negative. This is in line with Higgs and Worthington (2008) as well as Huisman et al (2007) and indicates that, as would be expected, prices are 6.9% lower on holidays than weekdays after holding all other factors constant, due likely to lower anticipated demand.

All monthly effects (β_2, β_3… β_{12}) were found to be significant and results show a close correlation to seasonal trends. Prices between February and April deviated less than 1% from January's (the reference category), indicating the still-present effects of summer characterised by lower demand. During the winter months of May and June when demand is at its peak, spot prices were between 2-3% higher than January. Interestingly, prices were significantly lower during the months July-December, despite July and August being considered winter months which are typically characterised by higher spot prices. A closer inspection revealed however that 2014 marked the warmest year for NSW, during which there were several significant heat waves and the warmest Spring maximum temperatures (September to November) were recorded (Australian Government: Bureau of Meteorology, 2015). In light of this, the fact that results indicate uncharacteristically low electricity prices throughout the latter half of the year is indeed feasible. Overall, the highest spot prices are experienced in June (3% higher than in January) and the lowest in December (22.45% lower than in January).

Intra-day seasonality is addressed with parameters $\tau_{1,}$ $\tau_{2\ldots}$ τ_{23} and with the exception of hour 8, all effects are significant at a 99.9% confidence. Overall, the standard errors associated with the hourly coefficients are less than 0.0025, indicating a fair degree of certainty. The estimates indicate that from 2AM-7AM, prices are between 3.20%-13.05% lower than hour 1 (the reference category). This is to be expected due to the correlation between lower spot prices and off-peak periods typically associated with the early hours of the morning. It is also corroborated by the work of Huisman et al (2007). 7AM-10AM is considered to be the peak morning period and prices steadily increase, likely as a result of residents waking up and preparing for the working day. Thereafter, hourly prices continue to decay until 2PM, but are still comparatively higher than tariffs in hour 1, before once again increasing in preparation for the peak evening period, from 5PM-9PM. As the evening wears on and residents end the day, prices steadily decrease once again. Based on these estimates, the highest spot prices are experienced at 7PM (14.75% higher than in hour 1) and the lowest at 4AM (13.05% lower than in hour 1). Based on these analyses it is clear that electricity spot prices follow the trends of demand, whether it be affected by annual, weekly or hourly seasonality.

Attention is now turned to the parameters governing stochasticity. Once again, all estimates are significant at a 99.9% confidence level, with the exception of the mean price level in regime 1 which is significant at a 95% confidence level. The rates of mean-reversion across all regimes is apparent, and is a clear indicator that this stylised feature is present in all states of hourly spot prices, but to differing degrees. As is expected of normal price dynamics (regime 0), the rate of mean-reversion is relatively low, indicating a slower return of prices to the long-run equilibrium due to its already 'normal' tendency. This corroborates Janczura and Weron (2012) who found that parameter estimates typically associated with high speeds of mean-reversion fell between 0.20 and 0.44. It appears that as prices shift out of state 2 they have already reverted to a large extent back to their equilibrium level as the rate of mean-reversion for the after-spike regime is relatively low at 0.0762, but still higher than that of the first state. Overall, the studies of Janczura and Weron (2012), Mari (2006) and Higgs and Worthington (2008) displayed the same relativity of each regime's mean-reversion parameters that is captured here. The magnitudes of mean-reversion are however significantly lower than those found by Higgs and Worthington (2008) who postulated that these rates depend largely on the nature of the spike regime. Specifically, higher rates are associated with incidents such as generator breakdowns or transmission failures that are quickly reparable but have a profound effect on supply in the market. Lower rates are likely the result of abnormal weather conditions (Blanco and Soronow, 2001) which take longer to stabilise.

The size of the price jump μ_1 is slightly lower (0.0735) than the long-run equilibrium. This is contrary to Huisman and Mahieu (2003) and Higgs and Worthington (2008), both of whom found the magnitude of price spikes to cause an increase in the mean price level. However, in light of the warm temperatures experienced by the region in 2013 and 2014 as well as the drop in spot prices observed after May 2014 (see Figure 4.1), these results once again seem feasible. Volatility of price changes in the normal state is extremely low (0.0006), significant and has an almost negligible standard error. This indicates that prices in the normal regime are relatively certain with little variance, which bodes well for the consumer as they are better able to manage their demand in the face of predictable price signals. These findings are also corroborated by Mari (2006) and Higgs and Worthington (2008). In stark contrast, almost 1000 times more volatility is experienced in the spike-regime, as well as higher uncertainty indicated by a standard error of 0.0126. Although volatility is also present in the after-spike regime, it is to a significantly lesser extent and with a higher degree of certainty.

A Markov transition matrix defines the probability, $\pi(i,j)$, of a switch from regime j to regime i in one time period. A steady-state probability, that is the probability of remaining in the same regime, is thus indicated by diagonal elements. Based on this, it is clear that hourly spot prices are most likely to occur, and stay, in the normal state, as indicated by γ_{11} (92.2%) and the expected duration of regime 0 (12.85 time periods). Interestingly, the probability of transitioning from the normal to the spike regime is extremely low at 0.0041. This indicates the poor likelihood of a spike occurring, which can be explained by the warm temperatures experienced in the region that keeps demand at a predictable minimum. Once in the spike regime however, spot prices primarily stay in this state (0.4060) or transition to the after-spike state (0.4434). This, coupled with the duration of the regime (1.68 time periods) indicates that spikes may stretch across more than one hour, which is feasible as repair times for system breakdowns may last longer than this. The strong presence of mean-reversion in the spike regime however would make more pronounced spikes short-lived, as is indicated in Table 4.1. Lastly, the probability of hourly prices staying in an after-spike state after one hour (0.7971) is higher than Mari (2006), due likely to the difference in time periods. The author uses a period of one day, whereas in this study it is better expected that the effects of one regime will still be felt an hour later.

Overall, it can be concluded that the magnitude, frequency and duration of jumps occurring in the NEM within NSW are extremely low, due likely to the warm temperatures experienced in the region as well as the stabilising effect of the coastline, but that the volatility and uncertainty associated with the spike state is fairly high. Finally, the maximised log likelihood value compares favourably to the work of Mari (2006), Huisman et al (2007) and Higgs and Worthington (2008). In fact, the highest log likelihood value achieved by these authors was by Mari (2006) who realised a value of 2167.4 with the use of a Huisman-Mahieu model extended to include Poisson jumps and applied to the Nord Pool. The AIC and BIC, both of which provide a means of comparing the explanatory power of models, are comparatively lower than the abovementioned works, once again indicating this models' superiority.

The above model was used to compute out-of-sample forecasts which were compared against actual observations for the same period. Hourly spot prices and simulated trajectories can be seen in Figures 4.2 and 4.3 respectively. Four measures of accuracy were used to validate the effectiveness of the model in predicting hourly spot prices and results are presented in Table 4.3. Deviations between predicted and actual observations are slight, as indicated by the comparison of these statistics to those of Higgs and Worthington (2008). This study outperforms the latter with regards to all measures of forecast accuracy. The TIC statistic, which is a measure of the effectiveness of the forecasting technique, is 0.0862 in Higgs and Worthington (2008), twelve times lower than the computed value for this study.

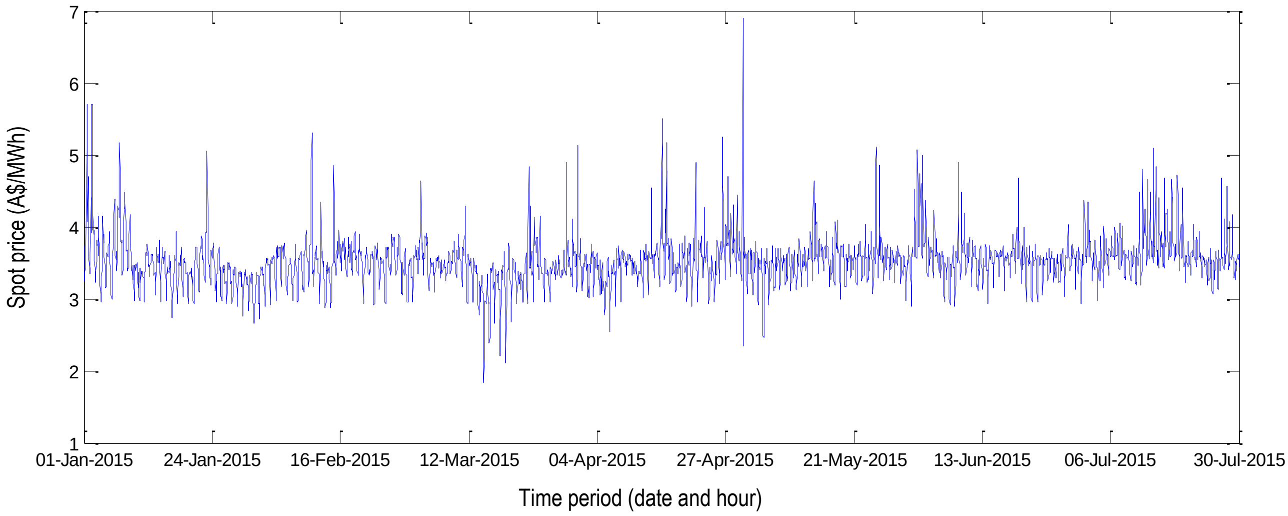


Figure 4.2: Natural logarithm of hourly spot prices for the period 1 January 2015 to 31 July 2015

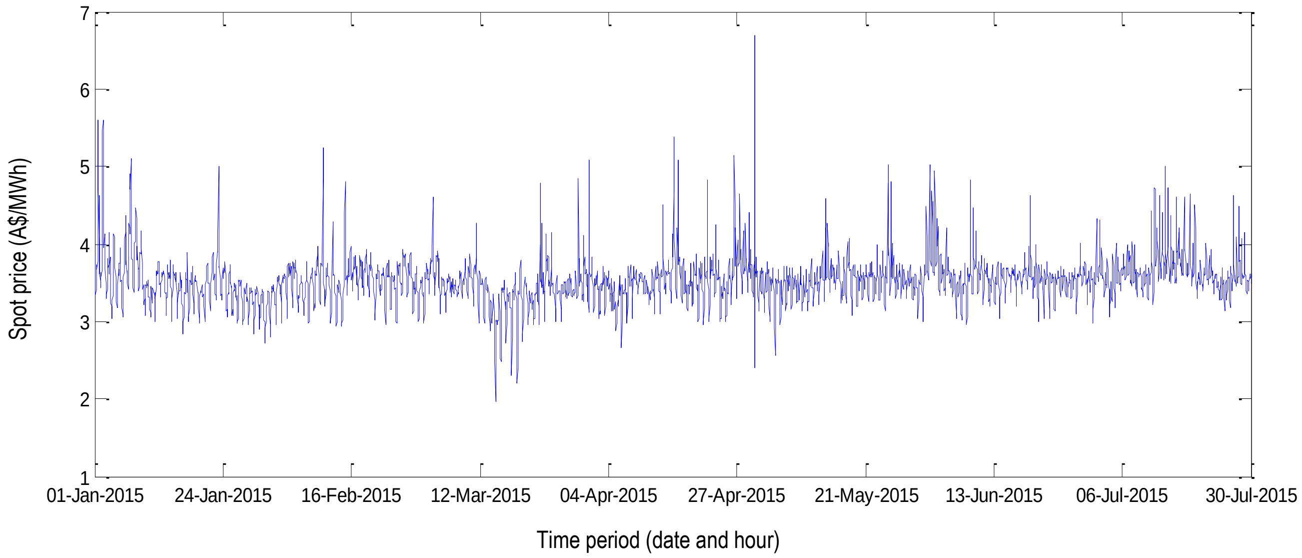


Figure 4.3: Simulated trajectory of hourly spot prices for the period 1 January to 31 July, generated from estimated parameters

Table 4.3: Measures of out-of-sample forecast accuracy

Statistic	
MSE	0.0310
MAE	0.0941
RMSE	0.1762
TIC	0.0071

MSE, MAE, RMSE and TIC are mean squared error, mean absolute error, root mean squared error and Theil inequality coefficient respectively

4.2.3 Summary

This study has confirmed that seasonality and uncertainty play important roles in predicting hourly spot prices. The three-regime Markov switching model proved superior in comparison to similar studies conducted, both with regards to parameter estimation and forecasting accuracy. This can likely be attributed to the lack of limitations placed on the spike behaviour of hourly spot prices, as it is clear that there is no distinct pattern (or exclusion) of transition from one regime to another. The introduction of dummy variables model, despite it incurring higher parameter requirements, was also able to appropriately capture hourly, daily and annual seasonality, all of which had significant effects on results. It is thus an appropriate tool for estimating s_t, the hourly spot prices required as input to the retailer problem.

4.3 DATA

Typically in scheduling problems the number and types of variables selected have a significant impact on model complexity, computing time and whether an optimal or sub-optimal solution can be derived. In load scheduling, the selection of parameters and input data play an especially important role. In fact, the presented solutions are only optimal for the set of gathered data relevant to the study in hand (Mohsenian-Rad and Leon-Garcia, 2010, Setlhaolo et al, 2014). This should not be confused with a lack of scalability however; demand responsiveness is consumer and load-profile specific which means that varying levels of success will always be achieved amongst residential users. The selection of arbitrary (but realistic) parameters, such as appliance data and baseline schedules for example, demonstrates a model's capability as well as its flexibility for real-life applications. With this in mind, the work that follows presents the data that was used for solving the consumer-retailer problem.

For the purposes of this study, time is discretized into 1 hour intervals over a 24-hour period. This is in keeping with Sou et al (2011) who reported huge computational burdens but little improvement in results when smaller time segments were used. Ten users are considered, each of which is assigned with a predefined probability to a specific income bracket. Such a strategy has been employed so as to accurately reflect the divergent socio-economic status of South African residents. For example, users who are classified as indigent are extremely unlikely to own a dishwasher, whereas affluent users would likely own all of the appliances considered in the study. Furthermore, according to Leadbetter and Swan (2012), the size of a battery used for electricity storage in a residence is dependent on the household in question. Specifically, energy-poor homes would find a smaller storage system sufficient for their consumption needs in comparison to an energy-intense home. Low-income users would also be more flexible to adjusting their schedules for the sake of a lower electricity payment in comparison to higher-income residents for which bill payments may be less important than the inconvenience they incur. This is corroborated by Chicco et al (2004) who indicated that qualitative data regarding the socio-economic standards of users was necessary to make significant and realistic classifications of residential load profiles. Along these lines, according to Finn et al (2012), a South African homeowner has a 66.2% likelihood of falling in a low income (or energy poor) bracket, 29.9% likelihood of falling in the middle income bracket, and a 3.9% probability of being a high income earner (or an energy intensive user). These probabilities have formed the basis upon which appliances, battery size and waiting flexibility have been allocated to each user, and have been summarised in Table 4.4. It should be noted that an inclusion such as this is what makes the formulation of this consumer problem novel and uniquely contextualised to the South African market. In fact, other studies have failed to create this link between energy intensity (or poverty) and consumer classification but have nevertheless acknowledged its importance (Chicco et al, 2004).

Table 4.3: User-specific data

User type no.	**User type**	**Probability that user *j* is assigned to a user type**	**Number of owned appliances**	**Rated battery capacity (KWh)**	**Importance of waiting, λ_j**
1	Lower	0.662	3	5	2
2	Middle	0.299	6	10	1
3	Upper	0.039	10	20	0.5

In a load scheduling problem, the data governing appliance usage must be realistic and comprehensive so that the generated results appropriately reflect the demand profiles of users. Table 4.5 presents this data. It is clear that the selected appliances range from those that would be found in low to middle income residents, such as a stove and microwave respectively, to those that would only be purchased by more affluent consumers, such as a dishwasher and tumble dryer. Power ratings can be obtained from manufacturers or on the device in question (Setlhaolo et al, 2014). The consumption patterns, depicted by the beginning and ending time intervals, indicate that most household activities occur in the morning (such as preparing breakfast and getting ready to start the day) and after work (when chores such as cooking and cleaning, and relaxation occur). Some appliances may be used more than once a day such as the electric water heater which is scheduled for at least 3 hours between 4AM-9AM and again between 3PM and 11PM. In such instances, the device is modelled as two separate appliances. Those listed in Table 4.5 are schedulable and non-interruptible, whilst still reflecting the discretionary and non-discretionary activities typical of a residence, as outlined in Schweppe et al (1989). Following the same authors, non-schedulable end-use devices have not been included due to their reported non-meaningful impacts on results. Finally, the number of considered appliances is in keeping with the work of Mohsenian-Rad and Leon-Garcia (2010), who recommend a consideration of 10-20 end-use devices. However, this results in the classification of the problem as being large scale in that the number of variables in the model exceeds one thousand (Benson et al, 2003), as can be seen in Table 4.6.

Table 4.5: Appliance data

No.	**Appliance, *i***	**Power rating, P_i**	**Duration of operation, D_i**	**Beginning of interval, e_i**	**End of interval, f_i**
1	Stove	3	1	6	9
			1	16	21
2	Microwave	1.23	1	15	21
3	Kettle	1.9	1	6	9
			1	15	21
4	Toaster	1.01	1	6	9
5	Steam iron	1.235	1	15	24
6	Vacuum cleaner	1.2	1	15	24
7	Electric water heater	2.6	3	4	9
			3	15	23
8	Dishwasher	2.5	2	16	24
9	Washing machine	3	1	16	24
10	Tumble dryer	3.3	1	16	24

Table 4.6: Number of decision variables in consumer problem

Decision variable description	**Classification**	**Number of variables**
$u_{i,t}^{j}$	Binary	3120
$z_{i,t}^{j}$	Binary	3120
b_{t}^{j}	Binary	240
d_{t}^{j}	Binary	240
B_{t}^{j}	Continuous	240
G_{t}^{j}	Continuous	240
Total		**7200**

Table 4.7 indicates which of the above-listed appliances are owned by a low, middle or high-income user, as well as the baseline or preferred schedule for the operation of each appliance. It also presents the degree of inflexibility associated with an appliance. High income users are seen to own all of the appliances under study, whereas low-income homeowners own the bare essentials, such as a stove, kettle and electric water heater. Lastly, a higher degree of inflexibility, such as that of the stove in comparison to a toaster, indicates that a user has a higher predisposition to shifting the usage of one appliance over another. This reflects the reality that a stove is required to prepare a meal whereas the non-immediate use of a toaster would be less devastating.

Table 4.7: Appliance scheduling data

No.	**Appliance, *i***	**User types owning appliance *i***	**Baseline schedule, $ubl_{i,t}^{j}$**	**Degree of inflexibility, δ_i**
1	Stove	1,2,3	7	1.25
		1,2,3	19	2
2	Microwave	2,3	17	1.25
3	Kettle	1,2,3	7	1.75
		1,2,3	17	1
4	Toaster	2,3	7	1

5	Steam iron	3	20	1.25
6	Vacuum cleaner	3	21	1.25
7	Electric water heater	1,2,3	5-7	2
		1,2,3	17-19	1.75
8	Dishwasher	3	20-21	1.5
9	Washing machine	2,3	20	1.5
10	Tumble dryer	3	21	1.5

As per the guidelines of Chen et al (2012b), battery-specific data is presented in Table 4.8.

Table 4.8: Battery data

Charging efficiency, ω_c	**Discharging efficiency, ω_d**	**Maximum battery capacity, E^j_{max}**	**Minimum battery capacity, E^j_{min}**
0.8	0.89	0.85*rated battery capacity	0.15*rated battery capacity

Attention is now turned to input parameters for the retailer's problem. As was previously stated, because the local energy market is regulated, data reflecting the dynamics of electricity prices in the wholesale and spot markets is not available. As such, the Australian NEM was seen to be a fair representation of a hypothetical South African competitive industry for reasons discussed in Section 2.3.3. According to the NSW Parliamentary Research Service (2014), the average first quarterly futures contract price for 2015 is A$37.5/MWh. This, associated with a variance $\sigma^2_{q,t}$ that is 10% of the average cost of contract prices, and subject to an arithmetic month's average exchange rate of R9.328 to the Australian Dollar, is used to derive the wholesale price q_t. The prediction of the spot market price s_t with the Markov switching model in Section 4.2 is also incorporated into the retailer's problem with the use of the monthly average exchange rate, with data selected from forecasted results for an arbitrarily chosen working day, 15 January 2015.

Figure 4.4 presents the contract and spot market prices for the selected 24-hour period. It is clear that the spot market price follows the trend of energy consumption in a 24-hour period, with peaks experienced in the morning and then in the evening after work with a roughly uniform profile in-between, and troughs experienced during the early hours of the morning when demand is typically low. The volatility of contract prices can be attributed to the variance $\sigma^2_{q,t}$. The behaviour of contract and spot prices relative to one another can be explained by economic theory (Shahidehpour and Alomoush, 2001). Commodity markets such as stock options and crude oil are known to demonstrate *contango*, an effect in which shorter-dated contracts (where delivery of the commodity is almost immediately after purchase) are priced lower than longer-dated contracts. This is due to the higher holding and inventory costs associated with delayed delivery. Whilst this is not the case with electricity as production can only occur in real-time and no cost to carry is incurred, a higher settlement risk is indeed associated with trading directly from the spot market. This is because supply is highly variable if not secured in advance and thus infrequently guaranteed, which means that retailers are better suited to service their consumers from forward contracts. As such, wholesale prices that are higher than spot prices are observed. Conversely, the electricity market also experiences the opposite of *contango*, called *backwardation*, in which prices increases as the time to delivery decreases, and contract prices are thus lower than spot prices. This trend is also necessary to drive down futures prices in order to attract speculators (buyers) to enter into trades with hedgers (sellers). Because both these phenomena are present in electricity trading they must be adequately represented, as is the case here (see Figure 4.4). For this study it is assumed that there is sufficient supply to service consumers from the spot market. Furthermore, it is noted that strategic bidding is not permitted and profits made by the retailer cannot be derived from manipulating the respective markets.

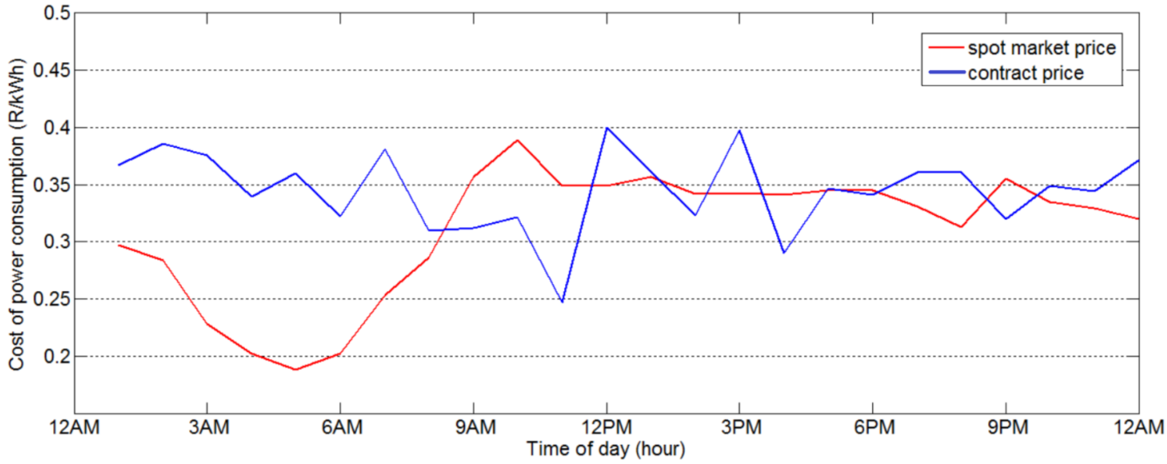


Figure 4.4: Wholesale and spot market electricity price data for the period 15 January 2015

Lastly, an appropriate feasible region for the selection of the retail price ρ_t must be defined. According to Eskom's Tariff and Charges Booklet for 2014/15, the *Home Power Standard* local authority rate is 101.12c/kWh for energy consumption less than 600kWh and 155.66c/kWh thereafter. These rates will form the minimum and maximum boundaries respectively for the retail rate as it reflects the decisions of local energy authorities. Whilst the implementation of minimum tariff policies is not regular practice, regulating bodies in competitive industries have placed ceilings on retail prices in order to stabilise the market and protect the exposed consumer from exorbitant fees during supply or transmission failures (Triki and Violi, 2009).

4.4 FIXED RATE TARIFF

In order to demonstrate the principle theory of the proposed problem formulation, a fixed rate scheme is rendered to the consumer by the retailer. This means that a flat rate is charged per kilowatt hour consumed, independent of time. The residents' and service provider's models were solved in MATLAB version 2013a with the YALMIP interface, and the instruction set can be found in Appendix A. The problem was solved using an academically-free version of Gurobi Optimizer, a state-of-the-art solver that can be applied to all major model types including linear programmes (LP), mixed integer linear programmes (MILP), mixed integer quadratic programmes (MIQP) and of most relevance, mixed integer nonlinear programmes (MINLP).

It should be noted that under a fixed pricing tariff, demand responsiveness is not achieved, despite the resident being equipped with tools such as a storage system and EMC to enable automated appliance shifting. This is because the selecting pricing scheme does not serve to communicate with the consumer as to when to reduce energy consumption based on peak periods. The primary reasons for evaluating this pricing scheme are thus three-fold. Firstly, it serves as a benchmark for future research that measures the performance of TOU and dynamic pricing strategies in terms of achieving demand responsiveness, reducing costs for the resident, generating revenues for the retailer and maximising social welfare. Secondly, it serves as a means of model validation in that the generated results for this tariff structure can be easily verified. Lastly, it is used as the basis for demonstrating the relationship of price elasticity between the consumer and retailer so as to achieve social welfare. Using more advanced pricing schemes to do this would not only be impractical for an immature deregulated market operating in South Africa, but would also be computationally expensive due to the resulting non-convex search space that results. It is thus highlighted as an area for future research but recommendations, preliminary findings and challenges are discussed in Chapter 5.

The effect of a fixed tariff on the identified key outcomes must first be quantified and analysed so that the price at which a maximum social welfare ratio is achieved can be established. In the formulation of (3.31), it is hypothesised that relative price-elasticity is a one-to-one ratio following assumptions in literature (Mohsenian-Rad and Leon-Garcia, 2010, Corradi et al, 2013) and indicated by the expression of both consumer and retailer interests as first degree polynomial functions of the tariff ρ_t. Under the conditions of a fixed rate tariff, preliminary results indicated that both consumer utility and retailer profits could be expressed as a linear function of the fixed rate charged. To confirm this, several rates within the established boundaries were evaluated for the consumer and retailer and have been presented in Figure 4.5 and Figure 4.6 respectively. In the case of the consumer, this linear representation is due to the fact that, regardless of the rate employed, residents are not financially motivated to deviate from their preferred schedules; in fact, the user has an incentive to maintain their preferred schedule so as to not incur an inconvenience penalty and increase their pain. This concept is illustrated in Figure 4.7 where it is clear that the baseline and optimal predicted schedules are identical. As a result, the PAR for aggregated loads is 6.716 and is also independent of the retail rate. Furthermore, it is reasonably assumed that at the commencement of the study period, all residences are fitted with storage systems that are at their minimum capacity. This means that a battery must first be charged and incur added expenses before it can dispense energy to end-use devices around the home. Also taking into account the charging and discharging efficiencies of a battery in which heat loss occurs, the derived benefit of it being used as an alternative energy source during expensive periods of high demand is not realised. As such, the residential storage systems too remain inactive which means that PAR does not experience any improvement under this pricing scheme and that the aggregated consumer utility can now be expressed as a function of the retail price charged, thus the shape seen in Figure 4.5. In the case of the retailer, revenues earned are once again independent of the consumer schedule due to the flat rate charged. This means that because the sum of all users' loads over a 24-hour period remains fixed regardless of the profile it adopts, profits may be expressed as a linearly increasing function, subject to trades made in the spot market in which unpredicted demand is satisfied.

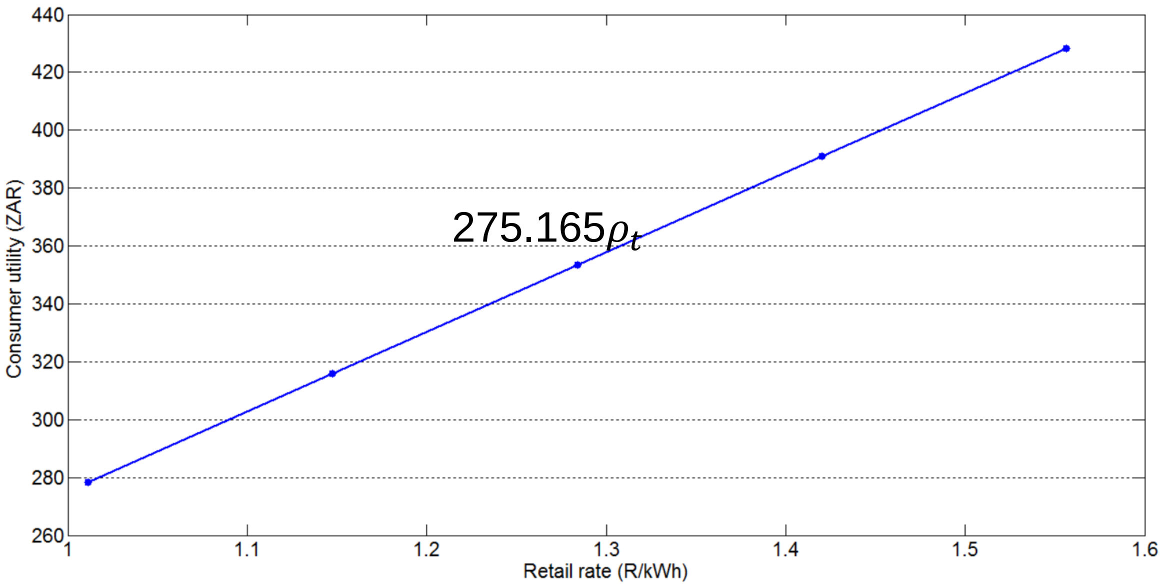


Figure 4.5: Consumer utility as a function of retail rate under a fixed pricing scheme

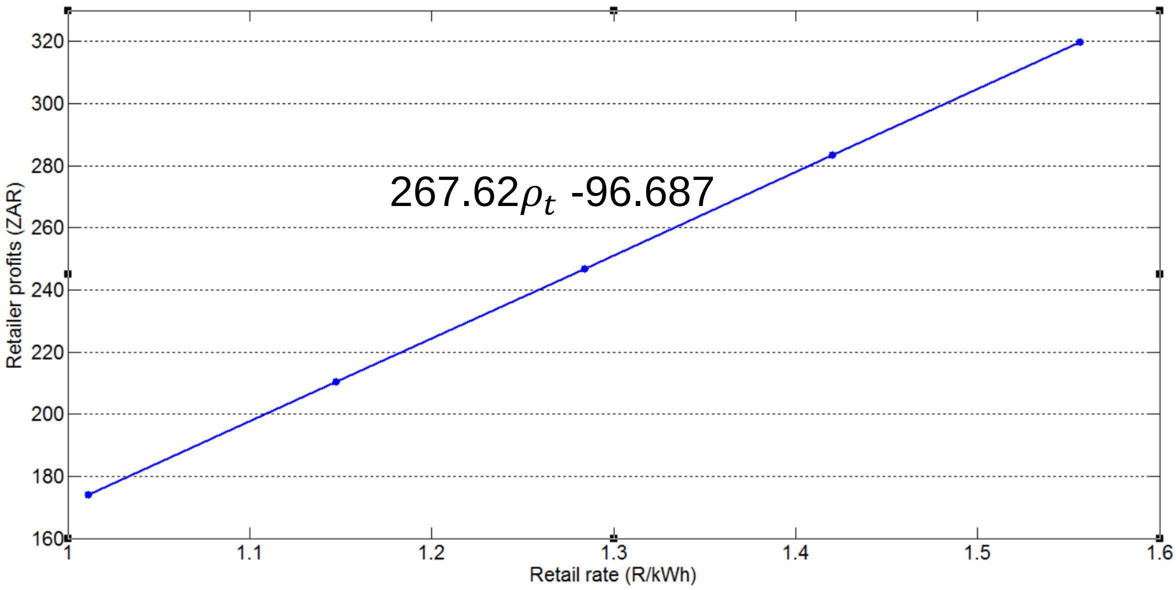
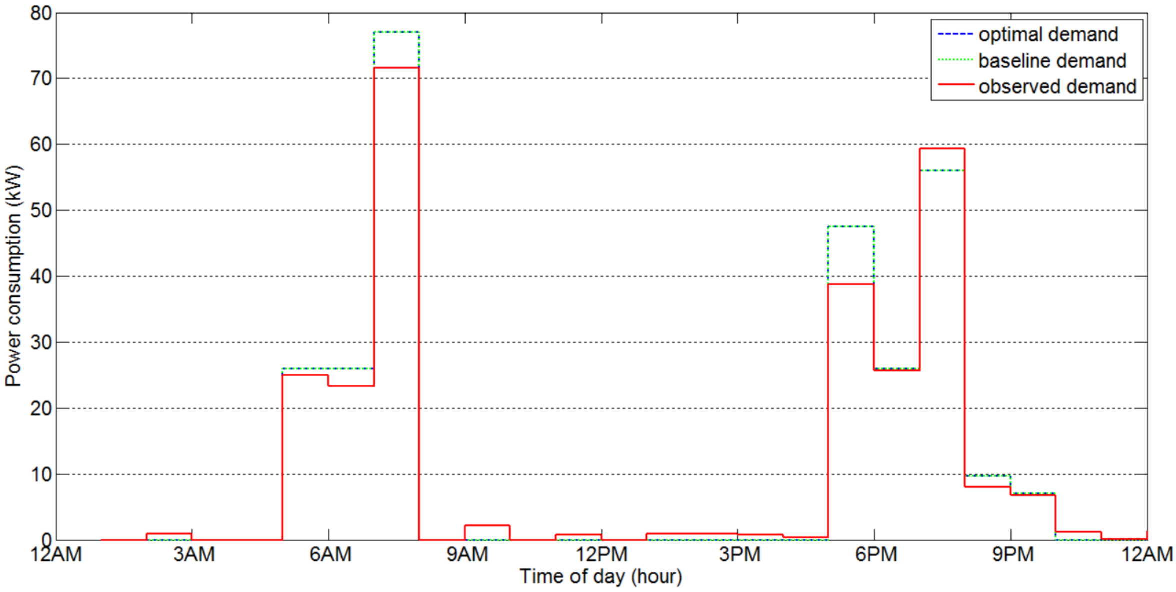


Figure 4.6: Retailer profits as a function of retail price under a fixed pricing scheme



*Observed demand is a function of uncertainty and optimal demand, calculated by the piecewise function in (3.22), and indicates that both positive (increases) and negative (decreases) deviations in expected demand may occur

Figure 4.7: One-day aggregated load profile of predicted and observed demand under a fixed rate tariff

Attention is now turned to the key interest of this study, social welfare. Under conditions of a fixed pricing scheme, this ratio can be generalized to a first degree polynomial rational function, as seen in (4.1). Under this formulation, a one-to-one relationship regarding price elasticity is assumed, that is an increase of R1/kWh in the retail rate would affect both parties equally. The asymptote is found to be ρ_t=R0.361/kWh, which means that as the retail price tends towards its asymptotic value, social welfare degrades. Figure 4.8 provides a graphical representation of the social welfare ratio within the floor and ceiling constraints, R1.0112/kWh and R1.5566/kWh respectively.

$$F = \frac{275.165\rho_t}{267.62\rho_t - 96.687} \tag{4.34}$$

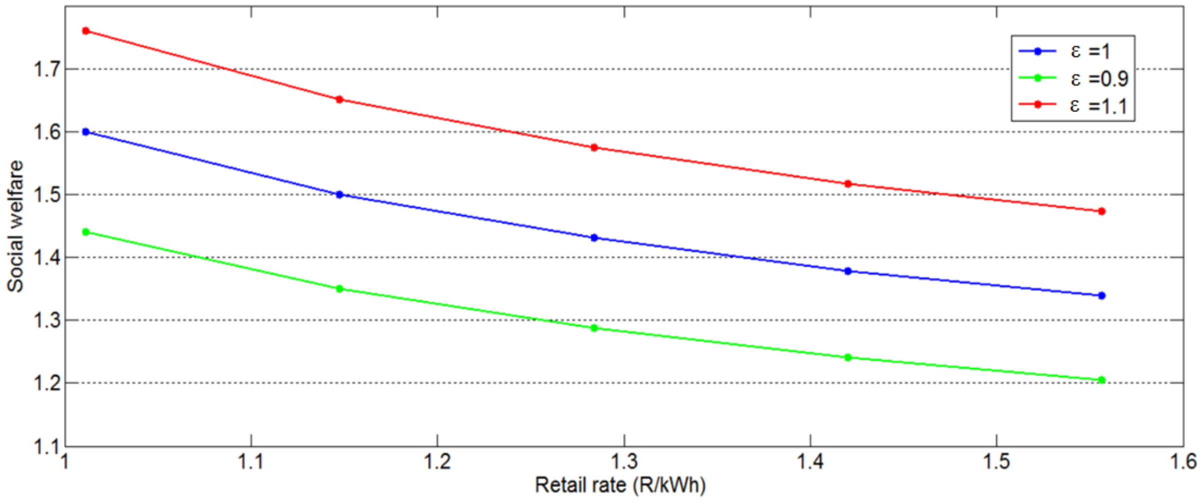


Figure 4.8: Social welfare as a function of retail rate under a fixed pricing scheme

A lower ratio indicates that a lower payment has been achieved by users whilst the retailer has simultaneously enjoyed higher revenues, which is the desired outcome. Results thus reveal that social welfare is maximised under a fixed rate tariff of R1.5566/kWh when the interests of the retailer are favoured slightly over those of residents, indicated by a weighting factor of 0.9. Similarly, social welfare is lowest when the retail rate is set to R1.0112/kWh, the lowest possible tariff, and the interests of the consumer take precedence. Based on the tendency of the plots in Figure 4.8, two conclusions can be drawn. Firstly, if data is extrapolated to extend beyond the predefined boundaries for ρ_t, then based on the laws of limits a maximum social welfare is achieved when social welfare tends to zero, that is, when the retail rate tends to infinity. Secondly, assigning various weighting factors to the consumer's utility does not serve to alter the profile of social welfare, but only stretches it vertically. Thus, it is clear that the current problem formulation favours the service provider excessively over the consumer because it is intuitively and realistically infeasible for social welfare to be achieved when retail prices are continuously on the rise. The sensitivity of the consumer to price changes relative to retailers is therefore not appropriately captured by (4.1), neither is the use of a weighting factor a sufficient tool for circumventing this problem. The initial problem formulation proposed in (3.31) is thus deemed an inadequate tool for capturing social welfare and relative price elasticity amongst parties and the hypothesis of a one-to-one relationship is disproved. A trial-and-error approach is now adopted and an alternative formulation is proposed, in which it is updated to

$$min\ F = \frac{U(\rho_t^n, \bar{x})}{R(\rho_t, \bar{y})} \tag{4.35}$$

where the power *n* is positive. This results in the re-formulation of (4.1) to

$$F = \frac{275.165\rho_t^n}{267.62\rho_t - 96.687} \tag{4.36}$$

The inclusion of a power term for consumers' sensitivity to price changes and not the retailer's proposes an alternative hypothesis that consumers suffer more with an increase in tariff than retailers enjoy the benefits. This is based on the understanding that utility providers around the globe typically enjoy relatively high profit margins due to their pseudo-monopoly of markets in regions where they operate. As such, they are less likely to experience profit cuts as harshly as other low-margin sectors such as the aviation industry. It should be noted however that there exists no literature studying the price elasticity of consumers relative to retailers in order to corroborate or disprove this theory. This, coupled with the knowledge of far-reaching socio-economic poverty in South Africa (Finn et al, 2012), intuitively supports the newly proposed problem formulation that consumers be categorised as far more sensitive than their service providers to price deltas. Figure 4.9 presents a graphical illustration of (4.3) with varying degrees of price sensitivity for the consumer, that is, when *n* is subject to change. It is clear that as the power of the retail rate for residents increases beyond a global minimum, the rate of social welfare degrades and this is confirmed by the rules of transformation (in which translating a function in various ways results in shifting its graph). This is far more representative of reality than the initial formulation presented in (4.1) as the presence of a global minimum for powers of $n > 1$ means that an optimal solution for social welfare may indeed be established. Table 4.9 demonstrates the use of the necessary condition to find these global minima over the selected domain as well as their associated prices for varying degrees of *n*.

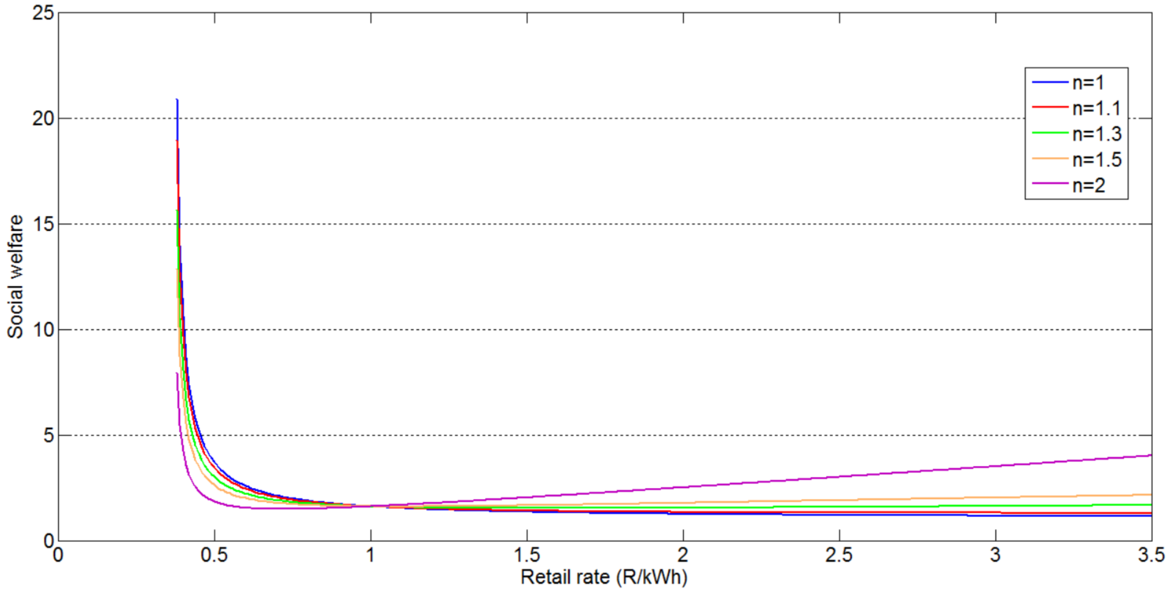


Figure 4.9: Social welfare as a function of the retail rate, subject to price-sensitive consumers

Table 4.9: Global minima for $F(\rho_t)$ over the domain $\rho_t \in (0.361,3.5]$

n	**F′(ρ_t)**	**Roots**
1	$-\dfrac{26604878355}{(267620\rho_t - 96687)^2}$	No real roots/zeros
1.1	$\dfrac{55033\sqrt[10]{\rho_t}\,(267620\rho_t - 1063557)}{2(267620\rho_t - 96687)^2}$	0 3.9741
1.3	$\dfrac{165099\rho_t^{\frac{3}{10}}(267620\rho_t - 418977)}{2(267620\rho_t - 96687)^2}$	0 1.5655
1.4	$\dfrac{55033\rho_t^{\frac{2}{5}}(535240\rho_t - 676809)}{(267620\rho_t - 96687)^2}$	0 1.2645
1.5	$\dfrac{165099\sqrt{\rho_t}\,(267620\rho_t - 290061)}{2(267620\rho_t - 96687)^2}$	0 1.084
2	$\dfrac{550330\rho_t(133810\rho_t - 96687)}{(267620\rho_t - 96687)^2}$	0 0.7226

Of the identified roots which represent the retail rates at which social welfare is optimised, those of *n=1.3*, *n=1.4* and *n=1.5* are the only roots to fall within the predefined boundaries for ρ_t. Under the guidelines of ceiling and floor constraints for the retail price set out by Eskom, social welfare can be optimised in the South African energy market under the conditions of n and ρ_t listed in Table 4.10. For example, when consumers' sensitivity can be quantified by a power of 1.4 relative to the retailer, social welfare is optimised when a rate of R1.2645/kWh is charged. Under this tariff structure, consumer payments amount to R347.95 and retailer profits amount to R241.72. It can be seen that the less elastic a consumer is to price, the higher the retail rate that is charged. At this stage the importance of the selection of n should be emphasised. In the absence of price boundaries, the n can take on a multitude of values, several of which are indicated in Table 4.9. In the presence of these price boundaries, still there are several options from which retailers may choose based on results in Table 4.10, each of which is based on consumer sensitivity. This dependency of price setting on demand elasticity is also testified to in other studies by Nwokoye (1975) and Gourville and Koehler (2004). Furthermore, research by Chicco et al (2004) indicates that demand elasticity is a highly qualitative field that depends on the market in question. Based on this literature and findings, it is recommended that the selection of parameter n be well justified by an extensive socio-economic profile of the consumer market in question as well as their demonstrated price elasticity in other commodity and retail sectors.

Table 4.10: Optimal social welfare and stakeholder outcomes for varying degrees of relative price elasticity, n

n	**Retail price, ρ_t**	**Social welfare ratio**	**Consumer utility (ZAR)**	**Retailer profits (ZAR)**
1.3	R1.5655/kWh	1.529	R430.77	R322.27
1.4	R1.2645/kWh	1.581	R347.95	R241.72
1.5	R1.084/kWh	1.672	R298.279	R193.42

4.5 CONCLUSION

The aim of this chapter was to demonstrate the effectiveness of the developed consumer and retailer models in accurately depicting reality through its application to a South African case study. It also demonstrated the ability of the proposed problem formulation to represent user price elasticity relative to the retailer so that a tariff that achieves optimal social welfare could be identified. A fixed pricing scheme was used for its computational ease, but more advanced structures can also be evaluated with no change to the problem setup and with the use of a metaheuristic that is discussed in Chapter 5. It was found that price boundaries and the selection of parameter n are of key importance when establishing a tariff that achieves social welfare, and both of these are identified to be areas for future study as they are specific to the market in question and require extensive field research.

CHAPTER 5

A SOLUTION STRATEGY FOR TIME-VARYING TARIFFS: TRIAL-AND-ERROR ALGORITHM

5.1 INTRODUCTION

Until now, the relative price elasticity of two stakeholders was a concept that was neither theoretically nor numerically explored in energy literature. This study has developed the basis for its application to the South African context, but to do so, a static tariff was employed for its computational ease and ability to express social welfare as a first degree polynomial. It also satisfied the requirement of an easy-to-implement pricing scheme for an immature deregulated market operating in South Africa. However, because of the lack of communication between the retailer and resident under this tariff, it is not able to derive the benefits of demand response such as reduced PAR for the retailer, and lower bill payments for the consumer as a result of peak load shifting and storage facilities. Because of this, consideration must be given to future research that may pave the way for the achievement of these benefits under more advanced tariffs such as TOU and dynamic pricing (Mohsenian-Rad and Leon-Garcia, 2010, Rastegar et al, 2012). When a static tariff was employed the effects of battery charge/discharge were nullified and optimal and consumer baseline schedules were seen to be identical. Under a dynamic tariff this is not the case. Furthermore, when ρ_t assumes a multitude of values, the size of the decision variable, and dimension of the consumer problem, increases. All of these factors result in greater model complexity and an alternative solution strategy to the one applied for a fixed rate tariff must be considered. To do this, an overview of existing strategies is required. Based on this literature, a local search greedy heuristic is found to be an appropriate tool for demonstrating the capabilities of the model that were overlooked under a static regime. This algorithm is applied under a TOU tariff, and results are presented in Section 5.3. Section 5.4 discusses challenges and recommendations for improvement and Section 5.5 concludes this chapter.

5.2 AN OVERVIEW OF SOLUTION STRATEGIES

The literature review presented in Chapter 2 found that a number of strategies have been applied to resolve the consumer's load scheduling problem. More generally, Feoktistov (2006) classifies approaches as those that deliver optimal or approximate solutions. From this wide array of options, the suitability of a method depends heavily on the complexity of a problem which is in turn, amongst other factors, a function of the number and type of variables that affect whether it may be solved in polynomial time (Christensen, 2007).

5.2.1. Optimal solution strategies

According to Grobler (2008), optimal solution strategies garnered popularity in the 1960s, and the overwhelming algorithm of choice soon became the branch-and-bound approach. Here, nodes are assembled to form a tree, each of which represents a subset of the solution space, and the algorithm systematically searches through these nodes for the optimal solution via implicit enumeration (Land and Doig, 1960). The branch-and-bound method has formed the basis for many exact solvers of LPs, MILPs and MINLPs. This approach, as well as derivatives of dynamic programming which follow a similar but less general 'divide-and-conquer' strategy, have been used by Mohsenian-Rad et al (2010), Chen et al (2011) and Chen et al (2012a) to determine Nash Equilibriums, and Huang et al (2004) and Ramanathan and Vittal (2008) to determine other forms of optimal solutions. The algorithm is however computationally expensive and performance is highly sensitive to the selection of initial upper and lower bounds. The objective of optimising social welfare, a function of conflicting utilities, is also novel and no literature is available to corroborate the use of these optimal approaches. Furthermore, Christensen (2007) shows load scheduling problems to be NP-complete which means that their optimal solutions cannot be found in a polynomial time. This is further supported by Garey and Johnson (1979) who state that such an algorithm does not exist and that NP-complete problems can only be solved optimally in reasonable time frames when their sizes are reduced. Unfortunately, this is not a viable option as the solution strategy must be capable of managing the appliance schedules of entire grids of smart homes. Approximate techniques have thus become an attractive alternative and have been applied to scheduling problems by Gomes et al (2007) and Pedrasa et al (2009). Despite the uncertainty of obtaining a global optimum, larger problems can be solved more efficiently (Grobler, 2008) making them more relevant to real-world applications.

According to Engelbrecht (2005), an algorithm is said to converge to a local optimum and can be classified as an approximate solution strategy if it satisfies the *algorithm* *condition* and *convergence condition.* This means that the new solution suggested by an algorithm's function must be no worse than the current solution, and that with every time step, the algorithm moves some variable **x** closer to the optimality region. To accomplish this, both problem-dependent and problem-independent techniques have been developed and these are known as heuristics and metaheuristics respectively. Because the purpose of applying such a technique in this study is purely to demonstrate the effect of time-based tariffs on demand responsiveness and consumer and retailer outcomes, and because metaheuristics are typically more complex, only heuristics will be discussed further.

5.2.2 Heuristics

Heuristics aim to obtain "good enough" solutions by iteratively progressing to a superior solution under a given instruction set. Criticisms of heuristic methods include their tendency of being too greedy in taking advantage of the specificities of a problem, resulting in entrapment in a local optimum and limited likelihood of finding a global solution. However, because the purpose of applying a heuristic in this study is to demonstrate the effects of demand responsiveness as well as the behaviours and shortcomings of the model when operating under a TOU tariff, these criticisms can be addressed in future research and with more advanced metaheuristics. In support of this theory, several heuristics, some of which have been tailored to the problem under study, have been applied with great numeric success in energy-related literature. Among these are the generic cost model, the local search (greedy) heuristic and the trial-and-error algorithm.

Generic cost model

In the proposed algorithm appliances are scheduled sequentially based on a greedy strategy without back-tracking (Ogwumike et al, 2015). The cost is then evaluated for each feasible start time and this is then fixed before continuing to the next un-scheduled appliance. Simulations revealed negligible cost differences between the heuristic and exact method, but in the worse-case scenario deviations were closer to 32%.

Local search (greedy) algorithm

Priority-based rules specified by the resident are used as the basis for appliance scheduling (Ogwumike et al, 2014). Priority can be assigned in terms of constraint satisfaction or minimum cost incurred based on start times. Findings indicated that costs from the algorithm fell within 5% of the optimal cost and that computational growth was linear with the number of controllable appliances.

Trial-and-error algorithm

According to Bei et al (2015), the trial-and-error algorithm is a key technique in problem solving and knowledge acquisition. Although there may exist other more refined or advanced solution strategies, it can be used as a relatively simple, time-effective means of achieving insights into problems that are otherwise untested. In the energy sector, it has been applied by Aalami et al (2008) in order to obtain the value of a parameter controlling peak loads, and by Chicco et al (2004) to calculate the distance between two representative load profiles.

The objective of the selected heuristic is primarily to explore the behaviour and shortcomings of the consumer-retailer model when operating under a time-based tariff. It is also to demonstrate the model’s capability in capturing and measuring demand response and battery activity. Because these tasks are knowledge-finding in nature and little attention is paid to solution refinement at this stage, the trial-and-error algorithm is found to be an appropriate tool for satisfying these objectives in a reasonable time frame.

5.3 TIME-OF-USE TARIFF

In Section 4.4 a flat rate tariff was used to demonstrate the interaction between the consumer and retailer as well as the impact on their respective outcomes and social welfare. However, it was found that under this regime the effects of demand response and battery charge/discharge status were nullified due to the lack of communication between the parties. In this section a time-varying programme is implemented to demonstrate these key features of the proposed problem formulation. In order to do this, the retail rate ρ_t is randomly selected within the given price range for peak, shoulder and off-peak periods of demand in the day. With each iteration of the local heuristic (in which $\rho_t$varies), the optimisation problem is then resolved. With this technique, the consumer-retailer problem approaches a better solution as time progresses, as will be seen further on.

Table 5.1 provides a comparison of the peak, shoulder and off-peak periods as per Eskom's tariff booklet (2014) against the periods used in this study. Whilst these blocks are comparable, an alternative regime to the one used by Eskom was employed to provide greater flexibility for retail price prediction, and better opportunity for demand responsiveness and battery charge/discharge. As an example for illustrative purposes, the first iteration which represents an initial guess for ρ_t over a 24-hour cycle can be seen in Figure 5.1. Here, it can be seen that higher (lower) retail rates correspond with the approximate periods of higher (lower) demand in Figure 4.7. At this point it should be noted that TOU prices in policy typically reflect the production costs associated with a specific period, but this has not been considered in this study. Specifically, Figure 4.4 does not indicate higher and lower production (wholesale) costs associated with times of peak and off-peak demand, as the determining factors of generation costs fall beyond the scope of this dissertation. As a result, ρ_t had the freedom to take on any value within the given price range (R1.0112-R1.5566/kWh), regardless of the state of demand.

Table 5.1: Comparison of selected TOU block periods against Eskom block periods

TOU block	**Eskom time periods**	**Selected time periods**
Peak	8AM-10AM 6PM-8PM	6AM-10AM 6PM-10PM
Standard	6AM-7AM 10AM-6PM 8PM-10PM	10AM-6PM
Off-peak	12AM-6AM 10PM-12AM	12AM-6AM 10PM-12AM

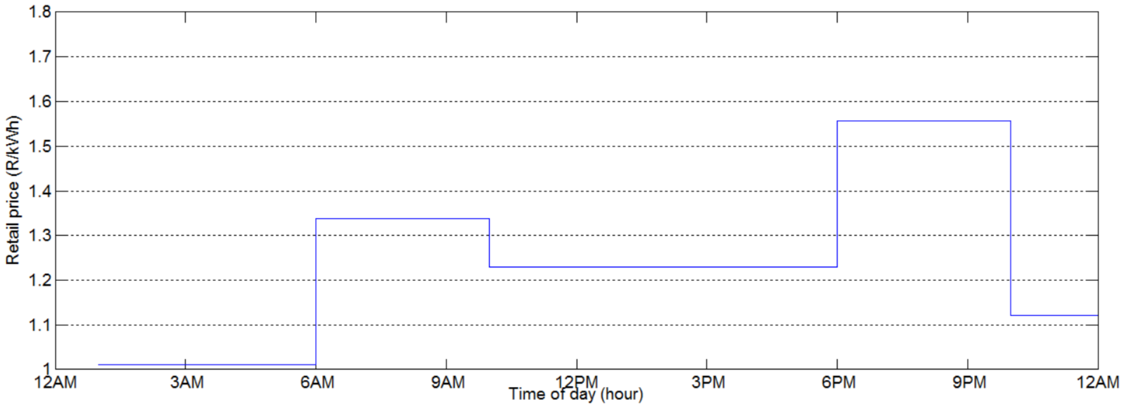


Figure 5.1: Initial guess for $\rho_t$over a 24-hour cycle

Due to the nature of the algorithm, a local heuristic experiences greater success as its number of iterations increases (Ogwumike et al, 2014). To achieve this within a reasonable computational time, a more powerful machine than the one used for a fixed tariff was employed. The consumer-retailer problem was run over a 30-hour period on an i7-3970X Core processor with 8 CPUs and a speed of 3.50GHz. 176 iterations were resolved in this time frame and their respective social welfare ratios can be seen in Figure 5.2. Because $\rho_t$was randomly generated with each iteration, social welfare performance is irregular, neither increasing nor decreasing with time, which is to be expected. It has a social welfare range of 0.2635 (1.6284 to 1.3649), comparable to 0.2609 when a fixed rate tariff (1.59976 to 1.3380) is applied, and this serves to validate the integrity of the model under a time-varying tariff. For ease of analysis, iterations have been re-ordered in Figure 5.3 to illustrate the improvement of social welfare. The corresponding retailer profits and consumer utilities have also been captured for comparison purposes and will now be analysed further.

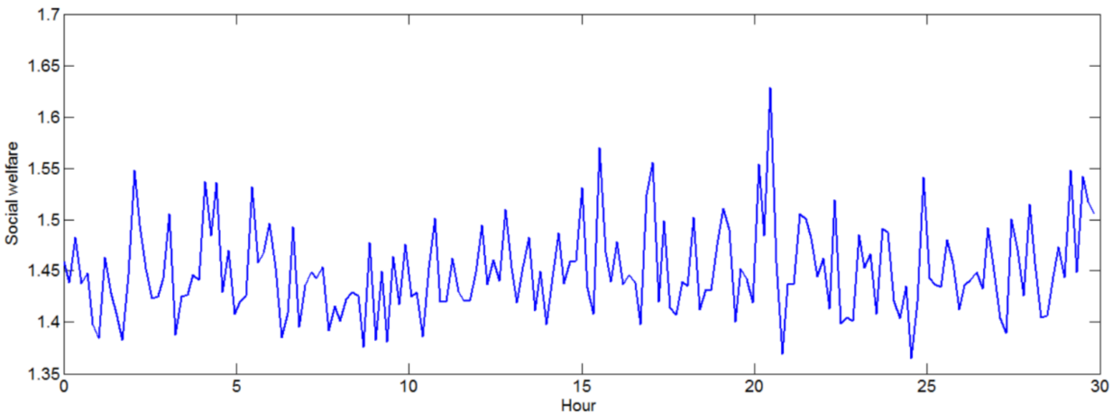


Figure 5.2: Social welfare over a 30-hour period when greedy algorithm is applied

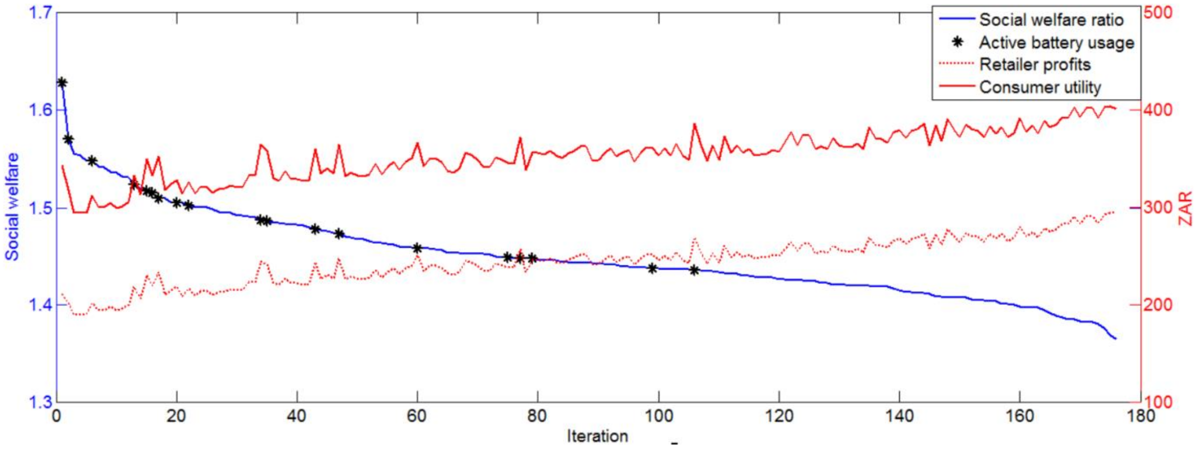


Figure 5.3: Iterations re-ordered from worst-to-best performing in terms of social welfare

Several observations can be made from Figure 5.3. Under the current problem formulation, a locally optimal solution is realised when social welfare is at a minimum of 1.3649, retailer profits are at a near maximum of R293.55, and consumer losses are also at a near maximum of R400.67. From these findings, it can be concluded that, similar to when a fixed rate tariff is applied, the best social welfare is achieved when this ratio tends to zero, that is, when retailer profits tend to infinity. This also implies that consumer utility would continue to increase, as can be seen from the trend in data. Under the current problem formulation, even when operating under a time-based tariff, the model continues to favour the service provider excessively over residents. This is clearly an intuitively infeasible representation of social welfare.

Upon closer inspection of the locally optimal solution, it is found that the consumer does not incur any inconvenience, that is, $\left|ubl_{i,t}^{j} - u_{i,t}^{j}\right|=0$, or the optimal and baseline schedules are once again equal. As such, PAR remains at a maximum of 6.716 as there is no load shifting to ensure a reduction in peak loads, and the residential battery remains inactive. Finally, Figure 5.4 demonstrates that the retail price ρ_t adopts relatively high values within the tight pricing boundaries, no less than R1.4/kWh, regardless of the time of day. It is clear that higher retail prices do not directly and clearly correspond to peak periods of demand. In fact, across the 24-hour cycle the retail rate deviates very little and the desired ‘block periods’ of demand are not mirrored by a diversified pricing regime. Despite being locally optimal, these findings indicate poor communication between the parties and a lack of demand responsiveness from the consumer due to poor price signals delivered by the retailer. Promising recommendations for improvement have however been made in Section 5.4.

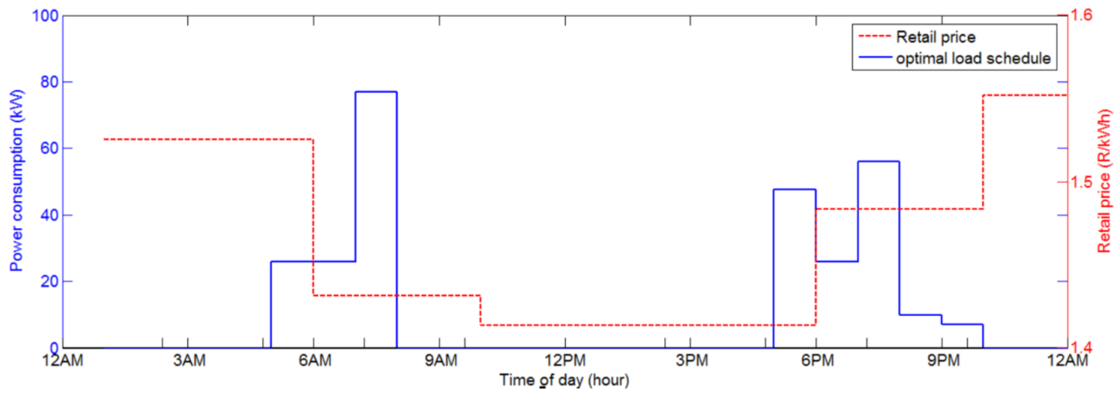


Figure 5.4: Locally optimal solution demonstrating retail price and load schedule over a 24-hour cycle

Due to the nature of results, the effect of the battery on consumer and retailer outcomes could not be appropriately addressed until now. As can be seen from Figure 5.3, only 19 of 176 iterations saw activity in residential battery usage. This can likely be attributed to the fact that a costlier bill from charging a battery could not compensate for the energy savings from it being discharged to appliances. This in turn was due to, firstly, the loss of energy as a result of charging/discharging inefficiency (parameters ω_c and ω_d). It could also be attributed to the lack of designated peak and off-peak periods characterised by higher and lower retail rates respectively, which could have otherwise enabled battery activity. For improvements to the current formulation in order to achieve higher levels of demand responsiveness through energy storage, the reader is referred to Section 5.4. Nevertheless, a comparative study of those computations in which battery activity was present will be conducted, and Table 5.2 presents the best, median and worst performing iterations.

Table 5.2: Comparison of the best, worst and median performing computations with battery activity in terms of social welfare

Category	**Best performing iteration with battery usage**	**Median performing iteration with battery usage**	**Worst performing iteration with battery usage**
Social welfare,	1.4358	1.4875	1.6284

min F			
Consumer losses	R385.22	R363.99	R342.47
Retailer profits	R268.30	R244.70	R210.31
Inconvenience factor, $\left\|ubl_{i,t}^{j} - u_{i,t}^{j}\right\|$	4	34	50
PAR	5.885	6.402	6.410
% reduction in PAR	12.37%	4.66%	4.56%

It is clear from the data that battery usage results in a reduction in PAR, which in turn assists the service provider in managing production planning and costs. This is corroborated by the findings of Chen et al (2012a) and Chen et al (2012b), who also reported significant cost savings. Although the high levels of reduction reported by Mohsenian-Rad and Leon Garcia (2010) of 38% cannot be boasted, results are still comparable to that of Mohsenian-Rad et al (2010) who were able to achieve a 17% decrease. With the implementation of improvements suggested in Section 5.4, it is of the belief that the current consumer problem formulation is still a powerful tool to achieve a lower PAR. It is also interesting to note that a reduction in PAR is not necessarily achieved through an increase in scheduling inconvenience, as one would assume intuitively. This is due to higher levels of dissatisfaction being experienced by a consumer in shifting a more essential appliance over another, as modelled by the parameter δ_i. For example, using a kettle at a less convenient time than a water heater would still reduce PAR, but would also have a less significant effect on consumer dissatisfaction. As such, a resident may select low-priority appliances to shift whilst still achieving a more uniform load profile and reducing their incurred inconvenience. To further demonstrate this, Figure 5.5 presents the optimal schedule in response to the given retail price of the best performing iteration with battery usage (column 1 of Table 5.2), and Figure 5.6 illustrates the energy state of each user type's battery. From here, it can be seen that user type 3's (poorest energy user) battery is charged at 2AM, and user type 1 and 2's batteries are charged at 5AM when the retail price is at its lowest. The electric water heater is used between 4AM-6AM instead of 5AM-7AM, thus accounting for the slight inconvenience factor incurred. All users' batteries are then discharged at 5PM when the tariff is at its highest to reduce the power drawn from the grid and thereby save costs. A similar phenomenon occurs at 8PM for user type 2 and at 9PM for user type 1 until all batteries reach their minimum energy state once again. As a result of this load shifting and battery activity, power consumption is reduced by 17.64kW (or 4.52%), which translates to R5.86 in cost savings for the consumer base, or a reduction of 1.5%. Whilst this is fairly insignificant in comparison to the work of Mohsenian-Rad and Leon Garcia (2010), Rastegar et al (2012) and Chen et al (2012b), under this problem formulation the constraining factors that reduce performance, such as strict pricing boundaries, have been able to be identified and are addressed in Section 5.4. Finally, it cannot be concluded that a reduction in PAR is accompanied by an increase in consumer losses and retailer profits, despite the results of Table 5.2. This is because in other computations in which the battery was inactive and PAR remained at a maximum, residents' and the service provider's objectives still continued to rise, as can be seen in Figure 5.3.

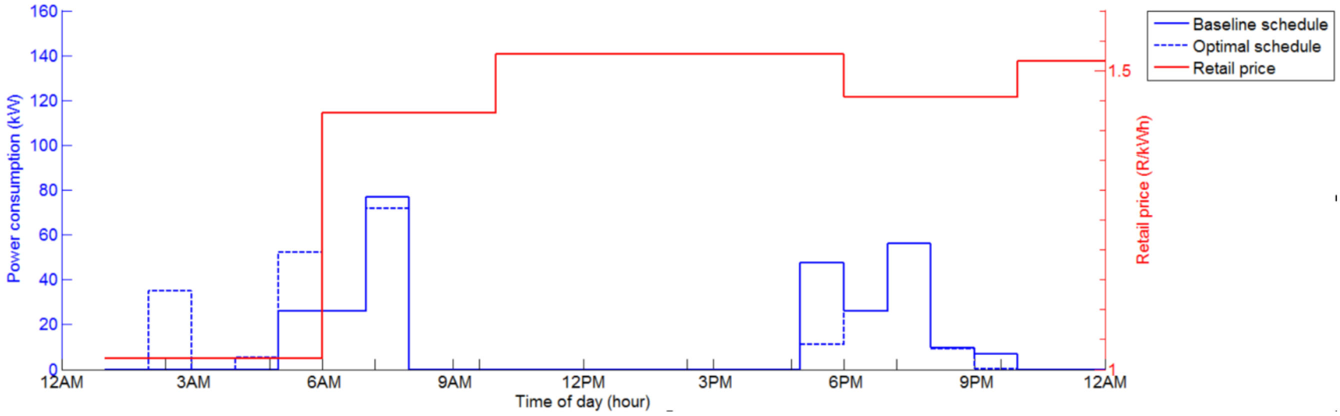


Figure 5.5: Optimal and baseline schedule as well as retail price over a 24-hour cycle when the battery is active

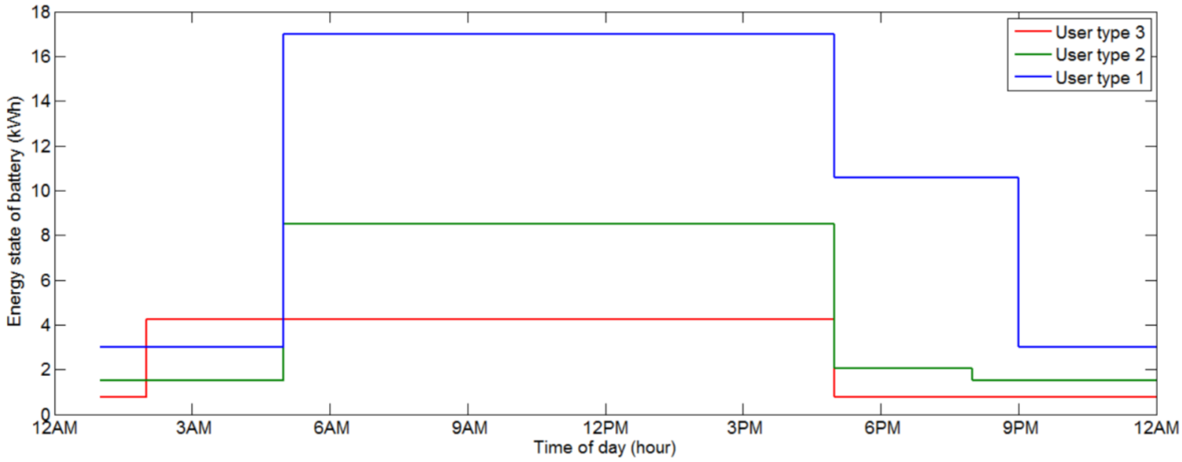


Figure 5.6: Energy state of user *j*'s battery over a 24-hour cycle

5.4 RECOMMENDATIONS FOR IMPROVEMENT OF SOLUTION STRATEGY

The above analysis indicated shortcomings in firstly, the current formulation of social welfare and secondly, in eliciting appropriate levels of demand response and battery usage from the resident, despite operating under a TOU tariff. The following recommendations can be made to improve these efforts in future research:

- An investigation of the power term n similar to the one conducted in Section 4.4 as a means of re-formulating the social welfare ratio. This will serve to once again propose, test and quantify the hypothesis that consumers suffer more than retailer's benefit during TOU tariff increases and that their relative sensitivity is indeed much higher. Because this technique has already been applied to a fixed rate tariff however, its implementation under a time-based tariff is considered to be fairly straight-forward and can be identified as an area for future research.
- Creating pricing boundaries for each block period instead of over a 24-hour cycle (for example, 12AM-6AM, 6AM-10AM and so on) would ensure that the retail price deviates sufficiently during anticipated high and low levels of demand. This would serve to represent the high (low) production costs associated with peak (off-peak) usage periods and would also encourage a shift in consumption by the user in order to reduce bill payments.

- As an alternative to focusing on the consumer, a weighting parameter could be assigned to the wholesale price, q_t, in the retailer problem so as to simulate the effects of higher generation costs during peak periods and lower costs during off-peak periods. This would serve to not only better represent reality, but would also assist in resolving the imbalanced price sensitivity (and associated social welfare skewness) of the consumer relative to the retailer as their profit margins would now be smaller.
- Findings revealed that a TOU tariff is not sufficient to prompt changes in consumption patterns. Rather, thought must also be given to the size of disparity of the retail rate during high and low demand periods; that is, a resident would only be demand responsive if the reduction in payments is larger than the inconvenience suffered in shifting their load. The pricing boundaries of R1.0112-R1.5566/kWh proved to be too tight in order to elicit a significant enough reduction in cost and as such, inconvenience suffered was always greater than payment. Thus, the optimal and baseline schedules were found to be equal and inconvenience was equal to zero. It is thus recommended that these floor and ceiling constraints be relaxed in future simulations.
- From an alternative perspective, a lack of demand responsiveness could also be attributed to the heavy weighting of inconvenience in comparison to the tight pricing boundaries. Reducing the size of parameters λ_j and δ_i, both of which control the importance of postponing or advancing the use of an appliance, would create a larger incentive for the resident to shift their load instead of incurring a high bill. To better investigate this, a sensitivity analysis could be conducted to identify the boundaries or values that best achieve this load shifting. It should be noted that such a strategy would have an impact on both retailer profits and social welfare, and that this too would require further investigation.
- Under the current regime pricing boundaries are too strict, scheduling inconvenience is weighted too heavily in comparison, and retail prices do not reflect peak and off-peak periods of demand. This resulting imbalance of the interacting variables caused battery inactivity in the large majority of evaluations

when the trial-and-error algorithm was applied (in 157 of 176 iterations, the battery was not used by any residents). It should be noted that no direct strategy can be applied to activate the battery as it acts only as a tool, at the disposal of the consumer, to behave optimally in reducing bill payments. As such, implementing any of the recommendations suggested above would help to rectify the imbalance currently present, which would in turn improve the performance of the battery in achieving demand responsiveness.

- The battery analysis revealed the possibility of new ‘peaks’ being created during the recharging process, as at 2AM and 5AM in Figure 5.5. In order to reduce the occurrence of peak-shifting, a sequential decision-making tool similar to that of Chen et al (2011) could be introduced.
- A metaheuristic, as referred to in Section 5.2.1, can be viewed as a higher-performing heuristic which take a more generalised approach to solving an optimization problem. As a result they are often applied to a variety of contexts and sectors, making their performance more reliable and verifiable. In the energy market, evolutionary algorithms and swarm intelligence computations have been used extensively. It is thus recommended that their merit in being applied to the problem under study be explored. For a further investigation of metaheuristics and specifically Particle Swarm Optimisation (PSO), the reader is referred to Appendix B.

5.5 CONCLUSION

A review of literature revealed that the trial-and-error algorithm was the most appropriate solution strategy for application to the consumer-retailer problem operating under a TOU tariff so as to satisfy the objectives highlighted above. It was able to demonstrate the effect of the battery in achieving demand responsiveness and cost savings for the consumer, as well as a reduced PAR for the retailer. Furthermore, the criteria under which a TOU tariff realises its benefits were identified as when there is a sufficient discrepancy between prices from one block period to another so as to delivery load shifting. It also found that an imbalance in the interacting variables of the consumer problem had a significant effect on the low levels of battery activity, load shifting and consumer savings. Once these conditions were satisfied, suggestions for refinement of the selected technique as well as the investigation and application of other superior solution strategies were identified as areas for future research. Pursuing these avenues will certainly contribute to the body of knowledge pertaining to the effect of time-varying schemes on service provider and residents' outcomes when social welfare is prioritised.

CHAPTER 6

CONCLUSION

The electricity market is a complex one with various stakeholders interacting with one another at different stages of the decision-making horizon. This dissertation highlighted a number of factors that must be considered when addressing the consumer and retailer's interests with specific focus on social welfare. This chapter summarises main findings of the study, comments on limitations encountered, and presents avenues for future research.

6.1 SUMMARY

The aim of this dissertation was to identify the tariff rate at which optimal social welfare for the consumer and retailer was achieved. Social welfare was defined as the stage at which these two stakeholder's conflicting objectives are satisfied without the excessive deterioration of the other. To do this, the South African energy market operating under a deregulated system was considered.

The consumer's problem was considered to be one of significant complexity due to its formation as a load scheduling problem. Interests lay in reducing bill payments and inconvenience levels whilst operating a minimum set of appliances in response to price signals received from the retailer. A review of literature revealed that six primary aspects required consideration, namely formulating the interaction between the consumer and retailer, discussing the role of smart technologies in DR, addressing consumer behaviour, managing load, addressing consumer inconvenience and evaluating the effect of energy storage facilities. Based on these findings, a model representing the load scheduling problem faced by the consumer fitted with a residential EMC was developed. In addressing each of these aspects in the model, consideration was given to the South African socio-economic climate and its effect on the energy poverty or energy-intense status of a household. This was thus used as the basis for resource allocation in terms of owned appliances, attitude towards scheduling inconvenience and the size of battery storage facilities.

The retailer's problem required the consideration of new competitive market dynamics. Two procurement options were considered, namely forward-contracts and trading in the spot market. A review of literature revealed that volatility, mean-reversion, seasonality and spike tendencies were apparent with spot market electricity prices and that these transactions had a significant effect on a utility's profits. The most challenging aspect of the retailer problem was identifying an appropriate tool that could accurately forecast these spot prices. To do this, a substitute market that mirrored the seasonal and supply trends of South Africa needed to be identified to serve as a source of historical data for the selected time series model. A three-regime Markov switching model with no parameter restrictions was applied to the Australian market. An analysis of results revealed that the selected method was capable of accurately predicting spot prices at a 95% confidence interval. This, coupled with the inclusion of a stochastic demand component and forward-contract trading, was used to develop the utility's decision-making reality.

The focus of this study was on the achievement of social welfare and not simply the optimisation of a singly party's outcomes. The consumer-retailer problem could not be classified as a Stackelberg game due to the independent optimisation of each party's interests, meaning that the consumer's satisfaction would be subject to the retailers. It could also not be classified as a MOOP due to the requirement of well-validated weighting coefficients that aim to quantify qualitative concepts but which are most often inadequate. A novel problem formulation that represented social welfare as a ratio of consumer and retailer utility was thus proposed.

The proposed model was applied to South African scheduling data in order to determine the fixed rate that would achieve optimal social welfare. Under the guidelines of Eskom's tariff booklet (2014), floor and ceiling constraints for the retail price were enforced. Results indicated that both consumer and retailer utilities could be represented as linear functions of the retail rate. In the first application of the social welfare formulation the resident and service provider responded with equal sensitivity to deltas in the retail rate. This was expressed as a first degree polynomial. This proved to be an inaccurate representation of reality as results indicated that optimal social welfare was achieved when the tariff continued to rise. Thus, the sensitivity of the consumer to price changes, relative to the retailer, could not be expressed as an equal relationship. The proposed problem formulation was thus amended to include a power term that increased if the resident was less price elastic. Based on this, the fixed rate tariffs that achieved optimal levels of social welfare for varying degrees of consumers' elasticity relative to the retailer were identified. The importance of a well-validated parameter that measures the impact of price changes on the resident with the retailer serving as point of reference was highlighted. This work has served to assist in establishing the evident relationship between the achievement of social welfare outcomes and relative price elasticity. Furthermore, neither of these concepts has thus far been explored in literature, and this work serves as a valuable foundation for this topic of research.

It was noted that the benefits of DR such as reduced PAR for the retailer and lower bill payments for the consumer could not be enjoyed under a fixed rate tariff. The determination of time-varying tariffs that achieved optimal social welfare could not be tackled in a similar fashion to that of the fixed rate however, due to added complexities such as the presence of discrete variables, nonlinear terms and a non-convex search space. To this end, a trial-and-error algorithm was proposed as a means of demonstrating the capabilities of the developed model in capturing demand response and battery storage. Results indicated a similar challenge to that experienced under a fixed rate tariff, that is, a continuously increasing retail price resulted in optimal social welfare under the current problem formulation. Recommendations for re-formulation and adjustment of key parameters were made, and attention was turned to the cases in which demand response and battery activity were achieved, resulting in reduced PAR and cost savings for the consumer. The algorithm was identified as a tool that fulfilled its knowledge-acquiring purpose in establishing the behaviour of the model and its effects on party outcomes under a time-based tariff.

6.2 RECOMMENDATIONS FOR FUTURE RESEARCH

This work has formed the basis for future efforts of optimising social welfare and considering the price elasticity of consumers relative to their utility. Above the recommendations presented in Section 5.4, the following opportunities to expand this research can be categorised into five main focus areas and are discussed further below.

- Expanding analysis to include more than one dataset so as to evaluate the effect of the number of users and appliances on model and algorithm performance.
- Validating the parameter controlling the relative price elasticity of the consumer and retailer: this parameter has a significant impact on tariff selection and social welfare achievement. Validation efforts would require qualitative research in the form of questionnaires, surveys and in particular, pilot studies.
- Validating floor and ceiling constraints for the retail tariff: these served as guidelines for determining the feasible regions from which optimal rates could be selected at varying degrees of relative price elasticity. Cross-validation techniques should be explored to ensure that both the parameter n and these pricing boundaries communicate the same social welfare outcomes and price sensitivities to the consumer.
- Investigating differentiating problem characteristics and their effect on consumer, retailer and social welfare outcomes as a form of sensitivity analysis. This may include the effect of various factors such as consumer flexibility, appliance-related inconvenience parameters, and forward-contract prices for the retailer, and should be quantified and analysed for their relative impacts on identified objectives.
- Investigating the ability of DR tools such as EMCs and battery storage facilities to reduce bill payments and improve PAR. Their respective effect on various user types require further investigation in order to determine if the asset management costs associated with their installation are recovered.
- Investigating multi-objective optimisation techniques as a means of addressing and combining the conflicting objectives of consumers and retailers such that social welfare is defined as a single utility function.
- Investigating the performance of alternative heuristics and metaheuristics when applied to the proposed problem formulation. Such an exploration, similar to the

one conducted in Appendix B on Particle Swarm Optimisation, may reveal complementary problem and algorithm characteristics that enhance performance.

6.3 LAST WORDS

South Africa's energy sector, in its current structure, is fast approaching its finite capacity to satisfy public demands for a reliable and cost-effective service. Fear of privatisation in the local climate has been attributed to a lack of knowledge regarding closed market systems as well as a lack of guarantee of social welfare achievement. This dissertation has attempted to depict an accurate reality for the consumer and retailer operating in this market. It has aimed to contribute towards the body of knowledge regarding the requirements and conditions under which social welfare can be achieved. In doing so, several opportunities for future research in the field of energy have been identified.

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APPENDIX A

MATLAB CODE: CONSUMER-RETAILER PROBLEM

```
% This programme models the consumer and retailer problems


%Sets
I=13;
T=24;
J=10;

%Decision variables
u=binvar(I,T,J);
b=binvar(T,J);
d=binvar(T,J);
z=binvar(I,T,J);
B=sdpvar(T,J);
G=sdpvar(T,J);
E=sdpvar(T,J);
C=sdpvar(T,J);
A=sdpvar(I,T,J);

%Parameters
      %Price of purchasing electricity in the day-ahead market
      %q=0.3498+normrnd(0,0.1*0.3498,T,1);
q=[0.367;0.386;0.375;0.339;0.360;0.322;0.381;0.310;0.312;0.321;
0.247;0.400;0.361;0.323;0.398;0.290;0.346;0.341;0.361;0.361;0.319;
0.348;0.344;0.372];

u_bl=zeros(I,T);
u_bl(1,7)=1;u_bl(2,19)=1; u_bl(3,17)=1; u_bl(4,7)=1; u_bl(5,17)=1;
u_bl(6,7)=1; u_bl(7,20)=1; u_bl(8,21)=1; u_bl(9,5:7)=1;
u_bl(10,17:19)=1; u_bl(11,20:21)=1; u_bl(12,20)=1; u_bl(13,21)=1;
u_bl=repmat(u_bl,[1 1 J]);
h=[7;19;17;7;17;7;20;21;5;17;20;20;21];
e=[6;16;15;6;15;6;15;15;4;8;16;16;16];
f=[9;21;21;9;21;9;24;24;9;23;24;24;24];
P=[3;3;1.23;1.9;1.9;1.01;1.235;1.2;2.6;2.6;2.5;3;3.3];
P=repmat(P,[1,24,J]);
D=[1;1;1;1;1;1;1;1;3;3;2;1;1];
gamma=[2;2;1.25;1.75;1;1;1.25;1.25;2;1.75;1.5;1.5;1.5];
alpha=zeros(I,T);
lambda=zeros(1,J);
      %Randomly generated numbers for assignment of users to classes
n=[0.995;0.3320;0.297;0.062;0.298;0.046;0.505;0.761;0.631;0.089];

E_cap=zeros(1,J);
E_max=zeros(1,J);
E_min=zeros(1,J);
n_c=0.8;
n_d=0.89;
      %Price of trading in the spot market, calculated by the Markov
      %switching model
s=[0.297;0.284;0.228;0.202;0.188;0.202;0.253;0.286;0.357;0.389;
0.349;0.349;0.357;0.342;0.343;0.341;0.345;0.345;0.331;0.313;0.355;
0.335;0.329;0.320];

%Consumer objective
```

```
obj=sum(ro.*sum(C'))+sum(lambda.*reshape(sum(sum(alpha.*A)),1,J));

%Subject to:
ro_t =1.0112:(1.5566-1.0112)/4:1.5566;

for o=1:5
      ro=repmat(ro_t(o), [1 T 1]);

      %Minimum usage of appliances
      min_usage=[];

      %Continuous operation of appliances
      cont_usage1=[];
      cont_usage2=[];

      for j=1:J
            if n(j)<=0.662
                for i=[1,2,4,5,9,10]
                  u_bl(3,:,j)=0;u_bl(6:8,:,j)=0;u_bl(11:13,:,j)=0;
                  min_usage=min_usage+[sum(u(i,1:T,j))>=D(i)];
                  cont_usage1=cont_usage1+
                  [sum(z(i,e(i):f(i)-D(i)+1,j))==1];
                    for t=e(i):f(i)-D(i)+1
                          cont_usage2=cont_usage2+
                          [sum(u(i,t:t+D(i)-1,j))>=D(i)*z(i,t,j)];
                    end
                end
                E_cap(j)=5;
                E_min(j)=0.15*E_cap(j);
                E_max(j)=0.85*E_cap(j);
                x(:,j)=E_max(j)-E_min(j);
                lambda(j)=2;
            elseif n(j)<=0.961
                for i=[1:6,9,10,12]
                    u_bl(7:8,:,j)=0;u_bl(11,:,j)=0;u_bl(13,:,j)=0;
                    min_usage=min_usage+[sum(u(i,1:T,j))>=D(i)];
                    cont_usage1=cont_usage1+
                    [sum(z(i,e(i):f(i)-D(i)+1,j))==1];
                    for t=e(i):f(i)-D(i)+1
                          cont_usage2=cont_usage2+
                          [sum(u(i,t:t+D(i)-1,j))>=D(i)*z(i,t,j)];
                    end
                end
                E_cap(j)=10;
                E_min(j)=0.15*E_cap(j);
                E_max(j)=0.85*E_cap(j);
                x(:,j)=E_max(j)-E_min(j);
                lambda(j)=1;
            else
                for i=1:I
                  min_usage=min_usage+[sum(u(i,1:T,j))>=D(i)];
                    cont_usage1=cont_usage1+
                    [sum(z(i,e(i):f(i)-D(i)+1,j))==1];
                    for t=e(i):f(i)-D(i)+1
```

```
                    cont_usage2=cont_usage2+
                    [sum(u(i,t:t+D(i)-1,j))>=D(i)*z(i,t,j)];
                end
            end
            E_cap(j)=20;
            E_min(j)=0.15*E_cap(j);
            E_max(j)=0.85*E_cap(j);
            x(:,j)=E_max(j)-E_min(j);
            lambda(j)=0.5;
        end
    end

%No appliance usage during non-valid hours
zero_usage=[];
for j=1:J
            for i=1:I
                zero_usage=zero_usage+[u(i,1:e(i)-1,j)==0];
                zero_usage=zero_usage+[u(i,f(i)+1:T,j)==0];
                zero_usage=zero_usage+[u(i,1:e(i)-1,j)==0];
                zero_usage=zero_usage+[z(i,f(i)+1:T,j)==0];
            end
        end

%Power balance
power_bal=[];
power_bal=power_bal+[C>=reshape(sum(P.*u),T,J)+B-(n_d*G)];

%Non-simultaneous battery charge/discharge
char_dis=[];
char_dis=char_dis+[b+d<=1];

%Battery's minimum charge of power at time t
min_charge=[];
%Battery's maximum discharge of power at time t
max_charge=[];

for t=1:T
    for j=1:J
        min_charge=min_charge+[iff(B(t,j)>=1e-5,b(t,j)==1)];
        min_charge=min_charge+[iff(B(t,j)<=1e-6,b(t,j)==0)];
        max_charge=max_charge+[iff(G(t,j)>=1e-5,d(t,j)==1)];
        max_charge=max_charge+[iff(G(t,j)<=1e-6,d(t,j)==0)];
    end
end

%Battery's energy balance
batt_bal=[];
batt_bal=batt_bal+[E(1,:)==E_min+n_c*B(1,:)-G(1,:)];
for t=2:24
    batt_bal=batt_bal+[E(t,:)==E(t-1,:)+n_c*B(t,:)-G(t,:)];
end

%Maximum energy state of battery
up_limit=[];
```

```
%Minimum energy state of battery
low_limit=[];
for t=1:T
    up_limit=up_limit+[E(t,:)<=E_max];
    low_limit=low_limit+[E(t,:)>=E_min];
end

%Creating the matrix for scheduler inconvenience
for i=1:I
     for t=e(i):f(i)
          alpha(i,t)=(gamma(i)^abs(t-h(i)))/(P(i)*D(i));
     end
end
alpha=repmat(alpha,[1 1 J]);

%Creating the dummy variable A
dum_var=[];
dum_var=dum_var+[A==abs(u-u_bl)];

%Non-negativity constraints
non_negative=[];
non_negative=non_negative+[B>=0];
non_negative=non_negative+[G>=0];
non_negative=non_negative+[C>=0];
non_negative=non_negative+[E>=0];

%Summation of constraints
constraints=min_usage+cont_usage1+cont_usage2+zero_usage+
power_bal+char_dis+min_charge+max_charge+batt_bal+up_limit+
low_limit+dum_var+non_negative;

%Solver
options=sdpsettings('solver','gurobi','verbose',0);
sol=optimize(constraints,obj,options);

%Analyze error flags
if sol.problem == 0
      obj=value(obj);
else
      display('Hmm, something went wrong!');
      sol.info
      yalmiperror(sol.problem)
end

%Retailer problem

%Uncertainty in observed demand: if demand=0 at time t, then
%uncrt~N(0,1), otherwise uncrt~N(0,total demand of baseline
%schedule/10);
uncrt=[-1.1750 0.8558 -0.3362 -0.2742 -0.9810 -2.7033 -5.4561 -
0.1192 2.1694 -0.3766 0.7987 -0.9420 0.8731 0.8960 0.7155 0.3716
-8.6624    -0.3682 3.3678 -1.7613 -0.2932 1.1975 0.0338 1.3563];

for t=1:T
      if sum(C')(t)+uncrt(t)<=0
```

```
            C_actual(t)=0;
         else
            C_actual(t)=dummy1(t)+uncrt(t);
         end
   end

   Q=sum(C’);
   S=C_actual-Q;
   ret=sum(ro.*C_actual)-sum(Q.*q')-sum(S.*s');
end
```

APPENDIX B

APPLICATION OF PARTICLE SWARM OPTIMISATION METHOD

METAHEURISTICS

Metaheuristics are typically effective because they do not exploit problem-specific parameters, they are able to avoid the entrapment of local optima, even allowing non-improving feasible moves around the search space to improve exploration, and can be seen as black box algorithms that only require fine-tuning of intrinsic parameters to improve performance (Puchinger and Raidl, 2005). Within this branch of optimization evolutionary algorithms and swarm intelligence algorithms have shown the most promise.

Evolutionary algorithms

Evolutionary algorithms (EA) are a population-based metaheuristic that simulate environmental pressures to cause natural selection and improve the fitness of the population (Eiben and Smith, 2003). These natural pressures can take the form of reproduction, mutation, recombination and selection, resulting in an array of subsidiary algorithms. Gomes et al (2007) applied an interactive evolutionary algorithm to capture the decision maker's changing preferences. The EA was also able to accommodate external changes such as profit and demand forecasts, but was not compared to optimum results and its ability in this regard remains untested. A genetic algorithm (GA) is used by Piccolo et al (2001) to optimally coordinate the charging of PHEVs using natural sequences. The selection of the metaheuristic is well-justified and the solution delivered is optimal. However, the repeated evaluation of the fitness function and the exposure of good solutions to destructive mutations can significantly slow down the simulation, especially for complex real-world applications in which a single evaluation may take hours or days. GAs have however enjoyed more success in optimal power flow problems.

Swarm intelligence algorithms

Swarm intelligence refers to the problem-solving social behaviour of a group of individuals who communicate with one another based on interactions with their local environment (Engelbrecht, 2005). This behaviour mirrors that of biological swarm systems such as ants (ant colony optimisation) and bird flocks (particle swarm optimisation). Of these metaheuristics, particle swarm optimisation (PSO) has been applied extensively in the energy and scheduling fields, and reported significant findings (Balci and Valenzuela, 2004, dos Santos Coelho and Lee, 2008, Pedrasa et al, 2009, Pedrasa et al, 2010). For Pedrasa et al (2009), results obtained were comparable to those derived from fuzzy dynamic programming. In Pedrasa et al (2010), the algorithm was enhanced to include static repulsion amongst particles which served to improve exploration of the search space and generated solutions more efficiently. It has thus established itself as an easy-to-implement and computationally efficient tool for optimisation, both of which are key requirements for the selected strategy. Of the approaches discussed, PSO has experienced significant improvements and adjustments since the basic algorithm was implemented by Kennedy and Eberhart (1995). Its application to a variety of problem contexts such as neural networks and manufacturing has proven its adaptability (Eberhart and Shi, 2001). The literature available on its application to scheduling and electricity also provides a sound basis for parameter selection, which has a significant impact on results, results comparison and validation. For these reasons, it is recommended that PSO be applied to the consumer-retailer problem when operating under time-varying tariffs.

PARTICLE SWARM OPTIMISATION

Basic algorithm

PSO has its foundations in the social sciences and was originally intended to graphically illustrate the graceful but unpredictable movements of a bird flock (Eberhart and Shi, 2001). After realising that conceptually it in fact acted as an optimiser, the algorithm was streamlined and first implemented for this purpose by Eberhart et al (1995). Recent modifications to the original algorithm have involved improving convergence and increasing the diversity of the swarm (Engelbrecht, 2005).

In the basic PSO algorithm a swarm (population) of disorganized particles (individuals) exists, each of which represents a potential solution. At each iteration or time step the movement of these particles through the search space is directed by its velocity. This velocity drives the optimisation process by including the personal experiences of the particle (known as the *pbest*), the experiences of other particles in the swarm (known as the *gbest*) gained through socially exchanged information and the velocity at time *t-1* (Engelbrecht, 2005). Herein lies the swarm intelligence of the algorithm. The position of a particle *i* in dimension *j* at time *t*, denoted by $x_{ij}(t)$, is then updated in the search space by adding a velocity that determines its magnitude and direction. This is given by

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \tag{5.37}$$

$$v_{ij}(t+1) = \omega v_{ij}(t) + c_1 r_{1j}(t)[y_{ij}(t) - x_{ij}(t)] + c_2 r_{2j}(t)[y_j^*(t) - x_{ij}(t)] \tag{5.38}$$

where $v_{ij}(t)$ is the velocity of the particle *i* in dimension *j* at time step *t*, ω is the weight of inertia, c_1 and c_2 are positive cognitive and social acceleration constants respectively, $r_{1j}(t)$ and $r_{2j}(t)$ are randomly generated values within the range [0,1] and following a uniform random distribution, *U(0,1),* and $y_{ij}(t)$ is a particle's personal best position. Two basic variations of the algorithm exist which account for the global best position, $y_j^*(t)$, by evaluating the entire swarm (*gbest* PSO) or subsets called neighbourhoods (*lbest* PSO). More detail regarding their differentiating features and neighbourhood formation can be found in Engelbrecht (2005).

The primary shortcoming of the original PSO algorithm is its inability to guarantee convergence (van den Bergh and Engelbrecht, 2002). This is because, when the current, personal best and global best positions of the particle are equal to one another, as seen in (5.3), position updates are solely based on momentum. This can lead to stagnation of the search process and convergence to a point that is not necessarily a local optimum.

$$x_{ij}(t) = y_{ij}(t) = y_j^*(t) \tag{5.39}$$

To overcome this, van den Bergh and Engelbrecht (2002) proposed a derivative of the original algorithm, called the Guaranteed Convergence Particle Swarm Optimiser (GCPSO), which forces the position of the global best particle to change when (5.3) holds true. GCPSO has proven its effectiveness in van den Bergh and Engelbrecht (2002) and Grobler (2008), and it is recommended that this derivative of the original algorithm be applied to the problem at hand when a time-varying tariff is employed.

Guaranteed Convergence PSO

In GCPSO, all particles' positions are updated as usual by (5.1) and (5.2), except that of the global best particle which is forced to search in a confined region for a better position. The *gbest* particle's trajectory is then governed by

$$x_{\tau j}(t+1) = y_j^*(t) + \omega v_{\tau j}(t) + \rho(t)(1-2r_j(t)) \tag{5.40}$$

$$v_{\tau j}(t+1) = -x_{\tau j}(t) + y_j^*(t) + \omega v_{\tau j}(t) + \rho(t)(1-2r_j(t)) \tag{5.41}$$

where τ is the index of the global best particle, $r_j(t)$ forces the random search of the space surrounding $y_j^*(t)$, and $\rho(t)$ is a scaling factor that controls the diameter of the confined region based on past performance. With this in mind, the algorithm for GCPSO is presented in Algorithm A.1.

Create and initialise an n_x-dimensional swarm of S particles

t=1

$\rho(t)=1$

$\zeta=0$ % number of successes

$\eta=0$ % number of failures

repeat

  for *each particle i* **do**

    if $f(x_{ij}(t)) < f(y_{ij}(t))$ **then** % set the personal best position

      $y_{ij}(t) = x_{ij}(t)$

    end

    if $f(y_{ij}(t)) < f(y_j^*(t))$ **then** % set the global best position

      $y_j^*(t) = y_{ij}(t)$

      $\zeta = \zeta + 1$

      $\eta = 0$

    else

      $\zeta = 0$

      $\eta = \eta + 1$

    end

  end

  for *each particle i* = τ **do**

    Update the *gbest* particle velocity with equation (5.5)

    Update the *gbest* particle position with equation (5.4)

  end

  Update the particle velocity with equation (5.2)

  Update the particle position with equation (5.1)

  $t=t+1$

until *stopping condition is true*

Algorithm A.1: The guaranteed convergence PSO algorithm (Grobler, 2008)

Modifications to the PSO algorithm and its derivatives have been made with the intention of improving the speed of convergence and quality of solutions. To do this, an optimal balance between exploration of different regions in the search space and exploitation of efforts in a promising area to refine the solution must be found. Several variations and aspects of the optimisation algorithm aim to control this trade-off and require further discussion.

- Particle dimension, size of the swarm, n_s, and maximum number of iterations, K_{max}, determine the computational burden of the algorithm. Larger swarms imply greater diversity but more complexity and a smaller number of iterations is less strenuous but risks premature termination.
- Clamping is applied to prevent velocities from exploding, resulting in particles leaving the boundaries of the search space. The maximum velocity, V_{max}, is typically set to larger values during initial iterations to enable exploration and is progressively reduced to facilitate concentrated searches of the optimal region.
- Inertia weight controls the memory of a particle’s previous flight direction (Engelbrecht, 2005). Values are problem-dependent with larger magnitudes favouring exploration. Dynamic variation can also applied with similar guidelines for velocity clamping followed here.
- The acceleration coefficients, c_1 and c_2, control the stochastic influence of the cognitive and social components on a particle’s velocity. Velocities with a stronger cognitive and weaker social influence update particle positions based almost exclusively on independent experiences.

Guidelines for these parameters can be found in Engelbrecht (2005). It is recommended that the GCPSO algorithm be applied with the inclusion of these variations so as to achieve higher levels of exploration and diversity during initial iterations, and higher levels of exploitation and solution refinement towards the latter stages of convergence.

PRELIMINARY FINDINGS

Time-varying tariffs introduce an additional complexity to the consumer-retailer problem. Nevertheless, when applied they are capable of achieving levels of demand response that assist the user and service provider in meeting their respective outcomes. These effects, as well as those on the objective of social welfare, must be quantified. The GCPSO algorithm was applied to the problem under study in order to determine the TOU tariff that would achieve optimal social welfare. Algorithm parameters were selected based on guidelines from Engelbrecht (2005) and have been summarised in Table A.1.

Table A.1: Parameters used in the application of GCPSO to improve performance

Parameter	**Value used**
n_s	15
K_{max}	200
V_{max}	0.5454
c_1	2.5 $\longrightarrow$ 0.5
c_2	0.5 $\longrightarrow$ 2.5
ω	0.9 $\longrightarrow$ 0.4

The instruction set, which can be found below, was modelled in MATLAB version 13a and run on an i7-3970X Core processor with 8 CPUs and a speed of 3.50GHz. Based on preliminary computations, the primary, and significant, limitation of applying this algorithm was found to be computational time:

- After a 72-hour period the run was terminated. It was found that only 36 iterations had been performed in this time, and literature suggests approximately 200 or more to be sufficient for convergence to an optimal solution. It should also be borne in mind that all computations were performed on an advanced 8 core processor with 12 threads. This means that it is superior to many regular personal computers both in terms of operating frequency and overall computational power.

- By extrapolating the computational time of 36 iterations and assuming a linear performance, running a full simulation would require approximately 16 days. This is however infeasible and realistically impractical for the current application as, in a deregulated market, tariffs require updates far more frequently. Furthermore, this time estimate is optimistic because when a programme runs on MATLAB, it uses a computer's memory, known as RAM, and this usage is exacerbated over time. This ultimately slows down the performance of a computer significantly, resulting in exponential increases in computational time.

To reduce the computational time, alternative avenues were explored and the following suggestions are made to improve the performance of the GCPSO algorithm applied to the consumer-retailer problem:

- The instruction set given in Appendix A was coded using syntax compatible with the YALMIP platform. This is a user-friendly and easy-to-interpret interface wherein constraints and objectives functions can be written as an operations research model. MATLAB however was designed for, and thus performs optimally, when data and constraints adopt a matrix format.

- Loops, in which certain instructions are performed repeatedly, are used extensively in the consumer-retailer problem. These calculations are only performed sequentially however, thus resulting in large computational times. An advanced technique to overcome this challenge is to apply parallel computing. Here, certain tasks in data-intensive problems are carried out simultaneously by operating on the principle that larger problems can be divided into independent sub-problems. Thus, the full power of multicore processors, grid computing services and computer clusters can be realised. In order to apply parallel computing, the Parallel Computing Toolbox, a product of Mathworks, is required.

```
% This programme models the GCPSO algorithm


% Initialise size of swarm
Ns=15;
T=24;
kmax=200;
k=1;

% Initialise GCPSO parameters
r1=rand(kmax,1);
r2=rand(kmax,1);
sf=zeros(1,kmax);
sf(1)=1;

% Scaling factor to control random search of area surrounding global
best position
no_successes=0;
no_failures=0;
f_prime=zeros(1,kmax);

% Calculation of w, inertia weight, equation (12.24)
para_null=0.9;
para(kmax)=0.4;
para=@(k) (para_null-para(kmax))*((kmax-k)/kmax)+para(kmax);
w=para(1:kmax);

% Calculation of c1 and c2, the cognitive and social constants
c_min=0.5;
c_max=2.5;
dummy1=@(k)((c_min-c_max)*(k/kmax)+c_max);
dummy2=@(k)((c_max-c_min)*(k/kmax)+c_min);
c1=dummy1(1:kmax);
c2=dummy2(1:kmax);

% Price formations
ro=zeros(Ns,T,kmax);
ro_min=repmat(1.0112,[Ns T kmax]);
ro_max=repmat(1.5566, [Ns T kmax]);
ro(:,7,1)=ro_min+rand()*(ro_max-ro_min);
ro(:,8:10,1)=repmat(ro_min+rand(Ns,1)*(ro_max-ro_min),[1 3 1]);
ro(:,11:18,1)=repmat(ro_min+rand(Ns,1)*(ro_max-ro_min),[1 8 1]);
ro(:,19:20,1)=repmat(ro_min+rand(Ns,1)*(ro_max-ro_min),[1 2 1]);
ro(:,21:22,1)=repmat(ro_min+rand(Ns,1)*(ro_max-ro_min),[1 2 1]);
ro(:,23:24,1)=repmat(ro_min+rand(Ns,1)*(ro_max-ro_min),[1 2 1]);

% Initialise particle position and particle personal best position at
k=0;
PBest=zeros(Ns,T,kmax);


% Initialise global best position
GBest=zeros(T,kmax);
GConBest=zeros(1,kmax);
GRetBest=zeros(1,kmax);
```

```
GFitBest=zeros(1,kmax);

% Measure the diversity of searches
diversity=zeros(1,kmax);
diff=zeros(Ns,T);


% Initialise velocity
v=zeros(Ns,T,kmax);
v_prime=zeros(Ns,T,kmax);

% Initialise velocity clamping
V_max=zeros(1,kmax);
velparam=1;
V_max(1)=velparam*(ro_max(1,1,1)-ro_min(1,1,1));
T_stroke=5;

% Calculation of beta, factor that decreases V_max
beta_null=1;
beta_param(kmax)=0.01;
beta_param=@(k)(beta_null-beta_param(kmax))*(kmax-(k/kmax))+
beta_param(kmax);
beta=beta_param(1:kmax);

% Run GCPSO
      while (k<kmax)||(f_prime(k)>0.0001)
            for m=1:Ns
                  diff(m,:)=(ro(m,:,k)-mean(ro(:,:,k))).^2;
                  save('C:\Users\User\yaj\PSOv2.mat');
                  [ff(k,m) cons(k,m)]=fitness(ro(m,:,k))
                  PBest(m,:,k)=ro(m,:,k);
                  [row,col]=find(ff==min(min(ff)));
                  GFitBest(k)=min(min(ff));
                  GBest(:,k)=ro(col(1),:,row(1));
                  GConBest(k)=cons(row(1),col(1));
                  if fitness(PBest(m,:,k))<=fitness(GBest(k,:))
                        no_successes=no_successes+1;
                        no_failures=0;
                  else
                        no_failures=no_failures+1;
                        no_successes=0;
                  end
            end
            if col==m
                  ro(m,:,k+1)=GBest(:,k)'+w(k)*v(m,:,k)+sf(k)*(1-
            2*r2(k));
                  v_prime(m,:,k+1)=-ro(m,:,k)+GBest(:,k)'+
                  w(k)*v(m,:,k)+ sf(k)*(1-2*r2(k));
                  v(m,:,k+1)=(abs(v_prime(m,:,k+1))<V_max(k)).*
                  v_prime(m,:,k+1))+(abs(v_prime(m,:,k+1))>=V_max(k)).*
                  (V_max(k)*tanh(v_prime(m,:,k+1)/V_max(k)));
            else
                  v_prime(m,:,k+1)=w(k)*v_prime(m,:,k)+c1(k)*r1(k).*
                  (PBest(m,:,k)-ro(m,:,k))+c2(k)*r2(k).*(GBest(:,k)'-
                  ro(m,:,k));
```

```
            v(m,:,k+1)=(abs(v_prime(m,:,k+1))<V_max(k)).*
            (v_prime(m,:,k+1))+(abs(v_prime(m,:,k+1))>=V_max(k)
            .*(V_max(k)*tanh(v_prime(m,:,k+1)/V_max(k)));
            ro(m,:,k+1)=ro(m,:,k)+v(m,:,k+1);
      end
      diversity(k)=(1/(max(ro(:,1,k))-min(ro(:,1,k))))*(1/Ns)
      *sum(sqrt(sum(diff')));
      if no_successes>15
            sf(k+1)=2*sf(k);
      elseif no_failures>5
            sf(k+1)=0.5*sf(k);
      else
            sf(k+1)=sf(k);
      end
      V_max(k+1)=V_max(k);
k=k+1;
end
```