ECONOMIC DESIGN IN THE VIRTUAL WORLD: THE FEE STRUCTURE AND SALES MECHANISM IN THE PLAYER-TO-PLAYER TRADING MARKET IN ONLINE VIDEO GAMES

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Abstract

Massively multiplayer online games (MMOGs), often referred to as online video games—which enable players to interact with each other in a virtual world—generate significant and growing economic value. The predominant business model is free-to-play, in which collecting fees from in-game player-to-player (P2P) trading markets is an important way for firms to generate revenue. Surprisingly, quantitative research on the free-to-play business model in this industry is limited and, in particular, a thorough empirical investigation of the design of the P2P market is lacking. This thesis examines the fee structure, followed by an initial investigation of the sales mechanism.

To understand the fee structure, we examine two commonly designs: listing fee (charged upfront) and commission (charged upon transaction). While it seems intuitive that listing fee—an entry cost—should lower the listing and transaction volumes (common industry indicators of long-run profitability) more than commission does, it is unclear which structure generates higher fee collection (short-run revenue). To answer this question, a structural model is developed to capture players' trading behavior and applied to a popular game. The model is empirically challenging due to several industry features including a large number of long-lived players with unobserved heterogeneity. We tackle the challenge by assuming that players don't use their identities as information to form the sale probability, which is innocuous when each player's impact on outcomes is small. We prove the existence of equilibrium, propose and implement a computationally light estimation procedure. Through counterfactuals, we find that there is a trade-off between them: listing fee (vs. commission) generates lower listing volume, similar transaction volume, and greater fee collection. We discuss the managerial implications.

To better understand the sales mechanism, we compare fixed-price posting and auction. Due to data limitation (only fixed-price posting is observed), our goal is to develop a tractable auction framework (for estimation with actual auction data) and to conduct preliminary analysis. Using parameter estimates from the fixed-price posting data, our initial counterfactual shows that there is a trade-off: auction generates higher fee collection and listing volume, but lower transaction volume. We discuss the limitations and managerial implications.

Lay Abstract

Massively multiplayer online games are increasingly popular. The predominant business model is free-to-play, in which collecting fees from in-game player-to-player (P2P) trading markets generates substantial revenue. Surprisingly, minimal research on the free-to-play business model exists. This thesis examines the impact of the fee structure and sales mechanism (auction vs. posted price) on economic outcomes in P2P markets.

For the fee structure, we compare listing fee (charged upfront) and commission (transaction-based) on listing and transaction volumes (long-run profitability indicators), and fee collection (short-run revenue). We develop a quantitative model of players' trading behavior and apply it to a popular game. We find: listing fee (vs. commission) generates lower listing volume, similar transaction volume, and greater revenue. For sales mechanism, our findings suggest that auction generates higher revenue and listing volume, but lower transaction volume. Managerially, the choice of fee structure and sales mechanisms involves a trade-off between long-run and short-run objectives.

Preface

I am the primary author of the work presented in this Ph.D. thesis. I am responsible for identifying the research questions, conducting the literature review, analyzing the data, modeling and coding the estimation and counterfactual procedures, as well as preparing the manuscript. Specific contributions for each chapter are described below.

Chapter 1: introduction. I am the primary author of this chapter with intellectual contributions from Xinlei (Jack) Chen and Charles Weinberg.

Chapter 2: fee structure. I am primarily responsible for identifying the research question, preparing the literature, developing the model and estimation strategy, implementing data clean-up, carrying out the estimations and counterfactuals, as well as preparing the manuscript. Xinlei (Jack) Chen and Charles Weinberg contributed by editing the manuscript, and identifying and positioning the research question. Xiaohua Zeng and Cheng Zhang provided the dataset.

Chapter 3: sales mechanism. I am primarily responsible for identifying the research question, preparing the literature, developing the model, carrying out the counterfactuals, as well as preparing the manuscript. Xinlei (Jack) Chen and Charles Weinberg contributed by editing the manuscript, and identifying and positioning the research question.

Chapter 4: conclusion. I am the primary author of this chapter with intellectual contributions from Xinlei (Jack) Chen and Charles Weinberg.

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Dedication

This thesis is dedicated to my parents Weidong and Yulan, my wife Cuiting, my son Hanxiao and daughter Hanrao.

Chapter 1: Introduction

Massively multiplayer online games (MMOGs), often referred to as online video games — in which players interact with each other in a virtual world—constitute a significant component (30%) of the global gaming market, which includes mobile/console games (Newzoo, 2015). Since the first commercial game was introduced in 1985, *Island of Kesmai*, more than a thousand games have been developed[1]. Some games, such as 2004's *World of Warcraft*, have more than 100 million accounts (Polygon, 2014). The industry's estimated global revenue in 2015 was $27.1 billion (almost 71% of the global box office in the same year; see MPAA, 2015). China (50% share) and the US (15%) are the two biggest markets (Newzoo, 2014, 2015). The industry attracts a large and wide base of gamers, e.g., roughly 168 million worldwide in 2013 (Newzoo, 2013). In 2011 the player base was estimated to be 70% males, with the breakdown of age groups 10–20, 21–35, and "36 or higher" roughly 30%, 50%, and 20%, respectively (Newzoo, 2011).

The industry has been striving for the best business model for years. Part of the challenge arises because gaming companies are concerned both about building their user base and earning profits. Initially, most games were pay-to-play, i.e., players paid a subscription fee in order to play the game. Gradually, the business model transitioned to free-to-play (*Wall Street Journal*, 2014), in which the game is free to download and access. The free-to-play games were estimated

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[1] The lists of English- and Chinese-title games can be found on the following sites (accessed on June 3, 2016):
(i) http://www.mmorpg.com/gamelist.cfm/show/all/All-MMORPG-Games.html
(ii) http://top.17173.com/list-2-0-0-0-0-0-0-0-0-0.html

to account for approximately 70% of the global revenue for all games in 2015 (Newzoo, 2014). In free-to-play, one way a company generates revenue is to sell virtual products to the players. It also collects fees from the player-to-player (P2P) trading market (a common design in the industry) embedded in the game, in which players can trade virtual products (that they earned inside the game)[2] with each other using virtual currency. Since most free-to-play games sell their virtual currencies and they can be used only for trading in the P2P market, the collection of fees becomes an indirect source of revenue which accounts for a significant share of the company's total revenue (Lehdonvirta and Castronova, 2014). For instance, for the focal game studied in this paper, such fee collection is approximately 25% of the total revenue.

Given the significance of the online gaming industry, there is surprisingly little research on its free-to-play business model. Although there are some qualitative studies (Guo and Barnes, 2012; Hamari, 2015; Lehdonvirta, 2009), they mainly focused on examining consumers' motivation to purchase virtual products. Few examined the supply side. An exception is Hamari and Lehdonvirta (2010)—a qualitative study investigating how marketing concepts can help firms improve sales of virtual products. To our best knowledge, there is no quantitative study on the free-to-play business model and, in particular, the economic design of the P2P market.

This thesis attempts to fill this gap by focusing on two important aspects of the P2P market: the fee structure and the sales mechanism. For the fee structure, in practice, most games adopt a two-part tariff (see Table 1.1 for a review of 10 popular games) with a listing fee

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[2] In practice, typically the group of virtual products that a game directly sells to players is mutually exclusive from the group of virtual products that players can earn inside the game and trade subsequently on the P2P market.

(charged as a percentage of price and upfront regardless of the sales outcome) and commission (charged as a percentage of price and only when a transaction is completed). Conditional on the same (percentage) level, the listing fee—being an entry cost by nature—is intuitively expected to lower the listing and transaction volumes (common industry measures of player engagement and therefore indicators of long-run profitability) more than commission. However, it is unclear which one can generate higher fee collection from the P2P market (a common industry measure of short-run revenue). Understanding the implications of these policies is both an interesting academic question and a practical effort for managers. Therefore, the primary research question of this thesis is whether listing fee or commission can generate higher fee collection in a fixed-price posting market, and we address it with an empirical investigation and delineation of the underlying mechanism.

For the sales mechanism, the industry practice (see Table 1.1) is to use either fixed-price posting (i.e., a product is listed at a fixed price before someone decides to purchase) or Buy-It-Now (BIN) English auction (i.e., a combination of English auction and fixed-price posting). Comparison between these two sales mechanisms on listing and transaction volumes (indicators of long-run profitability), as well as fee collection (short-run revenue) is not straightforward. Therefore, the secondary research question of this thesis involves preliminary investigation of this issue. Specifically, due to data limitation (only fixed-price posting is observed in the data), the goal is to develop a tractable auction framework as a starting point that might be useful for future studies and to conduct analyses relying on the estimates recovered from the fixed-price posting data.

The reason we focus on the design of the P2P market in MMOGs rather than the sales of virtual products is that the former is both theoretically and computationally more complex and much less studied. Conceptually, it is similar to many P2P platforms outside the gaming industry such as eBay and Stubhub, which have generated a lot of interest from academia. However, the P2P market in the gaming industry differs in several ways. For example, the products in the games are homogenous due to their digital nature. Conventional wisdom may suggest a perfect competition (for each product) with uniform price on the market. However, in reality we observe non-trivial price variation for the same product. This is mainly because within any given time period, supply and demand are limited. Therefore, sellers—entering the market at different time points—may exploit such supply-demand variation and set different prices from each other. In addition, in the game a player could sell and buy the product at different times, depending on her needs. This is very different from existing studies on P2P platforms outside the gaming industry, in which sellers and buyers are typically two separate groups of people. Therefore, it's necessary to endogenize the selling and buying decisions for a player. This requires endowment information for each player, and fortunately online gaming companies know that information. Furthermore, as suggested by previous studies, psychological factors related to the liking/disliking of buying and selling in a virtual world may have sizable impacts on the market outcomes. Therefore, we also need to incorporate them into the model (by latent costs). Empirically, P2P markets are characterized by a large number of long-lived players (repeat entries to the market) with unobserved heterogeneity in their utility functions. It is well known that solving the equilibrium when there is a large number of players with unobserved heterogeneity suffers from computational burden. In addition, long-lived players bring endogeneity in the evolution of the state space. Both these conceptually interesting aspects and empirical challenges make the P2P market a worthy target to pursue.

Given that the focus is the design of the P2P market, the two factors mentioned earlier—fee structure and sales mechanism—appear to be economically crucial. A review of the most popular games (Table 1.1) reveals large variation of fee structure in practice, suggesting it is not a straightforward decision. For example, some games charge both a listing fee and a commission, while some charge only one of them. Typically, a firm sets a low-high combination of them but the variation is large. In terms of sales mechanism, there is roughly an equal split between fixed-price posting and BIN English auction[3], suggesting there is no consensus as to which one is universally better. To make the business decision even more complicated, as mentioned earlier, typically the P2P market is seen as important both for generating short-term revenue (fee collection) and enhancing long-run profitability (listing and transaction volumes)[4]. That further complicates the comparison between the two types of fees and the two sales mechanisms. This thesis aims to provide an in-depth analysis for these issues.

Our contribution is two-fold. First, we study two important economic design issues—the fee structure and sales mechanism in P2P markets—in a large and fast-growing industry lacking systematic quantitative research. Second, from a broader perspective, this thesis contributes to the empirical literature of P2P trading platform design. In terms of fee structure, there is an

[3] For the ten popular games reviewed in Table 1.1, six of them adopts fixed-price posting.
[4] See Choi *et al.* (2007), Lehdonvirta and Castronova (2014), and Levitt *et al.* (2016).

abundant theoretical literature on two-part tariff[5] pricing design. However, as Schmalensee (1981) suggested, the relative effect (listing fee vs. commission) on revenue is an empirical question, depending critically on their relative impact on the market equilibrium price. Empirical studies on this issue, however, are limited. The closest one to ours is Yao and Mela (2008), who also compare listing fee and commission. This thesis differs in several important ways from that study. (1) While those authors consider an auction market, we examine fee structure under a fixed-price posting market. (2) They focus on the competition among bidders for each auction, while abstracting away from the competition across different auctions mainly due to computational concerns. By contrast, this thesis considers a different sales mechanism, which turns out to allow us to incorporate competition among sellers. To do this, we develop a distinctive conceptualization of the state space and, under reasonable assumptions, are able to derive a computable equilibrium. (3) We also consider a market in which each player can choose to become a seller or buyer at different time points, which arguably is a more general feature in many P2P trading platforms. (4) As found to be important from previous studies, this thesis incorporates psychological factors related to (liking or disliking) trading.

In terms of sales mechanism, there is a large theoretical literature that primarily focuses on the comparison from a seller's perspective. Empirical studies, especially from a platform's perspective, are limited. The closest ones are Hammond (2013) and Bauner (2015). Both papers' primary focus is explaining the coexistence of multiple sales mechanisms in some platform markets (e.g., eBay). In addition, Hammond (2013) compares the auction-only market and fixed-

[5] For a comprehensive review, see Iyengar and Gupta (2009).

price posting-only market in terms of seller and buyer surplus, while Bauner (2015) investigates the impact of the addition of auction and BIN auction to fixed-price posting on seller/buyer surplus and platform revenue. This thesis differs from these two in several important ways. (1) Neither of the other two papers gives a direct answer and in-depth analysis on the comparison of platform revenue between an auction-only market and a fixed-price posting-only market, which is one focus of this thesis. (2) Both papers consider a simultaneous game for sellers (i.e., sellers set prices simultaneously), in which all sellers face the same market information. By contrast, this thesis considers a dynamic market in which players move sequentially and therefore face different market conditions. This dynamic feature, inherent in most platform markets, is crucial in explaining pricing patterns across sellers, i.e., sellers set different prices at various time points partly because they face different market conditions. (3) Similar to the study of the fee structure in fixed-price posting markets, and unlike both existing studies, this thesis endogenizes the choice between selling and buying, and also incorporates psychological factors related to trading.

Chapter 2 studies the fee structure in a fixed-price posting P2P market. A structural model is developed to characterize players' listing/pricing and purchasing decisions. Players are heterogeneous in their utility functions. They enter the market sequentially and repeatedly, choosing among listing an item for sale and setting the price, purchasing an item, or doing nothing. To tackle the empirical challenges mentioned earlier, we make a novel assumption that the players act, *ceteris paribus*, as if the sale probability does not depend on who lists the item. We provide theoretical and empirical justifications for this assumption. Equilibrium existence is proved under this assumption. A computationally light two-step estimation procedure is developed. We apply the model to a popular MMOG (which allows only fixed-price posting and charges both listing fee and commission[6]) in China. We find that there is a trade-off between them: conditional on the same level, listing fee (vs. commission) generates lower listing volume, very similar transaction volume, but greater fee collection. Further investigation reveals that the underlying mechanism is driven by an over-supply of high-priced listings under commission. Managerial implications are discussed.

Chapter 3 compares two different sales mechanisms—fixed-price posting and English auction—under the status-quo fee structure of the focal game. As noted earlier, since we don’t observe the auction mechanism in the data, the focus is to develop a computable auction framework that captures the key aspects of such P2P markets and to conduct preliminary analysis. We therefore consider a simpler case, English auction, rather than the commonly adopted BIN English auction as a starting point. We extend the fixed-price posting model to an auction model in which players (heterogeneous in their utility functions) enter the market sequentially and repeatedly, choosing among bidding on an existing listing and setting the bidding price, listing an item for auction and setting the listing price (i.e., reserve price), or doing nothing. We make a similar assumption that players act, *ceteris paribus*, as if the sale probability (from a seller's perspective) and the winning probability (from a bidder's perspective) do not depend on the identity of the player who lists (or bids on) the item. As an initial step, we use the structural estimates recovered from the fixed-price posting data to compare the two sales mechanisms under the status-quo fee structure. We find a trade-off between them: English auction generates higher fee collection, larger listing volume, but smaller transaction volume

[6] The policy didn't change in the sample period.

than fixed-price posting. Further exploration reveals that such a pattern is driven by the fact that auction creates competition among bidders. Managerial implications are discussed.

The chapters proceed as follows. First, we study the fee structure in Chapter 2. We provide an overview in Section 2.1, review the relevant literature in Section 2.2, discuss institutional background and data description in Section 2.3, provide summary statistics and perform reduced-form tests for two key assumptions in the model in Section 2.4. We then develop the model in Section 2.5, discuss estimation and identification strategies in Section 2.6, perform counterfactual analysis in Section 2.7, and address limitations and managerial implications in Section 2.8. All technical details in the model development are provided in Section 2.9. Tables and figures are provided at the end of the chapter (Section 2.10). Second, we study the sales mechanism in Chapter 3. We provide an overview and relevant literature review in Section 3.1, develop the model in Section 3.2, perform counterfactual analysis in Section 3.3, and offer suggestions for future work and managerial implications in Section 3.4. Tables and figures are provided at the end of the chapter (Section 3.5). Chapter 4 gives a brief conclusion to the thesis.

1.1 Tables

Table 1.1: Popular (2017) Free-to-Play MMOGs in US and China—Sales Mechanism and Fee Structure

Name	Sales Mechanism	Listing Fee	Commission
Focal Game	Fixed-Price Posting	1%	3%
Guild Wars 2	Fixed-Price Posting	5%	10%
The Elder Scrolls Online: Tamriel Unlimited	Fixed-Price Posting	10%	15%
Blade & Soul	Fixed-Price Posting	2%	3%
TERA	BIN Auction	5%	0
Moonlight Blade (CH)	Fixed-Price Posting	2%	0
Dragon Oath 2 (CH)	Fixed-Price Posting	0	8%
Fantasy Westward Journey 2 (CH)	BIN Auction	0	1%
Rift (US)	BIN Auction	2%	5%
Lord of the Rings Online (US)	BIN Auction	6%	5%

Note. All of these games are available in both the US and China. The data sources are two popular MMOG websites: www.mmorpg.com (US) and www.17173.com (China). The first five games are on the top-10 lists of both websites. Games ending with "CH" are on the Chinese list but not the US list, and vice versa for games ending with "US". Each website has its own algorithm to determine the ranking, taking into account user base, playing time and reviews.

Chapter 2: Fee Structure

2.1 Introduction

This chapter examines the differential impacts of listing fee vs. commission on the market outcomes of interest (fee collection and listing and transaction volumes). The analysis takes several steps. A structural model is developed to characterize players' listing/pricing and purchasing decisions in the P2P market. Players are heterogeneous in their utility functions, and the heterogeneity is continuously distributed. They make trips to the market sequentially and repeatedly. During each trip, players choose one of the following decisions: listing an item for sale and setting the price, purchasing an item, or doing nothing. Players are strategic, taking into account the current and future market conditions before expiration[7] to form the sale probability.

Empirically, the model is difficult to estimate due to the challenges mentioned in Chapter 1. Not surprisingly, the few structural empirical studies (e.g., Sweeting, 2015; Yao and Mela, 2008) examining similar issues in real-world P2P markets are all constrained by similar challenges and therefore have to rely on various assumptions to make the model tractable. In our case, we show that the computational burden can be eased by focusing on the sale probability. Specifically, an item's sale probability depends on who lists it due to the unobserved heterogeneity. However, we argue theoretically and show empirically that the large number of "small" (in terms of market share) market participants allows them to act, *ceteris paribus,* as if

[7] In the game, a listing expires after 12 hours if no purchase occurs. In this case, the item automatically goes back to the seller's pocket. This is a common design in MMOGs.

the sale probability does not depend on who lists the item[8]. Under this assumption, we prove the existence of equilibrium with fairly weak conditions. In addition, we prove that the equilibrium can be represented on the space of sale probabilities, which leads to computational efficiency[9]. Specifically, since the sale probability can be directly recovered from the data, this leads to a two-step estimation procedure that successfully tackles the challenges and is computationally light.

We apply the model to a popular MMOG in China. Though analysis at the market level is ideal, the model quickly explodes with the number of products. Therefore, we study an important virtual product that is bought and sold in that game's P2P market. We demonstrate that our algorithm is computationally efficient and that our estimation provides a good fit and recovers the key parameters of interest.

We find that, conditional on the same level, listing fee (vs. commission)[10] generates a lower listing volume (as expected); interestingly, a very similar transaction volume; and more importantly, greater fee collection. This suggests a fundamental trade-off between these two types of fees in terms of short-run revenue (fee collection from the market) and long-run profitability (indicated by the listing volume). Further investigation reveals the underlying mechanism. In commission, there is an over-supply of high-priced listings, and therefore only the

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[8] This assumption follows a similar spirit in the literature as that of Weintraub *et al.* (2008), as well as Krusell and Smith (1998). Essentially, these papers assume that players don't use all information to make decisions.
[9] There are many policy functions due to the unobserved heterogeneity, but only one sale probability given this assumption, which simplifies estimation and counterfactual.
[10] The comparison is between two cases: at the same level, listing fee without commission vs. commission without listing fee.

limited number of low-priced listings contribute to the fee collection. In listing fee, although there are not as many listings in total, their prices are neither too high nor too low, and so a large portion of these medium-priced listings contribute to the fee collection. As a result, the fee collection under a listing-fee structure is greater than that under commission.

The insight gained from our analysis has important managerial implications. First, it provides an explanation for the large variation of fee structures in practice—different companies at different stages may want to trade short-term for long-term revenues or vice versa. Second, we show how this insight can serve as a conceptual roadmap to guide a company to experiment with new fee structures.

In the following sections, we review the relevant literature and provide institutional background and data summaries. We then develop the model and the estimation strategy, followed by a discussion of the identification strategy. Finally we conduct counterfactual analyses. Technical details are left for the end of the chapter.

2.2 Literature Review

In the online gaming literature, most papers use case studies and survey data to explore the demand side, focusing on what factors affect players' purchase of virtual products. Important determinants include (among others) gaming enjoyment (e.g., Hamari, 2015; Nojima, 2007), functional and hedonic attributes (e.g., Guo and Barnes, 2012; Lehdonvirta, 2009), social attributes (e.g., Hamari, 2015; Lehdonvirta, 2009; Lehdonvirta *et al.*, 2009), and different motivations such as extrinsic (e.g., economic in-game profit) vs. intrinsic (e.g., perceived fun) according to Choi *et al.* (2007)[11]. On the supply side, there are limited studies. Using case studies on two games, Oh and Ryu (2007) examine how to balance different groups of items such as those that can be purchased with real money and those that must be earned through game-play. Using case studies on several games, Hamari and Lehdonvirta (2010) look at how marketing concepts (e.g., segmentation and differentiation) can help the game better sell virtual products.

In the marketing literature, a few papers have investigated the pay-to-play business model in this industry. Xiang and Guo (2013) theoretically study the issue of third-party P2P trading, i.e., players trade their virtual products using real money through a third-party platform such as eBay[12]. Two other papers (Albuquerque and Nevskaya, 2015; Nevskaya and Albuquerque, 2015) examine different motivations of players' usage behavior. To our best knowledge, there is no study focusing on the in-game P2P market, which is a critical revenue source of many free-to-play games. Therefore, this chapter is the first to quantitatively examine the economic design (in particular, the fee structure) for the P2P market in this industry.

Although the model and analysis are derived in the online gaming industry, from a broader view, this chapter is related to the limited number of empirical works on the design of fee stucture for online trading platforms. As mentioned before, Yao and Mela (2008) examine fee structure for online auctions. Sweeting (2015) studies the commission structure for perishable

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[11] More references can be found in Hamari and Keronen (2016) and Hamari *et al.* (2015).
[12] The third-party P2P trading is different from the in-game P2P market in this paper. In the former, players trade with each other on a third-party platform using real money, which is often deemed illegal by the game. In the latter, players trade on the P2P market inside the game using virtual currency, which is a service provided by the game.

goods. Studies in this stream of literature are constrained by the empirical challenges mentioned earlier, and they do not consider a market with a large number of long-lived agents having unobserved heterogeneities[13], which is a key feature of many online platforms. Although the focal market of this thesis is inside an online game, several studies have shown that people behave "normally" (i.e., consistent with economic theories) in virtual worlds (Castronova, 2008; Castronova *et al.*, 2009; Chesney *et al.*, 2009; Sousa and Munro, 2012), and researchers started to use virtual worlds as an environment for economic laboratory experiments (Haruvy, 2001). Therefore, this paper also contributes to the empirical platform design literature by developing a tractable framework that can handle a large number of long-lived agents with unobserved heterogeneity.

Finally, our research is related to the empirical literature on games. It is well established that unobserved heterogeneity in players' utility functions creates computational burden—a burden that is exacerbated when the unobserved heterogeneity follows a continuous distribution (i.e., each player has a different utility function) and the number of players is large. Most existing methods assume discrete distributions for the unobserved heterogeneity, e.g., Aguirregabiria and Mira (2007) and Arcidiacono and Miller (2011). The idea is to first recover[14] the equilibrium choice probabilities from the data; based on this, the likelihood function can be evaluated without solving the game. However, such an idea does not apply in our context because even if

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[13] Yao and Mela (2008) consider unobserved heterogeneity in their study but don't model the competition among sellers, where unobserved heterogeneity creates significant challenges.
[14] Strictly speaking, equilibrium choice probabilities cannot be consistently recovered from the data when unobserved heterogeneity presents. Therefore, the authors use different ways to "update" the likelihood function, e.g., Aguirregabiria and Mira (2007) relies on the best-response iteration, while Arcidiacono and Miller (2011) adopt the EM algorithm.

the equilibrium choice probabilities are given, we are unable to evaluate the likelihood function due to the difficulty in simulating the sale probability (details are discussed in Section 2.5.3). We borrow the idea from Weintraub *et al.* (2008) and Krusell and Smith (1998) to make a novel assumption that players don't use their identities as information to form the sale probability. Under this assumption, we show that all computation collapses to a single sale probability despite the large number of policy functions, and our estimation method is developed on this basis.

2.3 Institutional Background and Data

2.3.1 Institutional Background

A typical game system (which the focal game adopts) is as follows. The game sells virtual products and currency to players through the game-store. The virtual products purchased from the game-store are used during the game-play. The virtual currency purchased from the game-store can be used only for trading virtual products in the P2P market, and the game charges certain fees per trade. Players can also gain virtual currency as rewards during game-play. There are two groups of virtual products: those sold by the game-store and those earned as rewards through successful game-play. Typically, they are mutually exclusive. In other words, only those virtual products gained as rewards can be traded on the P2P market. Therefore, firms make money from two sources: selling virtual products/currency and charging fees on the P2P market.

The data were obtained from an action-based role-playing game in China[15]. In the game, players can choose to portray different avatars. Each avatar has a distinct functionality, and players control their avatars to fight with the evil elements portrayed by the game masters, progressing through various challenges and socializing with others.

In this game, the P2P market allows fixed-price posting and charges 3% commission plus 1% listing fee. The policy doesn't change over the sample period. A player can list an item for sale by specifying its price[16]. A listing will last for 12 hours, after which the unsold product will be returned to the seller automatically. Potential buyers can search a specific product by typing its name in the search box. The search result will display all listings with the following information: quantity, name of seller[17], product level (indication of vertical differentiation), and price. However, the expiration time of a listing is not displayed. If deciding to purchase, a player can simply click a "buy" button and there is no price negotiation in this market. In the sample, the collection of fees, when converted into real money based on the price of virtual currency, accounts for approximately 25% of the total revenue.

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[15] The company requests to be anonymous.

[16] Players can put multiple units of an item in a listing. However, for the focal product that we study, more than 95% of its listings have quantity set at one. Therefore, we treat a listing as containing only one item in our model. We drop those listings with more than one unit from the estimation.

[17] Since there are thousands of players using some special Chinese/English/Japanese/Korean characters as their names, we assume that players pay no attention to the seller names in our model.

2.3.2 Data Description and Sample Construction

We collected two data files from the company. The first file includes all trading records on the P2P market. Each record contains: listing ID, listing time, seller's ID, product ID, product level, quantity, price, whether it is sold or not, and if so, purchase time and buyer's ID. The other file includes the list of products owned by all players over time. From that, we know the endowment of products for each player at each time point. Both data files last for 15 days across September and October 2012, three years after the game's launch.

For computational reasons, we need to focus on a single product. The selected product is a powerful belt that can be earned from game-play and traded on the P2P market, but cannot be purchased from the game-store. The belt can enhance a player's power during fights and is generally regarded as aesthetic. A player can wear more than one unit of this belt. The power increases with the units worn. During a fight, there is some probability that a belt will be damaged and therefore lost by the player.

This product is appropriate for our analysis for several reasons. First, it simplifies the model. By design of the game, it is stand-alone: its power does not depend on what other products a player owns. It has only one vertical level, and therefore no quality differentiation for the product, which also simplifies the model. Second, it is the leading product in the belt category. The number of listings for the second best-selling belt is less than 5% of the focal belt's. Third, it is an important product, accounting for 2.2% (the mean is 0.02%) of the total trading value.

We then choose the players for our analysis. We select those who have traded the belt at least once (either purchase or list) during the sample period, which gives us 3977 players. We next construct the estimation sample. In the game, a player needs to go to a specific place in the virtual world to launch the trading window, which we define as one “trip” to the P2P market. In the analysis, an observation is a trip. During a trip, a player can complete multiple trades for different products. We observe each trade but not each trip. Therefore, we need to infer trips from the trading data. We make two assumptions in this process. First, a player makes at least one trade (not necessarily for the focal product) during a trip. Since accessing the market is costly (e.g., a player needs to go to a designated place in the virtual world), we believe such an assumption is reasonable. Second, a player considers trading the focal product only once during a trip, an assumption that is driven by the data pattern (see Appendix A). Since all listings for the focal product have quantity equal to one in our sample, this implies that players can list/buy at most one unit of the focal product in a trip. We detail the construction of trips in Appendix A. In the end, we have 120,697 trips, with 13,536 of them containing a trading record (7369 listings and 6167 purchases) for the focal product.

2.4 Summary Statistics and Reduced-Form Evidence

In this section, we report some summary statistics, followed by an exploration of the data pattern to test some key assumptions in the model.

2.4.1 Summary Statistics

We first report the level of trading activity in Table 2.1. On average, a player makes 3.4 trades, 1.9 listings, and 1.6 purchases during our sample period. The players are "small" in that their contributions to the total market value are small (mean and max are 0.03% and 0.13%, respectively). In addition, 15% of the players both listed and purchased the focal product. This highlights the importance of endogenizing the listing and purchasing decisions, i.e., it is difficult to classify players as sellers or buyers *a priori*.

We examine the time variation for these activities. In Figure 2.1, we plot the total number of trips, listings, and purchases for each hour. Here hour 1 is from 12am to 1am. These activities are highly correlated (pair-wise correlation is at least 0.97). There is clear time variation in activities. The market is quiet in the morning, then gradually picks up till noon and stays at a similar level till evening. Activity picks up again and reaches its peak before midnight, then starts to fade. This pattern is largely consistent with people's daily routine.

Next, we examine the price dispersion. Since the products are homogenous, players always buy the listing with the minimum price[18]. Therefore, minimum price is a natural reference point to define price dispersion in our context. Specifically, we calculate two dispersion measures: percentage deviation of median price (or maximum price) to minimum price[19]. For

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[18] In the data, 95.7% of the transaction prices are the minimum prices. For those violations, the mean percentage deviation from the minimum price is 2.5%. Investigation of the reason is beyond the scope of this paper. We treat the minimum price as the transaction price for those observations with violation in estimation.

[19] The formulas are "(median price - min price) / min price" and "(max price - min price) / min price".

every 5 minutes in our sample, we extract all listings (on the market) from the data and compute the two measures. The means (across all 5-minute time points) are 0.062 (median price dispersion) and 0.159 (maximum price dispersion). This tells us that, on average, the median price and maximum price are 6.2% and 15.9% higher than the minimum price, respectively. This implies non-trivial price dispersion on the market.

2.4.2 Reduced-Form Tests

We use reduced-form evidence to support two of the critical assumptions in our model. First, players don't forward-look for their future trips. Second, during a trip, they consider the current market condition as well as what is going to happen in the next 12 hours (e.g., other players' actions). We test these two assumptions.

2.4.2.1 Are Players Forward-Looking for Their Future Trips?

When endowment is zero, a player has two options: purchase or do nothing. Since the product is homogenous, the purchase decision depends only on the current minimum price if a player is not forward-looking. Otherwise, she will use all current market information to form an expectation of the market condition in her next trip. This creates correlation between the purchase decision and some current market information other than the minimum price. To test this, we select trips in which the endowment is zero and regress the purchase decision onto the current market conditions, including minimum price, mean price, number of existing listings, and hourly dummies. We also control for player fixed effects. If a player takes future trips into consideration, we should observe a significant correlation between purchase and some variables other than the minimum price. Otherwise, minimum price should be the only significant factor.

We report the results in Table 2.2. Column 1 is the baseline case where only the minimum price is controlled, and it has a significantly negative impact on purchase. Column 2 shows that, after controlling for other contemporaneous market variables, only the minimum price is significant, which supports our assumption. In addition, if players consider future trips and have perfect foresight, the purchase decision would be correlated with variables in the next trip. Therefore, we ran a similar regression with variables in the next trip and report the results in column 3. It shows that none of the variables in the next trip has a significant effect. To provide a more comprehensive analysis, we also include market variables in both the current and the next trip (column 4). The main finding is robust. Furthermore, for variables other than the current minimum price, not only are they insignificant, the BIC also becomes larger (meaning a worse fit) after incorporating them. Taken together, these results suggest that forward-looking for future trips is unlikely to be a significant factor in the data.

It is worth noting that the same analysis doesn't apply to the trips in which the endowment is positive. In those cases, the choice set contains listing, purchasing, and doing nothing. Since the listing decision is likely to depend on all current market conditions, the purchase decision will correlate with all current market conditions through its correlation with the listing decision. Fortunately, for our purpose, nearly 71% of all trips have zero endowment. Therefore our results cover a large share of the observations.

2.4.2.2 Do Players Consider Current and Future (The Next 12 Hours) Market Conditions?

When the endowment is positive, a player can list, purchase, or do nothing. For purchase, future market conditions in the next 12 hours are an irrelevant factor because they don't change the current minimum price. For listing, since it will last for 12 hours, will a player anticipate the market change in the next 12 hours? To examine this, we regress decisions about trading (listing, purchase, or do nothing) and pricing on current market conditions, as well as the expected future market conditions in the next 12 hours. We construct the expected number of trips in the next 12 hours to capture future traffic[20] and report the results in Table 2.3. It shows that both listing decision and listing price are affected by the current market conditions. For example, the total number of listings will reduce the likelihood of listing as well as the listing price, possibly due to competition effect.

Table 2.3 further suggests that players do take the traffic in the next 12 hours into consideration when making listing decisions. Interestingly, when the future traffic is high, players become reluctant to list, which suggests that players realize that the increasing traffic may intensify competition. However, once they've decided to list, they tend to undercut the price in a more competitive market. This is true despite the fact that increasing traffic also implies

[20] Based on the idea that players use current market conditions to form expectations, we regress the observed number of trips in the next 12 hours on the current market conditions and use the fitted values. We tried other variables to capture future market conditions but they are highly correlated with current market conditions, e.g., expected mean price in the next 12 hours has 0.97 correlation with the current mean price. Thus, we don't use them.

higher demand. This suggests that future competition effect dominates future demand effect when players make listing decisions.

2.5 Model

The model aims to characterize the trading behavior of players in the P2P market. There is a single product. There are N long-lived players (i.e., they can re-enter the market). They make trips to the market sequentially and repeatedly. The system at t consists of three elements: (1) $B = \{p_j, e_j, w_j\}_{j=1}^{J}$ is the market-book[21] that includes information about all existing listings: p is price, e is time to expiration, w is ownership[22], and J is the total number of listings; (2) $D = \{n_i\}_{i=1}^{N}$ contains endowments of all players where n_i is the endowment for player i; (3) H is hour of the day (e.g., 1am or 10pm), which captures variation of traffic. In sum, in each period we define the system as $L = \{B, D, H\}$.

During period t, at most one player makes a trip to the market according to the trip process $\mathcal{P}_{\varpi}$, which is assumed to be exogenous. We believe the exogeneity assumption is reasonable. As mentioned earlier, players need to travel to a specific place in the virtual world to trade. Thus, it is unlikely that the majority of players would strategically pause their game-play from time to time and take several minutes to access the market, after which they return to their game-play. When a player makes a trip to the market, she checks the price for the focal product

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[21] For notational simplicity, we omit the time subscript.
[22] We have $w \in \{1, 2, \dots, N\}$, e.g., $w = 1$ indicates the listing is from player 1.

once[23]. She also knows the hour of the day and her own endowment. However, she doesn't know the expiration time (not displayed) or ownership for each listing, nor the endowments of the other players. Therefore, the market information observed by the player is $M = \left\{ \left\{ \tilde{p}_j, m_j \right\}_{j=1}^{\tilde{J}}, H \right\}$, where $\tilde{J}$ is the number of distinct prices and $\tilde{p}_j$ is the j^{th} distinct price (ordered from low to high by default) under which there are m_j listings. In addition, the player receives some idiosyncratic shocks (ϵ, ξ) with distribution $F_{\epsilon\xi}$.

When endowment is positive, the player can purchase a current listing at the minimum price, or list a product for sale, or do nothing. When endowment is zero, the player can only purchase or do nothing. If deciding to purchase, when there are multiple listings under the minimum price, the player will randomly select one. This is due to the fact that the product is homogenous[24]. If deciding to list, the player needs to pay a listing fee, and a commission will later be charged upon transaction. She will take into account the action of the incoming players in the next 12 hours. After making a decision, the player will leave the market and come back in the future according to the aforementioned trip process.

Time moves to the next period, *t+1*, during which two types of transitions occur: endowments and the market-book. For the player who made a trip to the market (say player 1),

[23] In the empirical application, if a trade of the focal product occurs during a trip, we know the time of checking the price. Otherwise, we assume that the exact times of checking the prices for the focal item are uniformly distributed within a trip. The empirical distribution of trading time of the focal product within trips in which trading of the focal item occurs is approximately uniform.

[24] The data show supporting evidence for this assumption. For example, when there are two listings under the minimum price, 53.8% of transactions occur at listings that were listed earlier. When there are three, 36.0% (28.7%) of transactions occur at listings that were listed earliest (the second earliest).

her endowment transits according to her decision. Other players' endowments transit based on the process $\mathcal{P}_D$, which captures the flow-in of additional products (i.e., rewards) and flow-out (e.g., damage during fights) during their game-play[25]. The book transits to the next period based on player 1's action and the expiration times in period t. For example, if player 1 decides to list an item for sale and one current listing expires after period t, then the book in the next period contains the new listing from player 1 and deletes the expired listing. In addition, the expiration times of all other listings are shortened by one period. Then a player (or no player) is randomly selected to make a trip to the market according to the trip process $\mathcal{P}_\varpi$ at time $t+1$. In sum, all player actions in the P2P market together with the endowment and trip processes generate the evolution of the system over time. In the next section, we specify a player's information set, action set, utility, and equilibrium.

2.5.1 Information Set, Action Set, and Utility

The information set upon entry[26] is $S = \{M, n, \epsilon, \xi\}$. Denote a_0 as doing nothing and a_b as buying an item at the minimum price. The action set (as a function of S) is

$$A(S) = \begin{cases} \{a_0, a_b\}, & n = 0, \\ \{a_0, a_b, p\}, & n > 0, \end{cases}$$

where p means listing an item for sale at price p. A strategy of player *i* is a function $\varrho_i : \mathcal{S} \to \mathcal{A}$, where $\mathcal{S}$ contains all possible information sets, and $\mathcal{A}$ contains all possible actions.

25 The endowment process is a Markov process, e.g., it gives the probability of the endowment at the next period conditional on the current-period endowment.

26 We use the terms "entry" and "trip" interchangeably.

A strategy profile is $\varrho = \{\varrho_i\}_{i=1}^{N}$. The trip process $\mathcal{P}_{\varpi}$, the endowment transition process $\mathcal{P}_D$, and the distribution of preference shocks $F_{\epsilon\xi}$ are all common knowledge.

We denote $V_i(n, \epsilon_n)$ as the utility of owning n units of the focal product for player *i* after receiving an idiosyncratic shock ϵ_n. As mentioned earlier, the item is generally regarded as aesthetic, and players become more powerful depending on how many belts are geared up. Therefore, we allow the utility to depend on the endowments. In addition, rather than being a flow utility, V_i is best viewed as the perceived sum of discounted utilities of owning the product over time. The subscript *i* emphasizes that different players value the item differently (i.e., unobserved heterogeneity), which is a key feature to capture variation across players in the data. The utility of purchase is

$$u_{ib}(S) = V_i(n+1, \epsilon_{n+1}) - \beta_i(\xi) \cdot \tilde{p}_1 - c_{ib} . \tag{2.1}$$

$\beta_i(\xi)$ is the individual-specific price coefficient, which depends on an idiosyncratic shock[27]. Since the virtual currency can be used only to purchase virtual products on the P2P market, it can be viewed as the value for the composite outside good in the virtual world. $\tilde{p}_1$ is the minimum price on the market.

[27] The reason to have a random shock in the price coefficient (in addition to the random shock in the product preference) is to smooth the choice probability, which is useful to guarantee the existence of equilibrium.

The last term c_{ib} reflects any latent cost of purchase and is used to capture some interesting and important features in the industry. First, as shown by Hamari (2015), players have different attitudes towards purchasing virtual products compared to earning them from game-play. Some view it as a justified behavior, but many are not proud of it. Often among social groups in a game, there is a negative view of purchasing virtual products as opposed to earning them through your own game-play. On the other hand, as shown by Choi *et al.* (2007) and Lehdonvirta and Castronova (2014), players perceive trading virtual products in a virtual world to be fun. We cannot separately identify these two effects and therefore unify them with the name "latent cost of purchase", allowing it to vary across individuals because different players may have different attitudes and perceptions about acquisition through purchase. Although termed a "cost", there is no restriction on its sign, according to the discussion above.

The utility of listing an item for sale at price p is

$$\begin{aligned} u_{is}(S,\, p; \varrho) = q_i(S,\, p; \varrho) \cdot \left[\beta_i(\xi) \cdot (1-\tau_1) \cdot p + V_i(n-1, \epsilon_{n-1}) \right] \\ + \left[1 - q_i(S,\, p; \varrho) \right] \cdot V_i(n, \epsilon_n) - \beta_i(\xi) \cdot \tau_2 \cdot p - c_{is}. \end{aligned} \tag{2.2}$$

τ_1 is commission and τ_2 is listing fee. The sale probability, $q_i(\cdot)$, is the probability that a newly added listing with price p by player *i* will be sold within the next 12 hours conditional on S and ϱ. We will discuss $q_i(\cdot)$ later. The first part of equation (2.2) captures the case in which the item is sold, including the gain of listing price and the utility of having only $n-1$ units. The second part captures the case of no sale: the utility of having n units. The first two parts are then weighted by the corresponding probabilities to form the expected utility. The third part captures the fixed cost of listing due to listing fee.

Similar to the purchase utility, the last part, c_{is}, is the latent cost of listing. In this case, in addition to capturing the fun of trading as mentioned above, it includes the transaction cost of listing, such as the effort to set up a listed item and find the optimal price (e.g., Choi *et al.*, 2007). We allow it to vary across players. Similarly, no prior restriction is imposed on its sign.

Finally, the utility of doing nothing is $u_{i0} = V_i(n, \epsilon_n)$. The distribution of preferences and the number of players are common knowledge.

2.5.2 Sale Probability

The computation of the model (estimation and counterfactual) is complicated by a large number of strategies and sale probabilities as consequences of a large number of players with unobserved heterogeneity. To resolve such complexity, we choose to work on the sale probability. We will first explain theoretically why the sale probability depends on the seller's identity, and why such dependence is empirically negligible when there is a large number of small players. We then present a key assumption based on this empirical feature and discuss how it helps to tackle the computational burden.

In principle, the sale probability of an item listed by a player depends on the behavior of other players, i.e., the opponent set of this player. Since the opponent set differs across players, and players are asymmetric because of unobserved heterogeneity, the difference in opponent sets implies difference in sale probabilities. However, if the market contains a large number of players, the opponent sets of any two players are largely overlapped. In addition, if players are "small" in that no one contributes a large share to the total trading value, then the difference between two opponent sets is empirically negligible compared to their overlap. Therefore, we assume that players don't use their identities to form the sale probability[28]. In other words, they all use a homogenous sale probability. Their strategies, however, remain heterogeneous because their utility functions are still different.

Although the assumption is intuitive, it turns out to be very powerful to tackle the computational burden if we can show that the equilibrium (which typically is expressed on the space of strategy profiles[29]) can be represented by the sale probability. This is the theoretical foundation of our computation algorithm, which hinges on two factors: (1) the existence of equilibrium, and if so, (2) its representation by the sale probability. Many existing proofs (Doraszelski and Satterthwaite, 2010; Rieder, 1979) for equilibrium existence of Markov games[30] rely on an assumption that the utility function is continuous on the strategies. In our model, such a continuity assumption depends on whether the sale probability is continuous on

[28] This follows the spirit of Weintraub *et al.* (2008) and Krusell and Smith (1998), who assume that players don't use all information to make decisions. We also use the data to test this assumption. We estimate the sale probability with a random individual intercept controlling for other factors. The *p*-value for the standard deviation of the random intercept is 0.63, suggesting that individual difference is unlikely to have significant impact on the sale probability.
[29] A strategy profile contains N different functions where N is large.
[30] The current game is a Markov game because players' strategies only depend on their current-period states rather than the entire history of the game.

players' strategies ϱ. It turns out that identifying conditions under which such a continuity property holds in our model is non-trivial. In what follows, we examine the continuity property of the sale probability, illustrate what the challenges are, and intuitively explain our solution.

Before that, following Yao and Mela (2008), we assume the random shocks are independent across players. Therefore, the incoming players' behavior is independent of ϵ and ξ, which implies that the sale probability does not depend on them. In sum, equation (2.2) becomes

$$\begin{aligned} u_{is}(S, p; \varrho) = q\left(\tilde{S}, p; \varrho\right) \cdot \left[\beta_i(\xi) \cdot (1-\tau_1) \cdot p + V_i(n-1, \epsilon_{n-1})\right] \\ + \left[1 - q\left(\tilde{S}, p; \varrho\right)\right] \cdot V_i(n, \epsilon_n) - \beta_i(\xi) \cdot \tau_2 \cdot p - c_{is}, \end{aligned} \tag{2.3}$$

where $\tilde{S} = \{M, n\}$, and $q\left(\tilde{S}, p; \varrho\right)$ is the probability that a newly added listing with price *p* by a player will be sold within the next 12 hours, conditional on $\tilde{S}$ and ϱ. The key difference from equation (2.2) is that in equation (2.3), the sale probability is conditional on a (randomly chosen) player while in equation (2.2), it is conditional on a specific player *i*.

We now examine the continuity property of the sale probability. For illustration purposes, we focus on the sale probability at the next period. Intuitively, *if* a player observes the following: (i) prices of all current listings, (ii) hour of the day, (iii) current endowments of all players, and (iv) expiration times of all current listings[31], then she can draw a path that can determine the probability that a newly added listing will be sold in the next period. The path consists of the identity and endowment of the player who will enter the market in the next period. The sale probability given a path would be an integration of the random shocks in the incoming player's

[31] If this is unknown, the player doesn't know what listings will remain on the market in the next period.

utility function, i.e., the choice probability induced by ϱ. An integration of all possible paths would be the sale probability *conditional* on (i) to (iv). In this case, continuity is straightforward: the sale probability is a linear function of the choice probabilities.

However, players don't observe (iii) and (iv). Therefore, one needs to further integrate (iii) and (iv), conditional on (i), (ii), and the strategy profile. The challenge to showing continuity brought by such integration is that the distributions of (iii) and (iv) are endogenous[32]: they depend on the strategy profile, and the functional forms are not obvious. The endogeneity of (iii) comes from the feature that players are long-lived (make repeated trades on the market), and thus their endowments at any given time point depend on their strategies. The endogeneity of (iv) is driven by the dependence of players' behavior on hour of a day[33]. While the endogeneity of (iv) may be a specific feature for some games in this particular industry, that of (iii) is likely to be a common feature for many online P2P platforms, as it essentially means that a repeat participant's current belongings depend on what she did in the past. Thus, the idea that we develop below may help future studies facing similar problems.

Our approach is to derive the stationary distribution of the system (which consists of (i) to (iv)), given a strategy profile. Based on that, we can derive the distribution of (iii) and (iv). Therefore, the questions now become: given a strategy profile, under what conditions does a

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[32] If they are exogenous (i.e., not dependent on the choice probabilities), such integration adds no difference.

[33] In an extreme example, if players never list their items for sale between 12am and 1am, then the expiration times of listings at 12pm are always less than or equal to 11 hours.

unique stationary distribution of the system exist, and most importantly, is it continuous with respect to the players' strategies? We show the results in the next section.

2.5.3 Stationary Distribution of the System and Equilibrium

Proposition 1 below shows the existence, uniqueness, and continuity of the stationary distribution of the system. We leave its proof and the derivation of the sale probability to Section 2.9.

Proposition 1. *Under certain conditions, given any strategy profile, a unique stationary distribution of the system exists. Moreover, it is continuous on the choice probabilities.*

We briefly discuss the conditions. First, we require the probability that no player making a trip to the market in any period is strictly positive, which is innocuous given that a period is defined to be one second in our application[34]. Second, we require the probability that any player obtaining any quantity of endowments through game-play without using the P2P market is strictly positive. This assumption does not dismiss the function of the P2P market in the game. In practice, such probability can be arbitrarily near zero, such that the P2P market plays an important role in facilitating circulation of virtual products. Last, we require the probability that endowments do not change in the next period to be strictly positive, which is reasonable given such a short time period.

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[34] During our sample period, only 9.3% of the periods contain a trip.

Proposition 1 serves as the building block to prove the existence of equilibrium. Equilibrium of this market, as defined below, is a strategy profile stipulating that every player's trading decision (purchase, listing, and pricing) is optimal given others behaving according to the focal strategy profile.

Definition. *An equilibrium is a strategy profile* $\varrho^* = \{\varrho_i^*\}_{i=1}^N$ *such that for any i and any* $S \in \mathcal{S}$ *,*

$$\varrho_i^*(S) = \arg\max_{A(S)} \left\{u_i(S, a_0), u_i(S, a_b), u_{is}\left(S,\ p; \varrho^*\right)\right\},$$

where $u_i(S, a_0) = u_{i0}(S)$*,* $u_i(S, a_b) = u_{ib}(S)$ *and*

$$\begin{aligned} u_{is}\left(S,\ p; \varrho^*\right) = q\left(\tilde{S},\ p; \varrho^*\right) \cdot \left[\beta_i(\xi) \cdot (1-\tau_1) \cdot p + V_i(n-1, \epsilon_{n-1})\right] \\ + \left[1 - q\left(\tilde{S},\ p; \varrho^*\right)\right] \cdot V_i(n, \epsilon_n) - \beta_i(\xi) \cdot \tau_2 \cdot p - c_{is}. \end{aligned}$$

Proposition 2 establishes the existence of equilibrium.

Proposition 2. *Under certain conditions, there exists an equilibrium.*

The proof is provided in Section 2.9. It turns out that we need only one more condition than those required by Proposition 1. Specifically, we assume that the optimal choice probability is continuous with respect to the sale probabilities. We do not provide general conditions under which it holds. Instead, we verify it (provided in Section 2.9) after specifying the functional forms in the model.

As mentioned earlier, the theoretical foundation of our computational algorithm depends on not only the existence of equilibrium, but also whether it can be represented on the sale probability or not. The reason why the representation of equilibrium on the strategy space is a computational hurdle is two-fold. First, there is a large number of strategies, which makes it challenging to solve the game in terms of time and computer memory. Second, even though the equilibrium strategy profile is given, in order to evaluate the likelihood function, one needs to simulate the sale probability. That, in turn, requires first solving the stationary distribution of the system. The large dimensionality of the system makes it a very challenging task. Fortunately, an important feature of the model—that all strategic interactions are embedded in the sale probability—provides a solution, as shown by the representation corollary below.

Representation Corollary. *The equilibrium can be represented by the sale probability*.

We provide the proof in Section 2.9. Intuitively, an equilibrium strategy profile induces a sale probability—let's call it an equilibrium sale probability. The corollary states that the set of equilibrium strategies is one-to-one to the set of equilibrium sale probabilities.

2.6 Estimation

We first specify the functional forms, then discuss estimation strategy, identification, and results.

2.6.1 Functional Form Specification

We assume the product utility $V_i(n, \epsilon_{int}) = \gamma_{in} + \epsilon_{int}$, where $\gamma_{i0} = \epsilon_{i0t} = 0$, γ_{in} is player *i*'s intrinsic preference for owning n units of the product, $\epsilon_{int} \sim N(0, \sigma_n^2)$ and $corr(\epsilon_n, \epsilon_{n-1}) = \rho$. We further assume that the random shocks are independent across players and time. For the price coefficient, we assume $\beta_i(\xi_{it}) = \beta_i + \xi_{it}$, where $\xi_{it} \sim N(0, \sigma_\xi^2)$ and is independent across players and time. We also assume that ξ and ϵ are independent from each other. To capture heterogeneity across players, we assume[35] $\gamma_{in} - \gamma_{i,n-1} \sim N(\gamma_n - \gamma_{n-1}, \sigma^2_{\gamma_n - \gamma_{n-1}})$ and $\beta_i \sim N(\beta, \sigma_\beta^2)$.

In Section 2.9, we verify that the optimal choice probability is continuous on the space of sale probabilities under these specifications.

2.6.2 Estimation Strategy

Based on the theoretical development presented earlier, we propose a two-step estimation procedure. In step one, we estimate the sale probability from the data. In step two, conditional on the estimated sale probability, we maximize the likelihood of the observed choices. We discuss each step in detail.

[35] Since we can only identify the differences of utilities, we work on the differences of parameters.

In step 1, the sale probability q depends on four elements: (1) M , price distribution on the market; (2) H , hour of day; (3) n , endowment of the seller; and (4) p , listing price. We observe all of them in the data. We also observe whether each listing is sold or not. Therefore, we use each listing event as an observation to run a logistic regression, with the dependent variable to be sold or not, and the independent variables to be (M, H, n, p) with a flexible functional form[36]. The output of this step is the fitted sale probability $\hat{q}(\cdot)$.

In step 2, we maximize the likelihood of the observed choices taking $\hat{q}(\cdot)$ as given. Specifically, for each player *i*, we observe the following data $\{y_{it}, \tilde{S}_{it}\}_{t=1}^{T_i}$, where y_{it} is the observed choice for the t^{th} trip, $\tilde{S}_{it}$ contains (M_t, H_t, n_{it}), and T_i is the total number of observed trips for *i*. The log-likelihood function is

$$\mathcal{Q}(\Theta) = \sum_{i=1}^{N} \log\left(\int_{\nu_i} \left[\prod_{t=1}^{T_i} \mathcal{W}\left(y_{it}; \tilde{S}_{it}, \nu_i, \Theta, \hat{q}(\cdot)\right) \right] dF(\nu_i; \Theta) \right),$$

where Θ is the unknown parameter, ν_i is the draw for individual parameters, $\mathcal{W}(\cdot)$ is the probability of observing choice y_{it} conditional on $\{\tilde{S}_{it}, \nu_i, \Theta, \hat{q}(\cdot)\}$, and $F(\nu_i; \Theta)$ is the cumulative distribution function for ν_i .

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[36] In practice, we discretize the state space and the action space (details are provided in Appendix B). We use a quadratic form for the independent variables in the logistic regression.

There are several merits of our proposed method. First, it is computationally light. The sale probability can be directly estimated from the data conveniently. In the second stage, once conditional on the sale probability, we don't need to solve the equilibrium (due to the representation corollary), nor do we need to solve the stationary distribution of the system. Therefore, the computational time is significantly reduced. Second, we have a large number of observations for the first stage (7369 listings), which provides a reasonable precision. Third, following Aguirregabiria and Mira (2007), we assume that the data are generated by a single equilibrium, even though at the true parameter the model permits multiple equilibriums. Since the equilibrium played in the data is already identified in the first stage due to the representation corollary, the potential issue of multiple equilibriums can be resolved in estimation.

2.6.3 Identification

The structural parameters include the distributions of product valuation V_i, price coefficient β_i, as well as the latent cost of purchase c_{ib}, and the latent cost of listing c_{is}. Identification of β_i comes from the correlation between purchase likelihood and the minimum price on the market (see equation (2.1)). In other words, variation in the minimum price helps to pin down the price coefficient. Once β_i is identified, variation in the listed prices helps to pin down V_i. Intuitively, conditional on listing, the higher the valuation for the product, the higher price a player will set. After both β_i and V_i are identified, the latent cost of purchase c_{ib} is identified from the likelihood of purchase conditional on endowment and the minimum price. Intuitively, if we treat purchase as a "brand", then the latent cost of purchase is a brand intercept. Therefore, aggregate share helps to pin down the latent cost. Similarly, the latent cost of c_{is} is identified from the aggregate share of listing. Since we observe multiple trips for each player, the variances of the distributions for all these parameters are identified from the variation across all players.

2.6.4 Estimation Results

The estimation results of the structural parameters are presented in Table 2.4. The results show that players exhibit non-linear, convex preference for the product with increasing endowments. That is, an average player has increasing marginal utility for the focal product. This is due to the design of the game, e.g., wearing two units of the product is more powerful than twice of wearing one unit. This is a common industry design, aiming to motivate players to spend more time playing the game.

The price coefficient has the expected sign that losing/gaining virtual currency has a negative/positive impact on the utility. The heterogeneity parameters highlight the importance of taking individual differences into account.

For the latent costs of purchase, the result shows that the negative view towards purchase dominates the fun of trading for an average player, reflected by c_b. However, the heterogeneity is large, reflected by σ_{c_b}. In addition, there is a clear segmentation along the latent cost for listing, reflected by c_s and σ_{c_s}. For some players, the perceived fun of listing dominates the transaction cost of listing, while the opposite holds for others.

We also assess our model fit, focusing on several important measurements. Table 2.5 shows that the full model fits the data well, but the fit of the model without latent costs is significantly worse. From the estimation result, we can see that the magnitudes of the latent costs are large. Therefore, without such latent costs in the model, players are much more willing to purchase, which dramatically drives up players' willingness to list and set higher prices. As a result, this greatly drives up the listing fee and commission collected by the game. This highlights the importance of incorporating the latent costs, which has already been shown to be important in the industry by the literature.

2.7 Counterfactual

We first examine the differential impact of listing fee vs. commission. We emphasize that our goal is not to find the "optimum" for the firm, since its objective function encompasses both volume and profit concerns; rather, we focus on understanding how the two types of fee work differently. We also reconcile the difference between our model and the extant research by investigating under what condition sellers and buyers would form distinct groups. We outline the steps of counterfactual equilibrium computation in Appendix C.

2.7.1 Listing Fee vs. Commission

2.7.1.1 Theoretical Discussion

The critical difference between listing fee and commission is that listing fee is charged regardless of the sales outcome, while commission is charged only for completed transactions. Conceptually, this is a two-part tariff, with listing fee being the fixed part and commission being the variable part. Thus, we draw insights from this literature, in particular Schmalensee (1981).

We focus on their differential impacts on the fee collection. One would naturally expect the listing volume to be lower in a listing-fee structure compared to the same level of commission. However, fee collection also depends on prices. Therefore, we discuss the possible impacts of fee structure on prices. When a commission is imposed (vs. no fees), we expect that players will increase prices simply because sellers will pass on (at least part of) an increased variable cost. For listing fee (vs. no fees), it is less clear. On one hand, it may increase prices due to less competition. On the other hand, players may want to decrease prices in order to increase the sale probability to recover the listing fee paid upfront. Therefore, their relative impacts on prices, hence the fee collection, is ambiguous.

2.7.1.2 Results

To isolate the effects of listing fee and commission, we conduct two series of counterfactual simulations. In each series, we assume that the game charges either a listing fee or a commission, but not both, with two levels: 5% and 10%. The levels are chosen to best detect their differences (if any). We also simulate a baseline case: no fees.

Table 2.6 shows their comparison[37]. At the same level, consistent with our expectation, listing fee leads to a lower listing volume (at least 13% lower) compared to commission. Interestingly, given this, listing fee generates a very similar transaction volume (at most 0.9% smaller). More importantly, listing fee generates a higher fee collection by at least 13.5%. This highlights a trade-off between them: listing fee is better in generating fee collection but worse in listing volume.

From our earlier discussion, it's not surprising that listing fee leads to a lower listing volume. However, how do we explain the comparison for transaction volume and fee collection? To answer this question, we explore how players make their listing and pricing decisions in listing fee/commission compared to the baseline case. We focus on one heterogeneity dimension—the latent cost of listing—for several reasons. First, it has strong correlation with the listing and pricing decisions, as will be shown later. Second, it is an important feature in the industry. Third, there is a clear segmentation in this dimension, as shown by the estimation result. Last, insights about this parameter are less likely to be product-specific, and so the implications may apply to the aggregate market level. As noted earlier, such latent cost consists of two opposing effects: the transaction cost and the perceived fun of listing. To facilitate illustration, we refer to players who have a negative latent cost of listing (perceived fun >

[37] Once the counterfactual equilibrium is solved, we simulate the market for a year to calculate the summaries.

transaction cost) as "fun-players", and players who have positive latent cost of listing (transaction cost > perceived fun) as "cost-players".

Figures 2.2 and 2.3 plot the number of listings and the mean listing price, respectively, against the latent cost of listing[38]. The results show very interesting patterns. The listing fee (vs. no fees) works as a "truncation" tool. It disproportionately discourages fun-players from listing. However, the cost-players basically maintain their listing frequencies. Therefore, listing fee "truncates" a specific segment of players (fun-players) from listing. Furthermore, the fun-players not only list less frequently, but once deciding to list, they become more practical by setting lower prices to increase sale probabilities (Figure 2.4). The cost-players, however, not only maintain their listing frequencies, they also increase their prices without sacrificing their sale probabilities (Figure 2.4) due to less competition. Consequently, the price dispersion becomes smaller.

By contrast, for commissions (vs. no fees) we get quite a different pattern. It works as a "random sampling" tool: no specific segment of players is disproportionately discouraged from listing. In addition, consistent with earlier discussion, all players increase their listing prices. Consequently, commission linearly shifts up all prices and so preserves the price dispersion.

How do these results explain the patterns in fee collection and transaction volume in Table 2.6? The key point is that the price dispersion is smaller in listing fee vs. commission.

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[38] For each player, we calculate those measures based on the simulation. In each figure, we fit a local regression to smooth the curves. We chose to investigate the case of 5%. For 10%, the effect is more pronounced.

Therefore, in listing fee, there are fewer high-priced listings. Though there are also less low-priced listings, overall prices are more "acceptable". As a result, listing fee drives up the sale probability (Figure 2.4). Thus, even though listing fee leads to a lower listing volume, its higher sale probability leads to a very similar transaction volume compared to the same level of commission. In terms of fee collection, under listing fee, it is

"listing volume × average listing price";

under commission, it is

"transaction volume × average transaction price".

The results show that the listing volume in listing fee is higher than the transaction volume in commission (average 11% higher). Also, the average transaction price under commission is lower than the average listing price under listing fee (average 3.8% lower). Therefore, commission loses to listing fee in both quantity and price. Thus, the greater price dispersion in commission can also explain its loss in fee collection: the limited number of low-priced listings disproportionately contributes to the fee collection.

In summary, there is a fundamental trade-off between the two types of fees. Listing fee generates higher fee collection and so it is better for short-term revenue; commission generates higher listing volume and thus it is better for long-term revenue. Such differential impacts stem from their different natures. Listing fee (vs. no fees) works as a truncation tool: fun-players are discouraged from listing, and those who do list decrease prices; while cost-players maintain their listing frequency and increase prices. Commission (vs. no fees) works as a random sampling tool: some players (but not a specific segment) list more frequently while the opposite holds for the others, and all players increase prices. As a result, prices are more "acceptable" in listing fee vs. commission. Consequently, listing fee leads to a lower listing volume, but a very similar transaction volume and a higher fee collection.

2.7.2 Mixed-Trader Market versus Buyer-Seller Market

Existing research separates sellers and buyers into distinct groups. In our context, an explanation for such separation is that some players are price-sensitive (hence sell more than buy), while the opposite holds for the others (hence buy more than sell). To explore this, we perform a counterfactual analysis as follows. We set all structural parameters to their estimated mean levels for all players except the price coefficient, which is assumed to follow a bi-modal distribution (with the mean equal to its estimated mean from the data). In other words, players are identical in all dimensions except for their price sensitivities, based on which there are two segments. Such a setting allows us to isolate the effect of heterogeneity in price sensitivities. We solve the equilibrium and then simulate the market for a year, based on the status-quo fee structure. We examine the individual-level purchase and listing probabilities to see if there is some degree of market separation. We demonstrate the results in Figures 2.5 and 2.6.

First, Figure 2.5 shows that there exists a group of players who seldom purchase, e.g., the spikes near the origin. The spread of the low spikes distant from the origin captures the other group, whose members have greater probability to purchase at any level of endowment. Note that we cannot show a similar graph for the listing behavior because when the endowment is zero, listing is not an option. Second, when both purchasing and listing are available (i.e., endowment is positive), Figure 2.6 shows a clear separation in terms of "sellers" (those with low probability of purchase but high probability of listing) and "buyers" (those with high probability of purchase but low probability of listing). It is worth noting that even for the "sellers", they still purchase but just less frequently than listing, and vice versa for the "buyers". This highlights in any given market, one may want to allow for such a seller-buyer possibility.

These findings suggest that our framework could be a general case to incorporate the traditional buyer-seller model, without the need for assigning individuals to one or another. At the same time, it allows individuals to choose between buying and selling based on their own endowments and market conditions.

2.8 Discussion and Managerial Implications

In this chapter, we examine the differential impact of listing fee vs. commission on the listing and transaction volumes (indicators of long-run profitability), as well as fee collection (short-run revenue) in the P2P market in the online video games industry. We build a structural model to characterize players' trading behavior. We prove the equilibrium existence and propose a computationally light two-step estimation procedure. Using data from a popular game, we estimate the model parameters and conduct counterfactual simulations to answer the research question. We find that there is a fundamental trade-off between listing fee and commission. Conditional at the same level, listing fee (vs. commission) generates higher fee collection but lower listing volume.

Our analysis has important managerial implications. First, it provides an explanation for the large variation of fee structures in practice: game developers may have different weights for their long- and short-term revenues, and these preferences may vary over time. Second, it provides a conceptual roadmap for firms to experiment with new fee structures and alternative fee levels. As an illustration example, we demonstrate how the insights gained from our analysis can help the company to improve its fee collection while not hurting the listing and transaction volumes too much. The simplest and (arguably) most practical options are: increasing either listing fee or commission while fixing the other based on the current policy. Though our analysis does not provide an immediate answer as to which one the company should try first[39], it does suggest that small steps should be used when increasing listing fee, and larger steps can be considered when increasing commission. To examine whether such an implication is reasonable, we run counterfactual simulations to verify this. We find: (1) increasing listing fee with the smallest step size (from 1% to 2%)[40] has a non-trivial negative impact on the listing volume (down by 3%), despite a large positive impact on fee collection (up by 28%); (2) increasing commission with a large step size (from 3% to 6%) raises fee collection substantially (up by 72%) while basically maintaining the listing and transaction volumes (only down by 0.6% and 0.3%, respectively)[41].

The exercise above suggests that our policy implication—small step size should be used in experimentation of listing fee while greater step size can be considered for commission—is

[39] This requires knowing the weights for the company's long- and short-term goals.
[40] To our best knowledge, no game charges a fee with decimal percentages.
[41] Charging a high commission in the P2P market is not uncommon in the industry. As shown in Table 1.1, two popular games, *Guild Wars II* and *Elder Scrolls Online*, charge "5% listing fee + 10% commission" and "10% listing fee + 15% commission", respectively.

reasonable, but the specific result in the illustration example is not conclusive. For example, we examine only one product but the P2P market has a common listing and commission fee structure for all products. In practice, firms may have more information, such as the correlation between listing/transaction volume and long-term revenue. Moreover, firms may know who those fun-players and cost-players are. Combining all this knowledge, we believe the insights gained from our analysis can help a firm to better design the fee structure of its P2P market.

2.9 Technical Details

Since there is a smallest unit of virtual currency in the game, price is discrete. To simplify our analysis, we assume that price is bounded by a sufficiently large number. Since the number of players is finite, this implies that the set of all possible systems, denoted as $\mathcal{L}$, is finite. The action set is also finite according to our earlier assumption.

Definition. A system $L = \left\{ B = \{p_k, e_k, w_k\}_{k=1}^{J}, D = \{n_i\}_{i=1}^{N}, H \right\} \in \mathcal{L}$ is said to be consistent with $\tilde{S} = \{M, n\} = \left\{ \left\{ \left\{ (\tilde{p}, m)_j \right\}_{j=1}^{\tilde{J}}, \tilde{H} \right\}, n \right\}$ if the following are satisfied:

(a) $\left\langle \{p_k\}_{k=1}^{J} \right\rangle = \{\tilde{p}_j\}_{j=1}^{\tilde{J}}$, where $\langle \cdot \rangle$ denotes the set of distinct elements;

(b) $\sum_{k=1}^{J} \left[\mathbb{I}\left(p_k = \tilde{p}_j \right) \right] = m_j$, for $\forall j \in \left\{1, 2, ..., \tilde{J}\right\}$, where $\mathbb{I}(\cdot)$ is the indicator function;

(c) $\exists i$ such that $n_i = n$;

(d) $H = \tilde{H}$.

Condition (a) requires the system to have the same set of distinct prices as that in $\tilde{S}$. Condition (b) requires that the number of listings under each distinct price of the system is the same as that in $\tilde{S}$. Condition (c) requires at least one individual in the system with the same endowment as that in $\tilde{S}$. Condition (d) requires times of the day to be identical. Denote $C(\tilde{S}) \subset \mathcal{L}$ as the set of systems that are consistent with $\tilde{S}$. It is important that this set does not depend on a strategy profile. Similarly, not every L_1, i.e., the system at the beginning of next period, is consistent with L_0. Specifically, we define the one-period-ahead system L' to be consistent with L as follows.

Definition 2. A one-period-ahead system $L' = \{B', D', H'\} \in \mathcal{L}$ is consistent with $L = \{B, D, H\} \in \mathcal{L}$ if

$$H' = \mathbb{I}(H' = H + 1) \cdot \mathbb{I}(H < T_d) + \mathbb{I}(H' = 1) \cdot \mathbb{I}(H = T_d),$$

where T_d is the total number of periods in a day[42], and only one of the following conditions is satisfied:

(a) no trade: $B' = B - 1$, meaning all listings deduct one period in their expiration times, and expired listings are eliminated;

(b) purchase: $B' = \{B - 1\} \setminus \{\tilde{p}_1\}$, meaning all listings in book B with one period less in expiration time and one listing under the minimum price $\tilde{p}_1$ is purchased;

[42] Here we treat *H* as a second of a day.

(c) sell: $B' = \{B-1\} \cup \{p', j\}$, meaning all listings in book B with one period less in expiration time and one new listing is added; say it is listed by player j with price p'.

Denote $C(L) \subset \mathcal{L}$ as the set of systems that are consistent with L. Denote $C_a(L)$ as the set of systems that are consistent with L and condition (a) is met. Denote $C_b(L)$ and $C_c(L)$ analogously. Also for type (c), denote $C_{cip}(L) \subset \mathcal{L}$ as the set of one-period-ahead systems, which are consistent with a new listing by i with price p that was added during the current period. These sets do not depend on a strategy profile.

2.9.1 Derivation of Sale Probability

2.9.1.1 Sale Probability at the Next Period

Denote $q_1(\tilde{S}, p; \varrho)$ as the probability that the listed item will be sold in the next period[43]. We first derive the distribution of L_0, i.e., the system at the beginning of the current period, conditional on $\tilde{S}$. Then we derive the distribution of L_1, i.e., the system at the beginning of the next period, conditional on L_0. Based on that, we compute the probability of the current added listing being sold in the next period.

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[43] There is a bit of abuse of notation here. The subscript "1" does not mean it is the sale probability for individual "1".

We can express q_1 in the following way:

$$q_1\left(\tilde{S},\ p; \varrho\right) = \sum_{L_0 \in C\left(\tilde{S}\right)} \sum_{i=1}^{N} \sum_{L_1 \in C_{cip}\left(L_0\right)} \Pr\left(L_0, i \middle| \tilde{S}; \varrho\right) \cdot \Pr\left(L_1 \middle| L_0, \left(i,\ p\right); \varrho\right) \cdot \Pr\left(p\text{-sold}_1 \middle| L_1; \varrho\right), \qquad (2.4)$$

where " p-sold$_1$ " denotes that the current newly added listing with price p is sold during the next period. The idea is as follows. Conditional on the market information $M \subset \tilde{S}$ and a player entering the market with endowment $n \in \tilde{S}$, we first compute the probability of a particular system at the beginning of the current period (i.e., L_0), and that of individual i entering the market during the current period, i.e., $\Pr\left(L_0, i \middle| \tilde{S}; \varrho\right)$. Then we compute the probability that the system at the beginning of the next period is L_1, conditional on L_0 and player i listing an item for sale with price p , i.e., $\Pr\left(L_1 \middle| L_0, \left(i,\ p\right); \varrho\right)$. Finally, we compute the probability that the focal listing is sold during the next period conditional on L_1, i.e., $\Pr\left(p\text{-sold}_1 \middle| L_1; \varrho\right)$. All probabilities mentioned above are conditional on the strategy profile ϱ. In the end, we sum over all possible paths. It is clear that not every system at the current period, i.e., L_0, is consistent with $\tilde{S}$. For example, if there are only two distinct prices in the current market, a system with three distinct prices would have zero probability conditional on the market information. Therefore, we only need to sum over the systems that are consistent with $\tilde{S}$, i.e., $C\left(\tilde{S}\right)$. Similarly, we only sum over $C_{cip}\left(\tilde{S}\right)$, i.e., the set of L_1 that are consistent with L_0 and player i listing an item for sale with price p . Detailed definitions of these sets can be found in the appendix. It is important to note that these sets do not depend on the strategy profile. We discuss each component in equation (2.4).

We start from the last component:

$$\Pr\left(p\text{-sold}_1 \middle| L_1; \varrho\right) = \sum_{j=1}^{N} \left[\mathcal{P}_{\varpi}\left(j \middle| L_1\right) \cdot \frac{1}{m_1} \cdot \int_{(\epsilon,\, \xi)} \mathbb{I}\left(\varrho_j\left(\tilde{S}_{j1}, \epsilon, \xi\right) = a_b\right) dF_{\epsilon\xi} \right] \cdot \mathbb{I}\left(p = \tilde{p}_1\right), \qquad (2.5)$$

where $\tilde{p}_1$ is the minimum price and m_1 is the number of listings under the minimum price, both in L_1. $\tilde{S}_{j1} \subset L_1$ contains j's endowments and the market information under L_1. Inside the bracket of equation (2.5) is the summation of each player's probability of purchasing the newly added listing when p is the minimum price in L_1. And the probability could be broken down into three parts: a) the probability of j entering the market $\mathcal{P}_{\varpi}\left(j \middle| L_1\right)$; b) the probability of buying exactly the focal listing, i.e., $\left(m_1\right)^{-1}$ due to randomization among listings with the same price; and c) the probability of purchase—the integration term. Otherwise, when p is not the minimum price in L_1, the probability of being sold is zero.

The second component in equation (2.4) is as follows:

$$\Pr\left(L_1 \middle| L_0, (i,\, p); \varrho\right) = \mathbb{I}\left(n_{i1} = n_{i0}\right) \cdot \mathcal{P}_D\left(D_{-i,1} \middle| L_0\right), \qquad (2.6)$$

where $n_{i0} \subset L_0$ and $n_{i1} \subset L_1$. Since we only limit $L_1 \in C_{cip}\left(L_0\right)$, the transition of the book is already implicitly taken into account. Equation (2.6) implies that the transition of i's endowment is deterministic given that her action and the transition of other players' endowments are captured by the second term.

Finally, the first component inside the summation of equation (2.4) is less obviously defined. It reflects the difficulties mentioned earlier, i.e., the conditional distribution of expiration times and the fact that all players' endowments are endogenous. Since these two factors are in L_0, specification of this piece essentially resolves those difficulties. We have

$$\Pr\left(L_0, i \middle| \tilde{S}; \varrho\right) = \Pr\left(L_0, i, \tilde{S}; \varrho\right) \cdot \left[\Pr\left(\tilde{S}; \varrho\right)\right]^{-1} = \Pr\left(L_0, i, n; \varrho\right) \cdot \left[\Pr\left(\tilde{S}; \varrho\right)\right]^{-1}. \tag{2.7}$$

The first equality of equation (2.7) holds by definition. The second equality holds because the market information $M \subset \tilde{S}$ is in L_0 since we require L_0 to be consistent with $\tilde{S}$. The numerator is the probability that the current system is L_0 and that i with endowment n enters the market given ϱ. The denominator is the probability that the current market information is $M \subset \tilde{S}$, and a player with endowment $n \in \tilde{S}$ enters the market given ϱ. To specify these objects, we utilize the stationary distribution of the system conditional on a strategy profile. We assume that the unique stationary distribution of the system is given by $\mathcal{F}_L\left(\cdot \middle| \varrho\right)$. We will show its existence and uniqueness later.

Given $\mathcal{F}_L\left(\cdot \middle| \varrho\right)$, the numerator of (2.7) is given by

$$\Pr\left(L_0, i, n; \varrho\right) = \mathcal{F}_L\left(L_0 \middle| \varrho\right) \cdot \mathcal{P}_{\varpi}\left(i \middle| L_0\right) \cdot \mathbb{I}\left(n_{i0} = n\right), \tag{2.8}$$

where $n_{i0} \subset L_0$. The first part is the probability of L_0 occurring, the second part is the probability that i enters the market conditional on L_0, and the last part requires i's endowment in L_0 to be n. The denominator of (2.7) is given by

$$\Pr\left(\tilde{S};\varrho\right)=\sum_{L\in C\left(\tilde{S}\right)}\left\{\mathcal{F}_L\left(L|\varrho\right)\cdot\sum_{j=1}^{N}\left[\mathcal{P}_{\varpi}\left(j|L\right)\cdot\mathbb{I}\left(n_j=n\right)\right]\right\}, \tag{2.9}$$

where $n_j \subset L$. The first part inside the bracket captures the probability that a system consistent with $\tilde{S}$ occurs. The second part captures the probability that a player with endowment $n \in \tilde{S}$ enters the market. The difference between (2.8) and (2.9) is that the latter needs to integrate across all possible systems that are consistent with $\tilde{S}$ and all players.

In sum, equations (2.4) to (2.9) characterize the probability of sale at the next period given a strategy profile.

2.9.1.2 Sale Probability Two Periods Later

Denote $q_2\left(\tilde{S}, p;\varrho\right)$ as the probability that a newly added listing with price p by a player who enters the market with endowment $n \in \tilde{S}$ will be sold two periods from now, conditional on the market information $M \subset \tilde{S}$ and the strategy profile ϱ. Following the same idea as before, we break down q_2 as

$$q_2\left(\tilde{S},\, p; \varrho\right) = \sum_{L_0 \in C(\tilde{S})} \sum_{i=1}^{N} \sum_{L_1 \in C_{cip}(L_0)} \sum_{L_2 \in C(L_1)} \Big[\Pr\left(L_0, i \middle| \tilde{S}; \varrho\right) \cdot \Pr\left(L_1 \middle| L_0, (i,\, p); \varrho\right) \cdot \Pr\left(L_2,\, p\text{-unsold}_1 \middle| L_1; \varrho\right) \cdot \Pr\left(p\text{-sold}_2 \middle| L_2; \varrho\right) \Big], \tag{2.10}$$

where $p\text{-sold}_2$ denotes the event that the current newly added listing with price p is sold two periods from now, and $p\text{-unsold}_1$ denotes the opposite event. Therefore, $\left(L_2,\, p\text{-unsold}_1\right)$ denotes a system at the beginning of the period two periods from now with the current newly added listing at price p unsold. The idea is very similar to the decomposition of q_1 with the exception that we require the focal listing not to be sold in the next period. Also, we require L_2 to be consistent with L_1. The first and second parts inside the summation have been specified through equations (2.6) to (2.9). The last part is specified in the same way as (2.5) with a change of subscript from "1" to "2". Therefore, we need only to specify the third part. There are three possible sets of L_2 in this part: (i) $L_2 \in C_a\left(L_1\right)$ which means L_2 is consistent with L_1 and no trade occurs during the next period; (ii) purchase occurs during the next period, i.e., $L_2 \in C_b\left(L_1\right)$; and (iii) a new listing is added during the next period, i.e., $L_2 \in C_a\left(L_1\right)$. We discuss them individually below.

When $L_2 \in C_a\left(L_1\right)$, i.e., no trade, we have

$$\Pr\left(L_2,\, p\text{-unsold}_1 \middle| L_1; \varrho\right) = \Pr\left(L_2 \middle| L_1; \varrho\right) = \left(1 - \sum_{i=1}^{N} \mathcal{P}_{\varpi}\left(j \middle| L_1\right)\right) \cdot \mathcal{P}_D\left(D_2 \middle| L_1\right) + \sum_{j=1}^{N} \left[\mathcal{P}_D\left(D_{-j,2} \middle| L_1\right) \cdot \mathcal{P}_{\varpi}\left(j \middle| L_1\right) \cdot \mathbb{I}\left(n_{j2} = n_{j1}\right) \cdot \int_{(\epsilon,\,\xi)} \mathbb{I}\left(\varrho_j\left(\tilde{S}_{j1}, \epsilon, \xi\right) = a_0\right) dF_{\epsilon\xi} \right], \tag{2.11}$$

where $n_{j1} \in \tilde{S}_{j1} \subset L_1$, $n_{j2} \in L_2$, and $D_2 \subset L_2$. The first equality holds because L_2 comes from a set that is consistent with L_1 and no trade occurs. Hence, the event p-unsold$_1$ is already incorporated. The first piece in the second equality captures the case where there is no player entry. The second piece captures the case where there is one player entry but no trade. Therefore, inside the bracket of the second piece, the first term captures the endowment transition of other players, the second term captures the probability that j enters, the third term captures the endowment transition for j, and the last term is the likelihood of no trade. Since we only consider L_2 that is consistent with L_1, the transition of the book and the hour of day are implicitly considered.

When $L_2 \in C_b(L_1)$, i.e., purchase occurs, we have

$$\begin{aligned}
&\Pr\left(L_2, p\text{-unsold}_1 \middle| L_1; \varrho\right) \\
&= \mathbb{I}(p \in L_2) \cdot \sum_{j=1}^{N} \Bigg[\mathcal{P}_D\left(D_{-j,2} \middle| L_1\right) \cdot \mathcal{P}_{\varpi}\left(j \middle| L_1\right) \cdot \mathbb{I}\left(n_{j2} = n_{j1} + 1\right) \\
&\cdot \frac{1}{m_1} \cdot \int_{(\epsilon,\xi)} \mathbb{I}\left(\varrho_j\left(\tilde{S}_{j1}, \epsilon, \xi\right) = a_b\right) dF_{\epsilon\xi} \Bigg],
\end{aligned} \tag{2.12}$$

where $n_{j1} \in \tilde{S}_{j1} \subset L_1$, $n_{j2} \in L_2$, $D_2 \subset L_2$, and the event $p_2 \in L_2$ denotes that the newly added listing with price p during the current period remains unsold at the beginning of two periods later. The terms in the summation represent the case in which one player enters the market and purchases a listing. The indicator function $\mathbb{I}(p \in L_2)$ makes sure that the focal listing is not sold.

When $L_2 \in C_{cjp'} \subset C_c(L_1)$, i.e., a new listing by j with p' is added in the next period, we have

$$\begin{aligned}&\Pr\left(L_2,\ p\text{-unsold}_1 \middle| L_1;\varrho\right)\\&=\Pr\left(L_2 \middle| L_1;\varrho\right)=\mathcal{P}_D\left(D_{-j,2}\middle|L_1\right)\cdot\mathcal{P}_{\varpi}\left(j\middle|L_1\right)\cdot\mathbb{I}\left(n_{j2}=n_{j1}\right)\cdot\int_{(\epsilon,\,\xi)}\mathbb{I}\left(\varrho_j\left(\tilde{S}_{j1},\epsilon,\xi\right)=p'\right)dF_{\epsilon\xi}.\end{aligned} \tag{2.13}$$

The first equality holds because no purchase occurs during the next period. The second equality describes the case in which j lists an item at price p', and it includes the endowment transition. In summary, the third part in equation (2.10) is given by equations (2.11) to (2.13). This completes the specification of q_2.

To summarize, we have specified the sale probability within the next two periods following the present one, i.e., $q_1 + q_2$, given a strategy profile. It shows that a strategy profile affects the sale probability in two ways: the choice probability and the stationary distribution of the system. Next we provide a general formula for the sale probability over 12 hours, which is a direct extension of the two-period case.

2.9.1.3 Sale Probability: General Case

Based on the earlier discussion, we present the general expression of the sale-probability function $q(\cdot)$ in this section. Specifically, we have

$$q\left(\tilde{S},\, p;\varrho\right)=\sum_{r=1}^{R} q_r\left(\tilde{S},\, p;\varrho\right), \tag{2.14}$$

where q_r is the probability that a newly added listing at price p by a player who enters the market with endowment $n \in \tilde{S}$ will be sold exactly r periods apart from now, conditional on the market information $M \subset \tilde{S}$ and the strategy profile ϱ. R is the total number of periods in 12 hours; q_1 and q_2 have already been specified in previous sections. The specification of q_r for $r > 2$ is not much different from the case of $r = 2$ conceptually. Therefore, we will omit the detailed explanation for cases with $r > 2$. Similar to (2.10), we have

$$\begin{aligned} & q_r\left(\tilde{S},\, p;\varrho\right) \\ & = \sum_{L_0 \in C(\tilde{S})} \sum_{j=1}^{N} \sum_{L_1 \in C_{cip}(L_0)} \sum_{L_2 \in C(L_1)} \cdots \sum_{L_{r-1} \in C(L_{r-2})} \sum_{L_r \in C(L_{r-1})} \Big\{ \Pr\left(L_0,\, j \middle| \tilde{S};\varrho\right) \times \Pr\left(L_1 \middle| L_0, (j,\, p);\varrho\right) \\ & \times \prod_{k=1}^{r-1} \left[\Pr\left(L_{k+1},\, p\text{-unsold}_k \middle| L_k;\varrho\right)\right] \times \Pr\left(p\text{-sold}_r \middle| L_r;\varrho\right) \Big\}. \end{aligned} \tag{2.15}$$

The first and second terms inside the bracket are specified through equations (2.6) to (2.9). The last term is specified in equation (2.5) with the corresponding change of subscript from "1" to "r". All other terms are specified through equations (2.11) to (2.13) with the corresponding change of subscripts from "1" to "k" and "2" to "$k+1$" for $k = 1, 2, 3, ..., r-1$. Therefore, without rewriting redundant equations, we claim that equations (2.5) to (2.9) and (2.11) to (2.15) characterize the sale probability $q(\cdot;\varrho)$ given a strategy profile ϱ.

2.9.2 Proof of Proposition 1

In this section, we show that a strategy profile can induce a unique stationary distribution of the system. Intuitively, a strategy profile induces a Markov chain of the system, i.e., $\{L_t\}$. Due to our model structure, transition of the book depends only on the current system and a strategy profile, the transition of hour of day is deterministic, and the transition of endowments depends only on current endowments[44]. Therefore, the distribution of L_{t+1} depends only on L_t, i.e., $\{L_t\}$ is a Markov chain given a strategy profile.

As discussed earlier, the set of all possible systems is finite. Therefore, the state space of $\{L_t\}$ is finite. Denote its transition probability matrix as $\mathcal{P}_L$. The discussion in Section 2.9.1 essentially specifies $\mathcal{P}_L$ for a given strategy profile. Given L_t, the probability of $L_{t+1} \notin C(L_t)$ is zero. For $L_{t+1} \in C_a(L_t)$ (or $L_{t+1} \in C_c(L_t)$), i.e., no trade (or listing occurs), the transition is given by (2.11) (or (2.13)) with the corresponding change of subscripts. The only difference is that (2.11) (or (2.13)) implicitly requires that the newly added listing with price p a period ago is in L_1 while it is an arbitrary $L_t \in \mathcal{L}$ here. For $L_{t+1} \in C_b(L_t)$, i.e., purchase occurs, the transition is given by (2.12) with the corresponding change of subscripts and the indicator term outside the summation, i.e., $\mathbb{I}(p \in L_2)$, being dropped because we deal with an arbitrary L_t here. Therefore,

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[44] This is not restrictive. If the transition of endowments also depends on past periods' endowments, we can define a set of current period variables that captures past periods' endowments. For example, if the distribution of n_{t+1} depends on both n_t and n_{t-1}, we can define $x_t = n_{t-1}$ to restore the Markov property.

without rewriting redundant equations, we claim that equations (2.11) to (2.13) characterize the transition probability matrix $\mathcal{P}_L$ given ϱ. Proposition 1—proved below—establishes existence of a unique stationary distribution for the system given any strategy profile. We first present some assumptions and lemmas that will be used later.

Assumption 1. For $\forall L \in \mathcal{L}$, $1-\sum_{i=1}^{N}\mathcal{P}_{\varpi}\left(i \middle| L\right) > 0$.

Assumption 2. For $\forall L \in \mathcal{L}, D^{'} \in \mathcal{D}$, there exists $k < \infty$ such that $\mathcal{P}_D^k\left(D^{'} \middle| L\right) > 0$, where $\mathcal{P}_D^k$ is the k-period ahead endowment transition probability.

Assumption 3. For $\forall L \in \mathcal{L}, D \subset L$, $\mathcal{P}_D\left(D \middle| L\right) > 0$.

Lemma 1. Under assumptions 1, 2, and 3, for any $\mathcal{P}_L$, $\mathcal{L}_{\varnothing} = \left\{L \in \mathcal{L} : B = \varnothing\right\}$ is irreducible.

Proof: For any $L, L^{'} \in \mathcal{L}_{\varnothing}$, by assumption 2, there exists k such that $\mathcal{P}_D^k\left(D^{'} \middle| L\right) > 0$. It indicates that all players can obtain endowments in $L^{'}$ from L within finite periods. By assumption 1, the probability of no entry throughout the periods during which players obtain their endowments in $L^{'}$ from L is strictly positive. It indicates that starting from L, it is strictly possible that the book remains empty after all players obtain their endowments in $L^{'}$. Once all individuals obtain their endowments in $L^{'}$, we let them stay with that and no entry to the market until the second of a day reaches H'. It occurs with positive probability by assumptions 1 and 3. Thus, the system can reach L' from L in finite periods. Since L and L' are arbitrary, $\mathcal{L}_{\varnothing}$ is irreducible. ■

Lemma 2. Under assumptions 1, 2, and 3, given any $\mathcal{P}_L$, for any $L \in \mathcal{L} \setminus \mathcal{L}_{\varnothing}$ and any $L_0 \in \mathcal{L}_{\varnothing}$, there exists $k < \infty$ such that $\mathcal{P}_L^k \left(L_0 \middle| L \right) > 0$, where $\mathcal{P}_L^k$ is the k-period ahead transition probability of the system.

Proof: Under assumption 1, it is strictly possible that starting from L, the book becomes empty (i.e., the system reaches $\mathcal{L}_{\varnothing}$) within finite periods. This is true because with long enough periods of no entry, the book becomes empty, i.e., all listings will expire. Since $\mathcal{L}_{\varnothing}$ is irreducible by Lemma 1, therefore it can reach any state in $\mathcal{L}_{\varnothing}$ from L. ■

Proposition 1. Under assumptions 1 to 3, for any $\mathcal{P}_L$, $\mathcal{L}$ can be uniquely partitioned as $\mathcal{T}_{\mathcal{P}_L} \cup \mathcal{C}_{\mathcal{P}_L}$, where $\mathcal{T}_{\mathcal{P}_L}$ is the set of transient states and $\mathcal{C}_{\mathcal{P}_L}$ is an irreducible closed set of non-null persistent states. The unique stationary distribution $\mathcal{F}_L(\cdot; \mathcal{P}_L)$ exists and is given by

$$\mathcal{F}_L(l; \mathcal{P}_L) = \frac{1}{\mu_{\mathcal{P}_L}(l)} \cdot \mathbb{I}\left(l \in \mathcal{C}_{\mathcal{P}_L}\right),$$

where $\mu_{\mathcal{P}_L}(l)$ is the mean recurrence time of state l under $\mathcal{P}_L$. Moreover, $\mathcal{F}_L(\cdot; \mathcal{P}_L)$ is continuous on $\mathcal{P}_L$[45].

[45] Proposition 1 stated here is slightly different from that stated in Section 2.5.3. However, as will be shown later, the transition probability is continuous with respect to the choice probability. Therefore, these two statements are, in fact, equivalent.

Proof: Define

$$\mathcal{L}_1 = \left\{ L \in \mathcal{L} \setminus \mathcal{L}_\varnothing : \exists L_0 \in \mathcal{L}_\varnothing, \exists k < \infty, \text{s.t. } \mathcal{P}_L^k \left(L \middle| L_0 \right) > 0 \right\}.$$

Since $\mathcal{L}_\varnothing$ is irreducible by Lemma 1, $\mathcal{L}_1$ contains all systems that do not have empty book and can be reached from any system with empty book. Define $\mathcal{T} = \mathcal{L} \setminus \left(\mathcal{L}_\varnothing \cup \mathcal{L}_1 \right)$. Thus, $\mathcal{L}$ is uniquely partitioned as $\mathcal{L} = \mathcal{T} \cup \mathcal{L}_\varnothing \cup \mathcal{L}_1$. By Lemma 2, any system in $\mathcal{T}$ can reach any system in $\mathcal{L}_\varnothing$. However, by definition, no system in $\mathcal{L}_\varnothing$ can reach a system in $\mathcal{T}$. Thus, $\mathcal{T}$ contains only transient states. By Lemma 2, any state in $\mathcal{L}_1$ can reach any state in $\mathcal{L}_\varnothing$. On the other hand, by definition, any state in $\mathcal{L}_\varnothing$ can reach any state in $\mathcal{L}_1$. Together with Lemma 1, $\mathcal{C} = \mathcal{L}_\varnothing \cup \mathcal{L}_1$ is irreducible. Since $\mathcal{L}$ is finite, $\mathcal{C}$ are non-null persistent[46]. We next show that $\mathcal{C}$ is closed. Suppose not, then at least one state in $\mathcal{L}_\varnothing$ can reach $\mathcal{T}$. This contradicts the definition of $\mathcal{T}$. Thus, $\mathcal{C}$ is closed. Based on this partition, the unique stationary distribution, both existence and expression, follows from Grimmett and Stirzaker (2001) Theorem 3 on page 227. Also, since there is a single irreducible set of systems, by Theorem 2 in Schweitzer (1968), the stationary distribution is continuous on $\mathcal{P}_L$. This completes the proof. ■

2.9.3 Proof of Proposition 2

The aim of proposition 2 is to show the existence of an equilibrium. We will work on the space of sale probability. First, we construct two mappings: (1) $\Gamma_{\varrho q}$, which maps from the space

[46] Grimmett and Stirzaker (2001), Lemma 5 on page 225.

of strategy profiles to the space of sale probabilities; (2) $\Gamma_{q\varrho}$, which maps from the space of sale probabilities to the space of strategy profiles given that individuals act optimally. Second, we show the existence of a fixed point to the following compound mapping $\Gamma_{qq} = \Gamma_{\varrho q}\left(\Gamma_{q\varrho}\right)$. This implies the existence of a fixed point to the mapping from the space of strategy profiles to the space of strategy profiles, i.e., $\Gamma_{\varrho\varrho} = \Gamma_{q\varrho}\left(\Gamma_{\varrho q}\right)$. This establishes the existence of equilibrium.

The discussion earlier essentially specified the functional form of how a strategy profile induces a sale probability. Equations (2.5) to (2.9) and (2.11) to (2.15) have shown that a strategy profile induces a sale probability in two ways: (1) the choice probability and (2) the stationary distribution of the system. Therefore, we will specify the mapping both from the space of strategy profiles to the space of choice probabilities and to the space of stationary distributions.

First, the mapping from the space of strategy profiles (denoted as Δ) to the space of choice probabilities, i.e., $\Gamma_{\varrho\sigma} : \Lambda \to \left(\Delta^{|\mathcal{A}|}\right)^{|\tilde{\mathcal{S}}|\cdot N}$ where $\Delta^{|\mathcal{A}|}$ is the $|\mathcal{A}|$-dimension simplex, is defined as

$$\sigma_i\left(\tilde{S};\varrho\right) = \left\{ \underbrace{\int \mathbb{I}\left(\varrho_i\left(\tilde{S},\epsilon,\xi\right) = a_b\right) dF_{\epsilon\xi}}_{\sigma_{ib}(\tilde{S})}, \left\{ \underbrace{\int \mathbb{I}\left(\varrho_i\left(\tilde{S},\epsilon,\xi\right) = p\right) dF_{\epsilon\xi}}_{\sigma_{ip}(\tilde{S})} \right\}_p, \underbrace{\int \mathbb{I}\left(\varrho_i\left(\tilde{S},\epsilon,\xi\right) = a_0\right) dF_{\epsilon\xi}}_{\sigma_{i0}(\tilde{S})} \right\},$$

for all i and any $\tilde{S} \in \tilde{\mathcal{S}}$.

Second, the stationary distribution depends on the transition probability matrix of the system by Proposition 1, which in turn depends on the choice probabilities as shown by (2.11) to (2.13). Therefore, the mapping from the space of choice probabilities to the space of transition matrices, i.e., $\Gamma_{\sigma\mathcal{P}_L} : \left(\Delta^{|\mathcal{A}|}\right)^{|\tilde{\mathcal{S}}|\cdot N} \to \left(\Delta^{|\mathcal{L}|}\right)^{|\mathcal{L}|}$, is defined as follows. For any $L \in \mathcal{L}$,

$$\mathcal{P}_L\left(L'\middle|L;\sigma\right) = \begin{cases} \left(1 - \sum_{i=1}^{N} \mathcal{P}_{\varpi}\left(j\middle|L\right)\right) \cdot \mathcal{P}_D\left(D'\middle|D\right) + \sum_{i=1}^{N}\left[\mathcal{P}_D\left(D'_{-i}\middle|D_{-i}\right) \cdot \mathcal{P}_{\varpi}\left(i\middle|H\right) \cdot \mathbb{I}\left(D'_i = D_i\right) \cdot \sigma_{i0}\left(\tilde{S}_i\right)\right], & \text{if } L' \in C_a(L), \\ \sum_{i=1}^{N}\left[\mathcal{P}_D\left(D'_{-i}\middle|D_{-i}\right) \cdot \mathcal{P}_{\varpi}\left(j\middle|H\right) \cdot \mathbb{I}\left(n'_i = n_i + 1\right) \cdot \frac{1}{m_1} \cdot \sigma_{ib}\left(\tilde{S}_i\right)\right], & \text{if } L' \in C_b(L), \\ \mathcal{P}_D\left(D'_{-j}\middle|D_{-j}\right) \cdot \mathcal{P}_{\varpi}\left(j\middle|H\right) \cdot \mathbb{I}\left(n'_j = n_j\right) \cdot \sigma_{ip'}\left(\tilde{S}_j\right), & \text{if } L' \in C_c(L) \text{ and } j \text{ lists } p'. \end{cases} \tag{2.16}$$

Proposition 1 defines a mapping from the space of all transition probabilities to the space of stationary distributions, i.e., $\Gamma_{\mathcal{P}_L\mathcal{F}_L} : \left(\Delta^{|\mathcal{L}|}\right)^{|\mathcal{L}|} \to \Delta^{|\mathcal{L}|}$. In sum, the mapping from the space of strategy profiles to the space of stationary distributions is defined as

$$\Gamma_{\varrho\mathcal{F}_L} = \Gamma_{\mathcal{P}_L\mathcal{F}_L}\left(\Gamma_{\sigma\mathcal{P}_L}\left(\Gamma_{\varrho\sigma}\right)\right) : \Lambda \to \Delta^{|\mathcal{L}|}.$$

As mentioned earlier, equations (2.5) to (2.9) and (2.11) to (2.15) define the mapping from the product space of choice probabilities and stationary distributions to the space of sale probabilities, i.e., $\Gamma_{\sigma\mathcal{F}_L,q} = \left(\Delta^{|\mathcal{A}|}\right)^{|\tilde{\mathcal{S}}|\cdot N} \times \Delta^{|\mathcal{L}|} \to [0,1]^{|\tilde{\mathcal{S}}\times\mathcal{Z}_p|}$, where $\left|\mathcal{Z}_p\right|$ is the set of all possible prices, which according to earlier discussion is finite. In sum, the sale-probability mapping $\Gamma_{\varrho q}$ is defined as follows:

$$\Gamma_{\varrho q} = \Gamma_{\sigma\mathcal{F}_L,q}\left(\Gamma_{\varrho\sigma}, \Gamma_{\varrho\mathcal{F}_L}\right) = \Gamma_{\sigma\mathcal{F}_L,q}\left(\Gamma_{\varrho\sigma}, \Gamma_{\mathcal{P}_L\mathcal{F}_L}\left(\Gamma_{\sigma\mathcal{P}_L}\left(\Gamma_{\varrho\sigma}\right)\right)\right) : \Lambda \to [0,1]^{|\tilde{\mathcal{S}}\times\mathcal{Z}_p|}. \tag{2.17}$$

The optimal strategy mapping $\Gamma_{q\varrho} : [0,1]^{|\tilde{S}\times\mathcal{Z}_p|} \to \Lambda$ is defined as follows[47]:

$$\varrho_i\left(\tilde{S},\epsilon,\xi\right)=\begin{cases} a_b, & \text{when } u_{ib}\left(\tilde{S},\epsilon,\xi\right)>\max\left\{\max\limits_{p}\left\{u_{is}\left(\tilde{S},\epsilon,\xi,p;q\right)\right\},0\right\},\\ p, & \text{when } u_{is}\left(\tilde{S},\epsilon,\xi,p;q\right)>\max\left\{\max\limits_{p'\neq p}\left\{u_{is}\left(\tilde{S},\epsilon,\xi,p';q\right)\right\},u_{ib}\left(\tilde{S},\epsilon,\xi\right),0\right\},\\ a_0, & \text{when } 0>\max\left\{\max\limits_{p}\left\{u_{is}\left(\tilde{S},\epsilon,\xi,p;q\right)\right\},u_{ib}\left(\tilde{S},\epsilon,\xi\right)\right\},\end{cases}$$

for all i, $\tilde{S}$, ϵ, and ξ. Define Γ_{qq} as follows:

$$\Gamma_{qq}=\Gamma_{\varrho q}\left(\Gamma_{q\varrho}\right)=\Gamma_{\sigma\mathcal{F}_L,q}\left(\underbrace{\Gamma_{\varrho\sigma}\left(\Gamma_{q\varrho}\right)}_{\equiv\Gamma_{q\sigma}},\Gamma_{\mathcal{P}_L\mathcal{F}_L}\left(\Gamma_{\sigma\mathcal{P}_L}\left(\underbrace{\Gamma_{\varrho\sigma}\left(\Gamma_{q\varrho}\right)}_{\equiv\Gamma_{q\sigma}}\right)\right)\right):\ [0,1]^{|\tilde{S}\times\mathcal{Z}_p|}\to[0,1]^{|\tilde{S}\times\mathcal{Z}_p|}. \quad (2.18)$$

It shows that Γ_{qq} is a compound mapping of $\Gamma_{\sigma\mathcal{F}_L,q}$, $\Gamma_{q\sigma}$, $\Gamma_{\sigma\mathcal{P}_L}$, and $\Gamma_{\mathcal{P}_L\mathcal{F}_L}$. Thus, the existence of a fixed point depends on the continuity of $\Gamma_{q\sigma}$. That in turn depends on the functional forms of $V_i(\cdot)$ and $\beta_i(\xi)$, as well as $F_{\epsilon\xi}$. Instead of showing the continuity of $\Gamma_{q\sigma}$ under some general conditions, we assume it is the case in what follows. We verify it later based on the functional forms specified in Section 2.6.1.

Assumption 4. The mapping $\Gamma_{q\sigma}$ is continuous.

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[47] We only consider pure strategies. When ties don't occur, the defined strategy is pure. When ties occur, we can put some structure on the choice to purify it as such even probabilities. Given that the idiosyncratic shocks are continuous random variables, ties occur with zero probability. Therefore, we omit equality in the definition.

The following Lemma 3 shows the existence of a fixed point to Γ_{qq}.

Lemma 3. Under assumptions 1 to 4, there exists a fixed point to Γ_{qq}.

Proof: Since Γ_{qq} is a compound mapping of $\Gamma_{\sigma\mathcal{F}_L,q}$, $\Gamma_{q\sigma}$, $\Gamma_{\sigma\mathcal{P}_L}$, and $\Gamma_{\mathcal{P}_L\mathcal{F}_L}$, it suffices to show that all of them are continuous. First, equations (2.5) to (2.9) and (2.11) to (2.15) show that $\Gamma_{\sigma\mathcal{F}_L,q}$ is continuous in both its arguments because only basic arithmetic operators are involved. Second, according to assumption 4, $\Gamma_{q\sigma}$ is continuous. Third, (2.16) shows that $\Gamma_{\sigma\mathcal{P}_L}$ is a linear function and so continuous. By Proposition 1, $\Gamma_{\mathcal{P}_L\mathcal{F}_L}$ is continuous. In sum, Γ_{qq} is a compound mapping of continuous mappings—hence itself is continuous. Since $[0,1]^{|\tilde{\mathcal{S}}\times\mathcal{Z}_p|}$ is compact and convex, by Brouwer's Fixed Point theorem, there exists a fixed point to Γ_{qq}. ■

Proposition 2. Under assumptions 1 to 4, there exists an equilibrium.

Proof: By Lemma 3, denote q^* as a fixed point of Γ_{qq}. Define a strategy profile $\varrho^* \equiv \Gamma_{q\varrho}(q^*)$. From (2.17), we have

$$\Gamma_{\varrho q}(\varrho^*) = \Gamma_{\sigma\mathcal{F}_L,q}\left(\Gamma_{\varrho\sigma}(\varrho^*), \Gamma_{\mathcal{P}_L\mathcal{F}_L}\left(\Gamma_{\sigma\mathcal{P}_L}\left(\Gamma_{\varrho\sigma}(\varrho^*)\right)\right)\right).$$

Substituting $\varrho^* = \Gamma_{q\varrho}(q^*)$ into the above equation, we have

$$\begin{aligned}\Gamma_{\varrho q}(\varrho^*) &= \Gamma_{\sigma\mathcal{F}_L,q}\left(\Gamma_{\varrho\sigma}\left(\Gamma_{q\varrho}(q^*)\right), \Gamma_{\mathcal{P}_L\mathcal{F}_L}\left(\Gamma_{\sigma\mathcal{P}_L}\left(\Gamma_{\varrho\sigma}\left(\Gamma_{q\varrho}(q^*)\right)\right)\right)\right) \\ &= \Gamma_{\sigma\mathcal{F}_L,q}\left(\Gamma_{q\sigma}(q^*), \Gamma_{\mathcal{P}_L\mathcal{F}_L}\left(\Gamma_{\sigma\mathcal{P}_L}\left(\Gamma_{q\sigma}(q^*)\right)\right)\right) = \Gamma_{qq}(q^*) = q^*.\end{aligned} \tag{2.19}$$

The first equality follows directly from the substitution. The second equality holds because $\Gamma_{q\sigma} = \Gamma_{\varrho\sigma}\left(\Gamma_{q\varrho}\right)$ as defined earlier. The third equality holds also by definition, i.e., equation (2.18). The last equality holds because q^* is a fixed point of Γ_{qq} by definition. Therefore, we have

$$\Gamma_{\varrho\varrho}\left(\varrho^*\right) = \Gamma_{q\varrho}\left(\Gamma_{\varrho q}\left(\varrho^*\right)\right) = \Gamma_{q\varrho}\left(q^*\right) = \varrho^*.$$

The first equality holds by definition. The second equality holds due to (2.19). The last equality holds by definition of ϱ^*. Therefore, ϱ^* is an equilibrium profile. ■

Representation Corollary. There exists a one-to-one mapping of the set $\mathcal{A}_\varrho = \left\{\varrho : \varrho = \Gamma_{\varrho\varrho}\left(\varrho\right)\right\}$ onto the set $\mathcal{A}_q = \left\{q : q = \Gamma_{qq}\left(q\right)\right\}$.

Proof: We will show that $\Gamma_{\varrho q}$ is a one-to-one mapping of $\mathcal{A}_\varrho$ onto $\mathcal{A}_q$.

(i) The mapping indeed maps $\mathcal{A}_\varrho$ to $\mathcal{A}_q$. For any $\varrho \in \mathcal{A}_\varrho$, define $q \equiv \Gamma_{\varrho q}\left(\varrho\right)$. By definition, we have

$$\Gamma_{\varrho q}\left(\varrho\right) = \Gamma_{\varrho q}\left(\Gamma_{\varrho\varrho}\left(\varrho\right)\right) = \Gamma_{\varrho q}\left(\Gamma_{q\varrho}\left(\Gamma_{\varrho q}\left(\varrho\right)\right)\right) = \Gamma_{\varrho q}\left(\Gamma_{q\varrho}\left(q\right)\right) = \Gamma_{qq}\left(q\right).$$

The first equality holds because $\varrho \in \mathcal{A}_\varrho$. The second equality holds because $\Gamma_{\varrho\varrho} = \Gamma_{q\varrho}\left(\Gamma_{\varrho q}\right)$ by definition. The third equality holds because $q = \Gamma_{\varrho q}\left(\varrho\right)$ by definition. The last equality holds because $\Gamma_{qq} = \Gamma_{\varrho q}\left(\Gamma_{q\varrho}\right)$ by definition. In sum, we have $q \equiv \Gamma_{\varrho q}\left(\varrho\right) = \Gamma_{qq}\left(q\right)$, which implies $q \in \mathcal{A}_q$.

(ii) It is a one-to-one mapping. Assume $\mathcal{A}_{\varrho}$ is not a singleton (i.e., there are multiple equilibriums); otherwise one-to-one is implied. Let $\varrho_1, \varrho_2 \in \mathcal{A}_{\varrho}$ and $\varrho_1 \neq \varrho_2$. Define $q_1 \equiv \Gamma_{\varrho q}(\varrho_1)$ and $q_2 \equiv \Gamma_{\varrho q}(\varrho_2)$. Suppose $q_1 = q_2 = \tilde{q}$. Define $\tilde{\varrho} = \Gamma_{q\varrho}(\tilde{q})$. Hence, we have

$$\tilde{\varrho} = \Gamma_{q\varrho}(\tilde{q}) = \Gamma_{q\varrho}(q_1) = \Gamma_{q\varrho}(\Gamma_{\varrho q}(\varrho_1)) = \Gamma_{\varrho\varrho}(\varrho_1) = \varrho_1.$$

The first equality holds by definition. The second equality holds by assumption $q_1 = \tilde{q}$. The third equality holds by definition of q_1. The fourth equality holds by definition, i.e., $\Gamma_{\varrho\varrho} = \Gamma_{q\varrho}(\Gamma_{\varrho q})$. The last equality holds because $\varrho_1 \in \mathcal{A}_{\varrho}$. Similarly, we have

$$\tilde{\varrho} = \Gamma_{q\varrho}(\tilde{q}) = \Gamma_{q\varrho}(q_2) = \Gamma_{q\varrho}(\Gamma_{\varrho q}(\varrho_2)) = \Gamma_{\varrho\varrho}(\varrho_2) = \varrho_2.$$

Therefore, we have $\tilde{\varrho} = \varrho_1 \neq q_2 = \tilde{\varrho}$, which is a contradiction. Hence, we must have $q_1 \neq q_2$. Therefore, $\Gamma_{\varrho q}$ is a one-to-one mapping.

(iii) It is onto. For any $q \in \mathcal{A}_q$. Define $\varrho \equiv \Gamma_{q\varrho}(q)$. First, we have

$$\Gamma_{\varrho\varrho}(\varrho) = \Gamma_{q\varrho}(\Gamma_{\varrho q}(\varrho)) = \Gamma_{q\varrho}(\Gamma_{\varrho q}(\Gamma_{q\varrho}(q))) = \Gamma_{q\varrho}(\Gamma_{qq}(q)) = \Gamma_{q\varrho}(q) = \varrho.$$

The equality holds because $\Gamma_{\varrho\varrho} = \Gamma_{q\varrho}(\Gamma_{\varrho q})$. The second equality holds by definition $\varrho \equiv \Gamma_{q\varrho}(q)$. The third equality holds because $\Gamma_{qq} = \Gamma_{\varrho q}(\Gamma_{q\varrho})$. The fourth equality holds because $q \in \mathcal{A}_q$. The last equality holds by definition $\varrho \equiv \Gamma_{q\varrho}(q)$. Therefore, we have $\varrho \in \mathcal{A}_{\varrho}$. Second, we have $\Gamma_{\varrho q}(\varrho) = \Gamma_{\varrho q}(\Gamma_{q\varrho}(q)) = \Gamma_{qq}(q) = q$. The first equality holds by definition. The second equality holds because $\Gamma_{qq} = \Gamma_{\varrho q}\left(\Gamma_{q\varrho}\right)$. The last equality holds because $q \in \mathcal{A}_q$. Therefore, for any $q \in \mathcal{A}_q$, there exists $\varrho \in \mathcal{A}_\varrho$ such that $\Gamma_{\varrho q}(\varrho) = q$, i.e., it is onto. (i), (ii), and (iii) complete the proof. ■

2.9.4 Verification of Assumption 4

We have

$$u_{ib}\left(\tilde{S}_{it}, \epsilon_{it}, \xi_{it}\right) = \underbrace{\gamma_{i,n+1} - \gamma_{in} - \beta_i \cdot \tilde{p}_{1t} - c_{ib}}_{U_{ibt}} + \underbrace{\epsilon_{i,n+1,t} - \epsilon_{int}}_{\tilde{\epsilon}_{i,n+1,t}} - \tilde{p}_{1t} \cdot \xi_{it},$$

and

$$\begin{aligned} u_{ip}\left(\tilde{S}_{it}, \epsilon_{it}, \xi_{it}\right) = & \underbrace{q\left(\tilde{S}_{it}, p\right) \cdot \left[\gamma_{i,n-1,t} - \gamma_{int} + \beta_i \cdot (1-\tau_1) \cdot p\right] - \tau_2 \beta_i p - c_{is}}_{U_{ipt}} \\ & - q\left(\tilde{S}_{it}, p\right) \cdot \underbrace{\left(\epsilon_{int} - \epsilon_{i,n-1,t}\right)}_{\tilde{\epsilon}_{int}} + p \cdot \left[q\left(\tilde{S}_{it}, p\right) \cdot (1-\tau_1) - \tau_2\right] \cdot \xi_{it}. \end{aligned}$$

To simplify notations, we drop $\tilde{S}_{it}$ in q and the subscripts i and t. The probability of no trade is:

$$\begin{aligned} \ell(a_0) = & \mathbb{I}(n>0) \cdot \int_{\xi} \int_{-\infty}^{-U_b + \tilde{p}_1 \cdot \xi} \left[1 - F_{\tilde{\epsilon}_n | \tilde{\epsilon}_{n+1}, \xi}\left(\max_p \left\{\frac{U_p + p \cdot \left[q_p \cdot (1-\tau_1) - \tau_2\right] \cdot \xi}{q_p}\right\}\right)\right] dF_{\tilde{\epsilon}_{n+1}|\xi} dF_{\xi} \\ & + \mathbb{I}(n=0) \cdot \Phi\left(-\frac{U_b}{\sigma_{\tilde{\epsilon}_{n+1} - \tilde{p}_1 \cdot \xi}}\right), \end{aligned} \tag{2.20}$$

where $\sigma_{\tilde{\epsilon}_{n+1}-\tilde{p}_1\cdot\xi} = \sqrt{1+\tilde{p}_1^2\cdot\sigma_\xi^2}$. The probability of purchase is as follows:

$$\ell(a_b) = \mathbb{I}(n>0)\cdot\left[1-\int_\xi\int_{\tilde{\epsilon}_n} F_{\tilde{\epsilon}_{n+1}|\tilde{\epsilon}_n,\xi}\left(\max\left\{\begin{array}{l}-U_b+\tilde{p}_1\cdot\xi, \\ \max_p\left\{\begin{array}{l}U_p-U_b-q_p\tilde{\epsilon}_n \\ +\left[p\cdot\left[q_p\cdot(1-\tau_1)-\tau_2\right]+\tilde{p}_1\right]\cdot\xi\end{array}\right\}\end{array}\right\}\right)dF_{\tilde{\epsilon}_n|\xi}dF_\xi\right] \tag{2.21}$$
$$+\mathbb{I}(n=0)\cdot\left[1-\Phi\left(-\frac{U_b}{\sigma_{\tilde{\epsilon}_{n+1}-\tilde{p}_1\cdot\xi}}\right)\right].$$

Equations (2.20) and (2.21) show that the integrand is continuous at q—hence the optimal choice probability is continuous at q. It suffices to show that the probability of listing at price p is continuous at q. Let

$$v_l(q) = \max\left\{\begin{array}{l}\dfrac{U_{p'}-U_p}{p\cdot\left[q(p)\cdot(1-\tau_1)-\tau_2\right]-p'\cdot\left[q(p')\cdot(1-\tau_1)-\tau_2\right]}, \\ \text{for }\left\{p'\neq q: q(p)=q(p'),\ p\cdot\left[q(p)\cdot(1-\tau_1)-\tau_2\right]>p'\cdot\left[q(p')\cdot(1-\tau_1)-\tau_2\right]\right\}\end{array}\right\}$$

and

$$v_u(q) = \min\left\{\begin{array}{l}\dfrac{U_{p'}-U_p}{p\cdot\left[q(p)\cdot(1-\tau_1)-\tau_2\right]-p'\cdot\left[q(p')\cdot(1-\tau_1)-\tau_2\right]}, \\ \text{for }\left\{p'\neq q: q(p)=q(p'),\ p\cdot\left[q(p)\cdot(1-\tau_1)-\tau_2\right]<p'\cdot\left[q(p')\cdot(1-\tau_1)-\tau_2\right]\right\}\end{array}\right\}.$$

Fix any q, for any $\tilde{S}$ and p such that $v_u(q)>v_l(q)$, the probability of listing at p is:

$$\ell(p) = \int_{v_l(q)}^{v_u(q)}\int_{\tilde{\epsilon}_{n+1}}\left[\max\left\{\ell_1(p,\tilde{\epsilon}_{n+1},\xi)-\ell_2(p,\tilde{\epsilon}_{n+1},\xi),0\right\}\right]dF_{\tilde{\epsilon}_{n+1}|\xi}dF_\xi,$$

where

$$\ell_1 = F_{\tilde{\epsilon}_n|\tilde{\epsilon}_{n+1},\,\xi}\left(\min\left\{ \begin{array}{l} \left(U_p + p\cdot\left[q_p\cdot(1-\tau_1)-\tau_2\right]\cdot\xi\right)\cdot q_p^{-1}, \left(U_p - U_b - \tilde{\epsilon}_{n+1} + \left(p\cdot\left[q_p\cdot(1-\tau_1)-\tau_2\right] + \tilde{p}_1\right)\cdot\xi\right)\cdot q_p^{-1}, \\ \min\limits_{p^{'}\in\left\{q_p - q_{p^{'}}>0,\ p^{'}\neq p\right\}} \left\{ \frac{U_p - U_{p^{'}} + p\cdot\left[q_p\cdot(1-\tau_1)-\tau_2\right]\cdot\xi}{q_p - q_{p^{'}}} - \frac{p^{'}\cdot\left[q_{p^{'}}\cdot(1-\tau_1)-\tau_2\right]\cdot\xi}{q_p - q_{p^{'}}} \right\} \end{array} \right\} \right)$$

and

$$\ell_2 = F_{\tilde{\epsilon}_n|\tilde{\epsilon}_{n+1},\,\xi}\left(\max\limits_{p^{'}\in\left\{q_p - q_{p^{'}}<0,\ p^{'}\neq p\right\}} \left\{ \frac{U_p - U_{p^{'}} + p\cdot\left[q_p\cdot(1-\tau_1)-\tau_2\right]\cdot\xi}{q_p - q_{p^{'}}} - \frac{p^{'}\cdot\left[q_{p^{'}}\cdot(1-\tau_1)-\tau_2\right]\cdot\xi}{q_p - q_{p^{'}}} \right\} \right).$$

Since v_u , v_l, and the integrand are continuous at q, $\ell(p)$ is continuous at q for $\tilde{S}$ and p such that $v_u(q) > v_l(q)$. For $\tilde{S}$ and p such that $v_u(q) < v_l(q)$, $\ell(p) = 0$. Since v_u and v_l are continuous, for any $q^{'}$ in a small enough neighbourhood of q, we have $v_u(q^{'}) < v_l(q^{'})$ which implies $\ell(p^{'}) = 0$. Therefore, $\ell(p)$ is also continuous at q for $\tilde{S}$ and p such that $v_u(q) < v_l(q)$. For $\tilde{S}$ and p such that $v_u(q) = v_l(q)$, we have $\ell(p) = 0$ because ξ is a continuous random variable. Consider a small perturbation of q, i.e., $q^{'}$, such that $v_u(q^{'}) > v_l(q^{'})$. We have

$$\ell(p) \leq \int_{v_l(q^{'})}^{v_u(q^{'})} f_\xi(\xi)\, d\xi = F_\xi\left(v_u\left(q^{'}\right)\right) - F_\xi\left(v_l\left(q^{'}\right)\right),$$

where F_ξ is the c.d.f. of ξ. Since F_ξ is continuous, for $\tilde{S}$ and p such that $v_u(q) = v_l(q)$, for any $a > 0$, there exists a small neighbourhood such that $\ell(p) < a$ for all $q^{'}$ within it. This implies that $\ell(p)$ is also continuous for $\tilde{S}$ and p such that $v_u(q) = v_l(q)$. This completes the verification.

2.10 Tables and Figures

Table 2.1: Summary Statistics Across Players

	Min	Q25	Median	Mean	Q75	Max
# of trades of the focal product	1	1	2	3.4	4	18
# of listings of the focal product	0	1	1	1.9	2	9
# of purchases of the focal product	0	0	0	1.6	2	14
Contribution to the total market value	0.004%	0.01%	0.02%	0.03%	0.03%	0.13%

Note. The total market value is the summation of total listing value and total purchase value.

Table 2.2: Fixed-effect Logistic Regression of Purchase or Not when Endowment is Zero

DV: purchase or not	Estimate (Std. Err)			
Minimum price at trip *t*	-0.8002*** (0.2100)	-1.3647*** (0.3865)	-2.3727*** (0.3478)	-1.7058*** (0.7124)
Mean price at trip *t*	--------	0.6438 (0.4455)	--------	-0.8965 (0.7608)
Total number of existing listings at trip *t*	--------	0.0005 (0.0012)	--------	-0.0014 (0.0016)
Minimum price at trip *t+1*	--------	--------	0.9722 (0.6761)	0.7551 (0.7158)
Mean price at trip *t+1*	--------	--------	0.8067 (0.6498)	1.2046 (0.8418)
Total number of existing listings at trip *t+1*	--------	--------	0.0013 (0.0012)	0.0021 (0.0015)
P-value of Wald test for hourly dummies	--------	0.6499	0.7339	0.6974
BIC	6189	6430	6249	6269

Table 2.3: Discrete Choice and Listing-Price Regressions when Endowment is Positive

Variables (baseline is *n* = 1)	Discrete choice regression		Listing-price regression
	Listing utility	Purchase utility	
Min price	--------	-0.5528*** (0.0875)	--------
Mean price	0.1200* (0.0684)	--------	0.8347*** (0.0175)
Total number of listings	-0.0007* (0.0004)	--------	-0.0005*** (0.0001)
Expected number of trips in the next 12 hours	-0.0415*** (0.0086)	--------	-0.0062*** (0.0022)

Note. For the discrete choice regression, an observation is a trip with positive endowment. We control individual random effects separately for listing and purchase intercepts. For the listing-price regression, an observation is a trip in which the player decided to list. We control for individual fixed effect.

Table 2.4: Structural Parameter Estimation Results

		Variable	Estimate	Std. Err
Mean parameters	Utility gains	$\gamma_1 - \gamma_0$	1.2341	0.0060
		$\gamma_2 - \gamma_1$	1.9644	0.0122
		$\gamma_3 - \gamma_2$	2.0631	0.0172
		$\gamma_4 - \gamma_3$	2.4606	0.0242
	Price coef.	β	0.9175	0.0011
	Latent costs	c_b	2.6392	0.0137
		c_s	0.0144	0.0003
Heterogeneity parameters	Utility gains	$\sigma_{\gamma_1-\gamma_0}$	0.4519	0.0058
		$\sigma_{\gamma_2-\gamma_1}$	0.1802	0.0120
		$\sigma_{\gamma_3-\gamma_2}$	0.2248	0.0195
		$\sigma_{\gamma_4-\gamma_3}$	0.1284	0.0246
	Price coef.	σ_β	0.4836	0.0004
	Latent costs	σ_{c_b}	0.9038	0.0091
		σ_{c_s}	0.0759	0.0007

Table 2.5: Model Fit

	Total fees	Commission	Listing fee	# of listings	# of transactions	Mean listing price	Mean transaction price
Full model	1.9%	2.7%	< 0.1%	3.6%	3.4%	-4.2%	-0.8%
Without latent costs	181%	195%	147%	90%	114%	28%	38%

Note. We simulate the market for a year under each model. Since the sample length is 15 days, we compute the average summary per 15 days. The numbers are percentage deviance of the simulation to the observed data. They are calculated as illustrated in the following example: for total fees, in the full model, 1.9% = (the total fees collected under simulation / the total fees collected observed in the data) - 1.

Table 2.6: Listing Fee vs. Commission: Fee Collection, Listing/Transaction Volumes

Ratio of Listing Fee to Commission - 1	Listing Fee/Commission	
	5%	10%
Fee Collection	17.4%	13.5%
Listing Volume	-13.0%	-15.4%
Transaction Volume	-0.1%	-0.9%

Note. The values are calculated as illustrated in the following example: in the case of 5%, 17.4% = (revenue with 5% listing fee only) / (revenue with 5% commission only) - 1.

Figure 2.1: Time Distribution of Activity

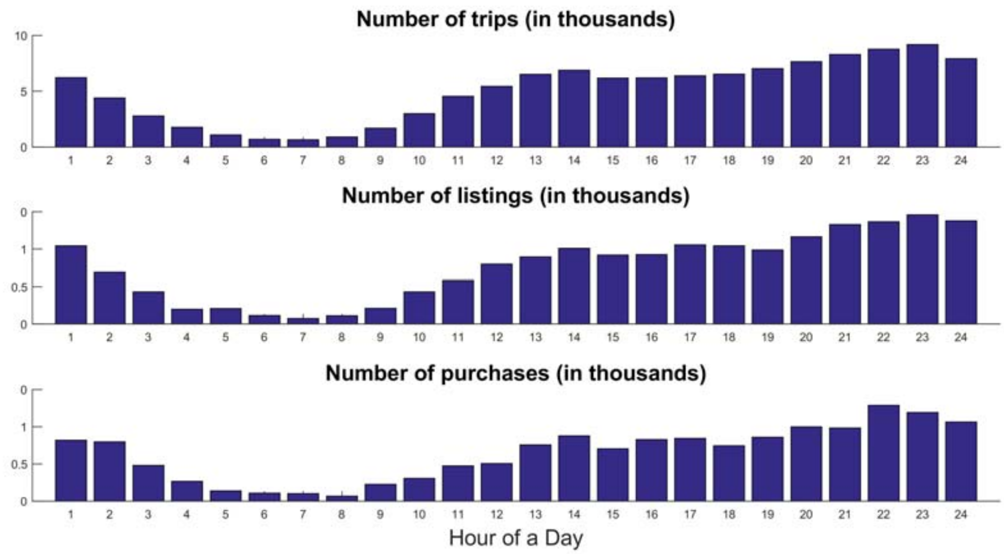


Figure 2.2: Number of Listings vs. Latent Cost of Listing

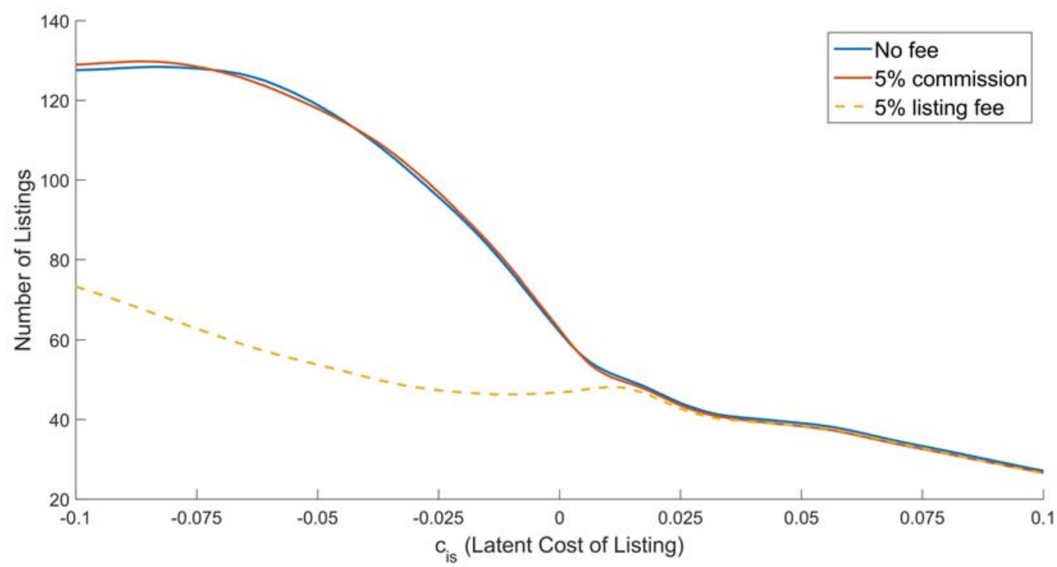


Figure 2.3: Mean Listing Price vs. Latent Cost of Listing

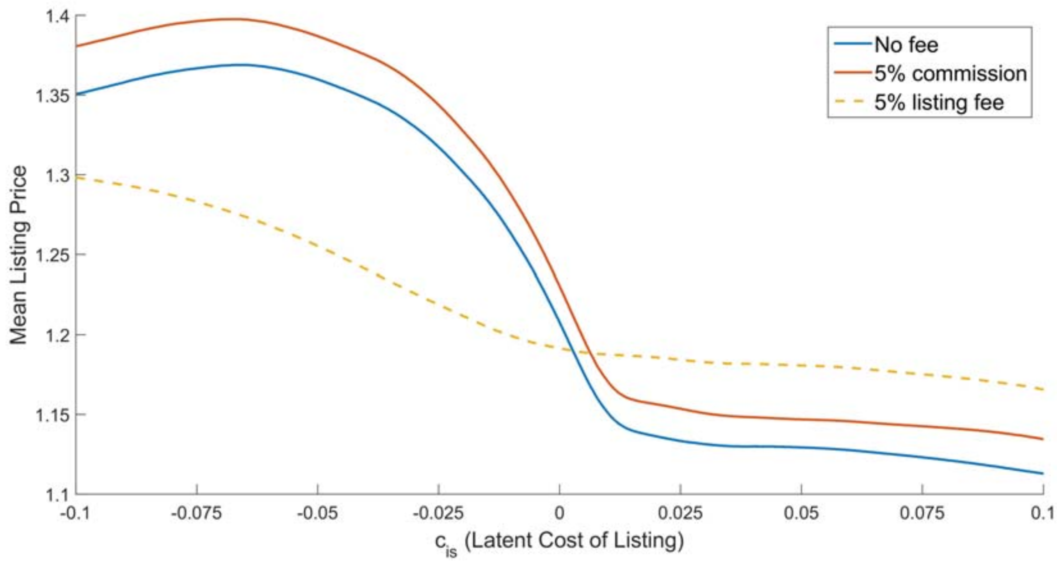


Figure 2.4: Mean Sale Probability vs. Latent Cost of Listing

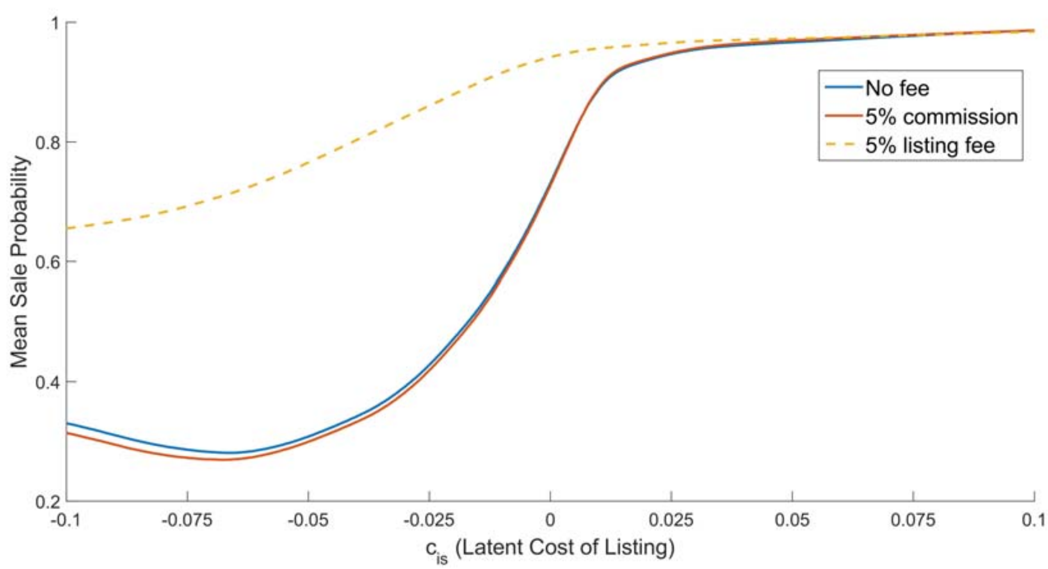


Figure 2.5: Histogram of Purchase Probabilities

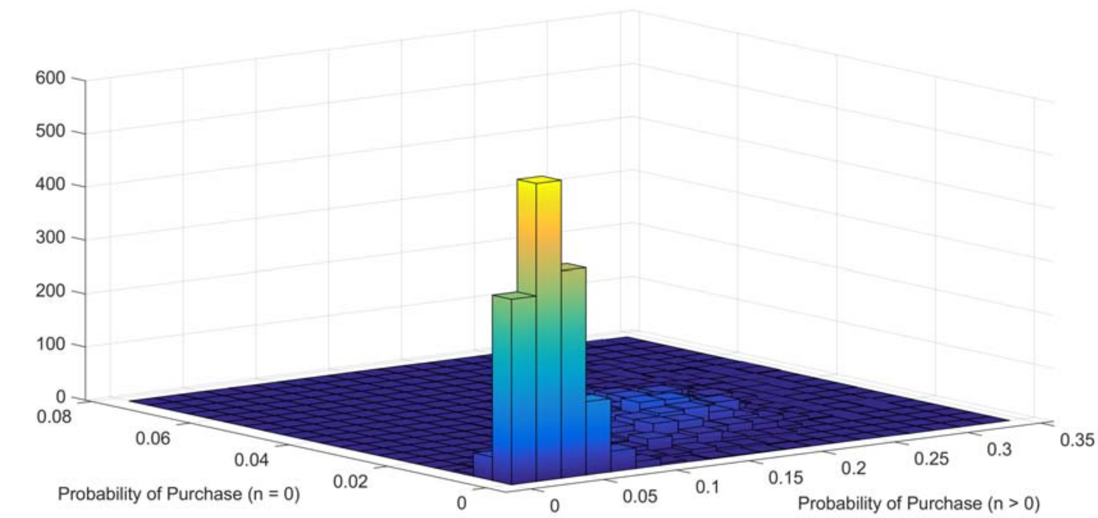
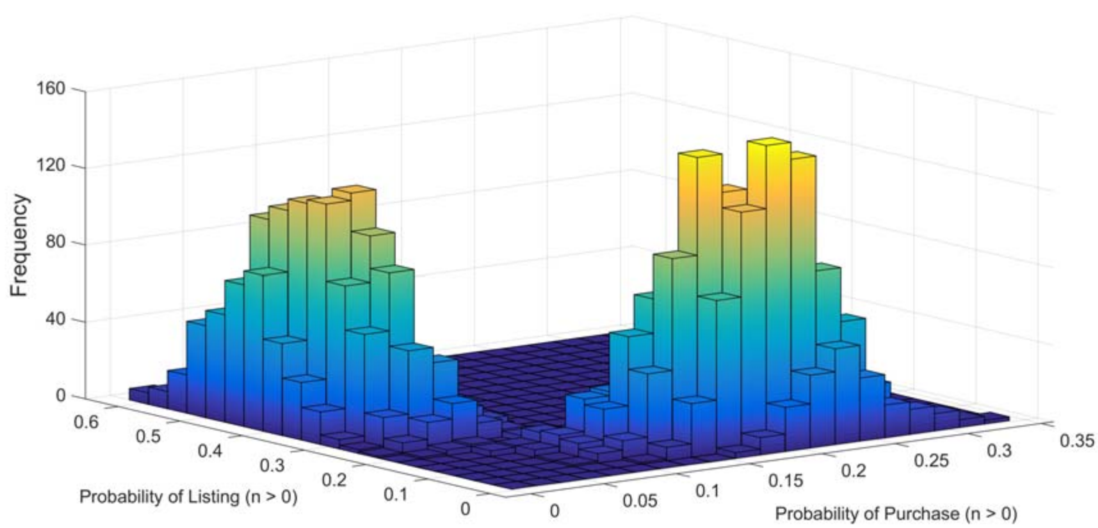


Figure 2.6: Histogram of Purchase and Listing Probabilities when Endowment is Positive



Chapter 3: Sales Mechanism

3.1 Introduction and Literature Review

Along with fee structure variation, MMOGs differ in terms of the sales mechanism in their P2P markets. The two most popular mechanisms are fixed-price posting and Buy-It-Now (BIN) English auction. The BIN English auction is a combination of English auction and fixed-price posting, in which a buy-now price can also be set by the seller, and the auction will end prematurely when a buyer pays the buy-now price. Since we observe only fixed-price posting in the data, the goal of this chapter is to develop a tractable auction framework and to conduct preliminary analysis on their differential impacts on market outcomes (listing/transaction volume and fee collection), conditional on the status-quo fee structure observed in the data for the fixed-price posting market. As an initial analysis, we explore a simpler case: English auction vs. fixed-price posting. We leave the examination of the BIN English auction for future research with appropriate data.

For the rest of this section, we first review the relevant literature and state our contribution, then discuss the conceptual distinction between auction and fixed-price posting.

3.1.1 Literature Review and Contribution

A number of theoretical studies examine the question of which sales mechanism generates higher profit for the seller. Under different settings and assumptions, the conclusions from this literature are mixed. Under the independent private value assumption, Wang (1993) shows that for a single seller, the two mechanisms—auction and fixed-price posting—are equivalent when the latent cost of listing in the fixed-price posting market is the same as the latent cost of posting an auction in the auction market. When the auction latent cost is greater, fixed-price posting can generate higher profit than auction if the dispersion of bidder valuation is large enough. Kultti (1999) considers a case with many sellers and buyers without heterogeneity: all sellers value the good equally and so do the buyers (though sellers and buyers value the good differently). He considers a market allowing both auction and fixed-price posting, where all sellers and buyers can choose which one to enter. In equilibrium, he shows that all agents are indifferent between auction and fixed-price posting, i.e., they are practically equivalent. Campbell and Levin (2006) consider that buyers' values are interdependent and show that auction is not always optimal for a single seller in terms of expected profits. These studies highlight two important factors in determining the relative performance of auction and fixed-price posting: mechanism-specific latent costs and heterogeneity among agents, which will be incorporated in the model.

Most of the empirical work on the use of different types of sales mechanisms is based on data from eBay. Hammond (2010) uses data on compact discs from eBay to show that the revenue equivalence hypothesis cannot be rejected. Hammond (2013) investigates the reasons for the co-existence of auction and fixed-price posting on eBay. He finds that seller-heterogeneity is more accountable than buyer-heterogeneity, and auction (vs. fixed-price posting) generates higher buyer surplus but lower seller surplus. Bauner (2015) considers a slightly different question, i.e., the welfare and revenue implications of allowing more sales mechanisms in a seller's choice set, using data from eBay. He finds that the addition of auction to fixed-price posting benefits some types of buyers (but not others), sellers, and the platform. Furthermore, the addition of BIN-auction to auction and fixed-price posting benefits all buyers but hurts sellers and the platform. Einav *et al.* (2017) examine the reasons for a recent trend (2003 to 2009) on eBay—the decline in the relative popularity of auction as compared to fixed-price posting when both options are available—and they find that a fall in the relative demand by bidders for auction (vs. fixed-price posting) is the most plausible explanation.

As indicated in Chapter 1, the key contributions of this chapter are as follows. First, we study the design of sales mechanisms in P2P markets in the online gaming industry, which is large and rapidly growing but lacks systematic quantitative research. Second, from a broader perspective, we contribute to the empirical literature on sales mechanisms, as mentioned above. We provide a systematic comparison between an auction-only market and a fixed-price posting-only market on platform revenues as well as listing and transaction volumes by developing a tractable model of a dynamic market in which players can choose between buying and selling.

3.1.2 Conceptual Discussion

Drawing from the literature, we note that a key conceptual difference between the two sales mechanisms is that auction creates competition among bidders. This has two important implications. First, from a seller's perspective, it is less costly to list in auction (vs. fixed-price posting) because the final transaction price (the highest bid) is no smaller than the listing price (which is the reserve price in auction), conditional on the item receiving at least one bid.

Therefore, under auction, players can set a low listing price to increase sale probability and avoid paying a high listing fee. More importantly, to do this, they don't need to sacrifice the final transaction price too much due to the competition among bidders. Therefore, players may have higher incentive to list. Second, from a buyer's perspective, *ceteris paribus*, bidding at an auction has lower expected return than buying a fixed-price listing. In auction, due to competition among bidders, a bidder is uncertain of winning while in fixed-price posting, paying the listing price ensures obtaining the item. As a result, in auction, players have lower incentive to bid (than to buy in fixed-price posting), but those who are willing to bid will bid aggressively to increase their chances of winning. Therefore, compared to fixed-price posting, auction may deter demand but drive up transaction price. Thus we expect auction to generate higher listing volume but lower transaction volume compared to fixed-price posting. However, given the interplay of the two factors mentioned above, in particular their impacts on prices, it is uncertain which one would generate a higher fee collection.

In what follows, we develop the auction model by adapting the fixed-price posting model detailed in Chapter 2. We then conduct counterfactual simulations based on the parameters recovered in Chapter 2 to compare the two sales mechanisms given the status-quo fee structure of the focal game. We provide managerial implications and close the chapter with conclusions.

3.2 Model

We consider a market with English auction as the only sales mechanism. We modify the framework in Chapter 2 so that it is similar to what is commonly used by other games that adopt auction while maintaining comparability with the status-quo policies. The auction rule is as follows. The player with the highest bidding price (equal to or greater than the listing price) within 12 hours[48] of listing wins the product and pays her bidding price. If no one bids within 12 hours, the auction expires.

Following the status-quo fee structure, we assume the game charges a 1% listing fee (here as a percentage of reserve price paid by the seller upfront) and 3% commission (here as a percentage of the final transaction price paid by the seller when the transaction occurs). To be more specific, the commission is deducted from the final transaction price, and the seller receives the highest bid price less the commission.

The model specification is as follows. In each period, at most one player makes a trip to the market according to the same trip process described in Chapter 2. The player observes the following information: (1) all current listings and their most current bidded prices (if a listing hasn't been bidded on yet, its most current bidded price is the listed reserve price)[49], (2) her own endowment, (3) hour of the day, and (4) idiosyncratic preference shocks. To be consistent with the focal game, time to expiration is not displayed[50]. The player chooses among three options: (1) selecting an existing listing to bid on and setting the bidding price, (2) listing an item for sale and setting the reserve price, and (3) doing nothing. Then the player leaves the market and comes back according to the trip process described in Section 2.5 of Chapter 2. Time moves on to the next period. All other players' endowments transit to the next period according to the same

[48] This is to be consistent with the current policy.
[49] Most MMOGs adopt such design: displaying only the most current bidding price but not the entire bidding history.
[50] This is adopted by many games that only allow for auction mechanism.

endowment process described in Chapter 2. We also assume that once a player bids on a listing, she cannot bid anymore (for all listings) until the outcome of her bidded listing is determined. Relaxing this assumption naturally requires a model that is forward-looking for future trips, which is outside the scope of this thesis and is left for future research.

The utility of bidding price p_b on the j^{th} existing listing is[51]

$$\begin{aligned} u_{ib}(p_b, j; \varrho) = q_{ib}\left(\tilde{S}, p_b, j; \varrho\right) \cdot \left[V_i(n+1, \epsilon_{n+1}) - \beta_i(\xi) \cdot p_b\right] \\ + \left[1 - q_{ib}\left(\tilde{S}, p_b, j; \varrho\right)\right] \cdot V_i(n, \epsilon_n) - c_{ib}, \end{aligned} \tag{3.1}$$

where $q_{ib}\left(\tilde{S}, p_b, j; \varrho\right)$ is the probability of winning the auction when player *i* entering the market with endowment $n \in \tilde{S}$ bids price p_b for the j^{th} existing listing, conditional on the market information $M \subset \tilde{S}$ and the strategy profile ϱ. Here the market information contains the most current bidded prices of all existing listings and hour of a day. Now c_{ib} is the latent cost of bidding. The utility of bidding is similar to the utility of purchase in the fixed-price posting model, except that the probability of winning needs to be incorporated. This reflects the uncertainty of winning in auction (vs. fixed-price posting), as discussed earlier.

The utility of listing an item with reserve price p_s is

[51] Most of the notations are adapted from Chapter 2. For brevity, we don't re-explain them in this chapter.

$$u_{is}(p_s;\varrho) = \sum_{p' \geq p_s} \left\{ q_s\left(\tilde{S},\, p_s,\, p';\varrho\right) \cdot \left[\beta_i(\xi) \cdot (1-\tau_1) \cdot p' + V_i(n-1,\epsilon_{n-1}) \right] \right\} + \left[1 - \sum_{p' \geq p_s} q_s\left(\tilde{S},\, p_s,\, p';\varrho\right) \right] \cdot V_i(n,\epsilon_n) - \beta_i(\xi) \cdot \tau_2 \cdot p_s - c_{is}, \tag{3.2}$$

where $q_{is}\left(\tilde{S},\, p_s,\, p';\varrho\right)$ is the probability that an auction with reserve price p_s by player *i* who enters the market with endowment $n \in \tilde{S}$ ends up with p' as the final highest bid, conditional on the market information $M \subset \tilde{S}$ and the strategy profile ϱ. The utility of listing is similar to that in the fixed-price posting model, except that all possible final transaction prices (greater than or equal to the listing price) need to be considered. This feature—the final transaction price is higher than the listing price—reflects the theoretical discussion earlier that, *ceteris paribus*, players may have a higher incentive to list in auction compared to fixed-price posting. However, (3.2) also shows a counterweight to this incentive, namely that the probability of no bidding $\left[1 - \sum_{p' \geq p_s} q_s\left(\tilde{S},\, p_s,\, p';\varrho\right)\right]$ cannot be too high.

Theoretically, the sale probability q_{is} and the winning probability q_{ib} both depend on *i* for similar reasons as those discussed in Chapter 2—the opponent sets across players are different and players are heterogeneous in their utility functions. Since there is a large number of players, this leads to computational burden. However, following the same reasoning as in Chapter 2, when each player is small, such dependence will be negligible in practice. Therefore, we make a similar assumption as that in Chapter 2: players act as if the sale probability does not depend on the seller's identity, and the winning probability does not depend on the bidder's identity. In other words, we assume $q_{ib}\left(\tilde{S},\, p_b,\, j;\, \varrho\right) = q_b\left(\tilde{S},\, p_b,\, j;\, \varrho\right)$ (the probability of winning the auction when a player entering the market with endowment $n \in \tilde{S}$ bids price p_b for the j^{th} existing listing, conditional on the market information $M \subset \tilde{S}$ and the strategy profile ϱ) and $q_{is}\left(\tilde{S},\, p_s,\, p^{'};\, \varrho\right) = q_s\left(\tilde{S},\, p_s,\, p^{'};\, \varrho\right)$ (the probability that a newly added auction with reserve price p_s by a player who enters the market with endowment $n \in \tilde{S}$ ends up with $p^{'}$ as the final highest bid, conditional on the market information $M \subset \tilde{S}$ and the strategy profile ϱ).

Equilibrium in the auction model can be defined analogously as that in the fixed-price posting model. We assume the existence of equilibrium and it can be represented on the space of $\left(q_b, q_s\right)$. We leave the proof for future research but speculate that it closely resembles that of the fixed-price posting model. For example, similar to the feature in the fixed-price posting model that all strategic interaction is embedded in the sale probability, all strategic interaction here is embedded in both the sale probability and the winning probability—once they are given, the model becomes a single-agent optimization problem. Therefore, we believe it is reasonable to deduce that the equilibrium can be represented by $\left(q_b, q_s\right)$.

Since we don't observe the auction mechanism in the data, we use the structural parameters estimated from the fixed-price posting data in the auction model. In other words, we directly solve the auction counterfactual equilibrium using the parameters estimated from the fixed-price posting data. Therefore, no estimation is performed in the auction model. The counterfactual algorithm is similar to the one described in Appendix C except that the fixed point being solved is (q_b, q_s) instead of just q_s. For future research where the structural parameters need to be recovered from the data, the estimation method can be adapted from the two-step estimation strategy developed in Chapter 2. The key difference is that in step 1, one also needs to estimate q_b in addition to q_s. If a researcher has access to the bidding record, then all elements in q_b are observable and therefore q_b can be consistently estimated from the data.

Since the parameters used in the auction counterfactual simulations are recovered based on the fixed-price posting data, we discuss the potential limitation here. It is worth noting that valuation of the product and price sensitivity (V and β) are unlikely to be mechanism-specific. However, the latent costs c_{is} and c_{ib} are likely to differ across the two sales mechanisms. For example, in terms of the latent cost of listing, the transaction cost of setting up an auction can be different from that of fixed-price posting, possibly due to the difference in costs of finding an optimal reserve price and an optimal fixed listing price. In terms of the latent cost of bidding, it may be greater than the latent cost of purchase in the fixed-price posting model for several reasons. First, as suggested by Einav *et al.* (2016), the relative popularity of auction compared to fixed-price posting is declining. In other words, the disliking (i.e., the latent cost) of auction is greater than that of fixed-price posting. Second, another difference between the two sales mechanisms is the timing of getting the product. In fixed-price posting, a player gets the product immediately after paying the price. In auction, not only is a player uncertain of getting the product, she is also uncertain when she will get the product. This may be undesirable if a player wants to have the product right away to play the game rather than waiting, say, for several hours. Though our model does not structurally incorporate such difference, the relative latent costs in the two sales mechanisms can capture some of this effect, i.e., the latent cost of bidding is greater. As mentioned earlier, we don't have auction data to estimate these parameters, and so we simply use those recovered from the fixed-price posting data in the counterfactual simulations of auction. Therefore, all our results below rest on this limitation, and they should be viewed as an initial investigation of the problem.

3.3 Counterfactual

Given the status-quo fee structure (1% listing fee + 3% commission), we solve the equilibriums under both sales mechanisms. We then simulate the market for a year and report the summary results in Table 3.1. It shows a trade-off between them. Auction generates greater fee collection by 12%, greater listing volume by 66%, but smaller transaction volume by 33% compared to fixed-price posting.

Earlier discussion suggests that it is not surprising to see a higher listing volume and a lower transaction volume in auction. What remains to be explained is the patterns in fee collections. The result shows that auction generates a higher collection in listing fees but a smaller collection in commissions. Overall, auction generates higher total fee collection. We explain these patterns as follows. First, the lower commission collection in auction is obviously due to the smaller transaction volume despite higher transaction prices. Both smaller transaction volume and higher transaction price in auction are consistent with the theoretical discussion earlier. Second, the higher listing-fee collection in auction is due to both higher listing volume and higher mean listing price in auction. The former has been explained before while the latter is less straightforward. According to the discussion above, e.g., players have incentive to set a lower reserve price to increase sale probability (and avoid paying a high listing fee) without sacrificing the final transaction price too much in auction, we should observe lower listing prices. While we do observe a much smaller 25-quartile of listing price under auction, we also observe a much higher 75-quartile of listing price that ultimately drives up the mean listing price. Why?

To answer this question, we further explore the pricing pattern at the individual level and present the result in Figure 3.1. It shows that auction spreads the pricing patterns across the individual latent cost of listing much more extremely than fixed-price posting does. Specifically, the fun-players (those whose latent cost of listing is negative) primarily account for the high-priced listings, while the cost-players (those whose latent cost of listing is positive) mainly account for the low-priced listings in auction. The pattern can be explained as follows. Cost-players have low tolerance of low sale probability and therefore list lower prices. Fun-players, however, have relatively high tolerance of low sale probability. Therefore, if there are enough players who are willing to bid on high-priced listings, then the fun-players are more likely to post high-priced listings. But who will bid on high-priced listings? One possibility is that players who have high bidding costs bid infrequently, but once deciding to bid, they will bid aggressively on listings with high currently bidded prices to ensure winning[52]. This is confirmed by our findings: the individual-level correlations between the latent cost of bidding and the number of bids/mean currently bidded prices from the bidded listings/mean bidding price are -0.77/0.34/0.39. The first number (-0.77) implies that the higher the latent cost of bidding, the less likely that the player

[52] One possible reason why those players choose to bid on listings with high currently bidded prices rather than on listings with low currently bidded prices is that the former have smaller expected time to expiration, which helps to ensure winning.

bids. The second number (0.34) implies that once someone has decided to bid, the higher the latent cost of bidding, the more likely that player chooses to bid on a listing with a high currently bidded price. The last number (0.39) implies that once a player decides which listing to bid on, the higher the latent cost of bidding, the higher the price that the player bids. In sum, those high-priced listings listed by the fun-players are used to target players with high bidding cost. This completes the explanation in Table 3.1.

To summarize, our preliminary results show that there is a trade-off between auction and fixed-price posting. Auction generates higher fee collection (due to higher listing-fee collection) and higher listing volume, but lower transaction volume compared to fixed-price posting, conditional on the status-quo fee structure. The driving force of such a pattern is the competition among bidders that is created by auction (vs. fixed-price posting), which leads to more listing but less bidding, as well as more high-priced listings and biddings (the leading reason why auction generates a higher fee collection).

3.4 Discussion and Managerial Implications

In this chapter, we extend the fixed-price posting model to an auction model, which may be suitable when appropriate auction data from an online video game are available. Using the parameters recovered from the fixed-price posting data, we conduct preliminary analysis to compare fixed-price posting and English auction. Conditional on the status-quo fee structure, we find that there is a trade-off between the two sales mechanisms: auction generates higher fee collection and listing volume but lower transaction volume.

This provides an explanation for why English auction is not popular in the industry. It is very likely that games have a minimum requirement for the transaction volume, i.e., they want to ensure that players can at least get what they want (in order to perform better in the game) above some level. The sizable lower transaction volume in English auction makes it an unfavorable choice. On the other hand, the English auction does generate higher fee collection and listing volume. These factors may ultimately motivate games to consider a combination of the English auction and fixed-price posting, i.e., the BIN English auction that is as popular as fixed-price posting in the industry. We again caution readers about the limitation of this study due to data limitation.

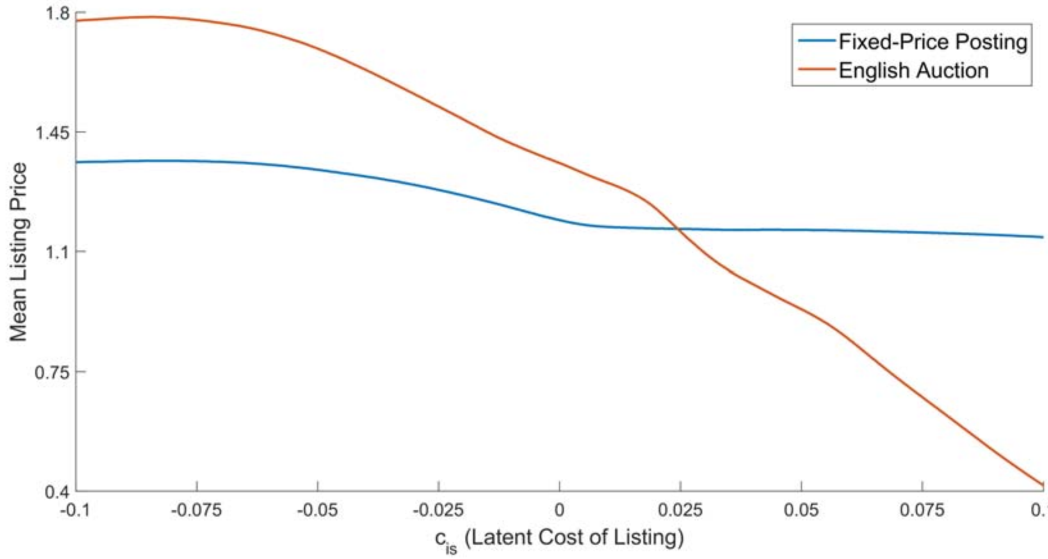
3.5 Tables and Figures

Table 3.1: Auction vs. Fixed-Price Posting (3% Commission + 1% Listing Fee)

		Fixed-price posting	English auction
	Total fees collected	7435	8343 (12%)
	Commission collected	5249	4404 (-16%)
	Listing fee collected	2186	3939 (80%)
	Number of listings (in thousands)	183	304 (66%)
	Number of transactions (in thousands)	153	103 (-33%)
Listing price	25-quartile	1.10	0.21 (-81%)
	Mean	1.19	1.30 (9%)
	75-quartile	1.21	1.93 (60%)
Transaction price	25-quartile	1.09	0.90 (-17%)
	Mean	1.14	1.42 (25%)
	75-quartile	1.17	1.80 (54%)

Note. The numbers in brackets are percentage deviances of English auction to fixed-price posting. For example, for the total fees collected, 12% = 8343 / 7435 - 1.

Figure 3.1: Auction vs. Fixed-Price Posting: Mean Listing Price



Chapter 4: Conclusion

In this thesis, we study the free-to-play business model in the online gaming industry, focusing on the economic design (fee structure and sales mechanism) of the P2P market. Specifically, we compare two different types of fees (listing fee vs. commission) and two sales mechanisms (fixed-price posting vs. English auction) and their effects on the fee collection (short-run revenue) and the listing/transaction volume (indicators of long-run profitability) of the P2P market.

Such a market is similar to many real-world platforms (e.g., eBay), with some unique and important features: non-uniform price for homogenous products, endogenous buying and selling, and psychological factors that can encourage or discourage trading. In addition, the market is characterized by a large number of long-lived players with unobserved heterogeneity, which complicates the analysis considerably. We develop a novel tractable framework under fixed-price posting to model players' trading behavior that can incorporate those features. To tackle the computational burden, we make a novel assumption that players don't use their identities as information to form the sale probability. The assumption is reasonable for a market with a large number of small players. We prove the existence of an equilibrium under this assumption. We develop a two-step estimation strategy that is computationally light and apply our framework to a popular online video game. We also extend the fixed-price posting model to a tractable auction model and conduct a preliminary comparative analysis of the two market structures.

We find that there is a trade-off between the two types of fees: listing fee (vs. commission) generates lower listing volume and very similar transaction volume, but higher fee collection. We also find that there is a trade-off between the two sales mechanisms: conditional on the status-quo fee structure, auction (vs. fixed-price posting) generates higher fee collection and listing volume, but lower transaction volume. We derive managerial implications from these findings.

While we have discussed the assumptions of our research throughout the thesis, we wish to highlight three key limitations that are particularly worthy of future research. First, we consider a single product. Therefore, we select a product that is relatively stand-alone. An interesting extension would be multiple products. The challenge is that the dimension of the price distribution grows exponentially with the number of products. Second, we don't consider forward-looking behavior. Specifically, we assume that players are not forward-looking for their future trips to the market. Although we demonstrate that such behavior is not significant in the current application, it would be interesting to extend the model in this direction while maintaining that players have unobserved heterogeneities. We speculate that in this case it’s possible to represent the equilibrium on the space containing the sale probability and the state transition. Further investigation is left for future research. Third, due to data limitation, we are unable to recover two key parameters in the auction model, namely the latent cost of listing (in an auction) and of bidding. Given such constraint, the goal of the present auction study is to develop a tractable framework that is suitable for the online video games industry. We hope it can provide insights to future studies with more appropriate auction data in this industry.

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Appendices

Appendix A Construction of Estimation Sample

We detail how to construct a trip from the data. We first assume that a player makes at least one trade (for any of the game's products) during a trip. Although each trade is observed, we don't know whether some of them happened in the same trip. Considering the time between two consecutive trades by a player, the data show that 86.5% of these trades occur within 5 minutes of each other. Therefore, we define two consecutive trades as happening in one trip if the time between them is less than 5 minutes. For example, if a player has trading records at minutes 1, 4, and 8 (possibly for different products), all three trades will be considered as happening within one trip. In this way, we can construct trips from the data.

Next, we explore how many possible trades for the focal product can happen in a trip. We focus on trips in which the focal product is traded and find that 95.6% of the time there is only one trade for the focal product. Thus, we assume that a player considers trading the focal product only once during a trip. We divided those trips with multiple trades for the focal product into separated trips such that each contains only one trade for the focal product. This gives us the final estimation sample.

Appendix B Discretization of State Space and Action Space

First, we need to discretize the market price distribution, which can be represented as follows. Fix the minimum price and then count the number of listings between (x%, y%] away from it. We discretize the minimum price to the following eight levels so that the empirical distribution for each range is approximately uniform: (0, 0.9], (0.9, 1], (1, 1.1], (1.1, 1.2], (1.2, 1.3], (1.3, 1.4], (1.4, 1.6], (1.6, infinity). We set the (x, y] pairs to be: (0, 0], (0, 2.5%], (2.5%, 5%], (5%, 7.5%], (7.5%, 10%], (10%, 15%], (15%, 25%], (25%, infinity). Then based on the empirical distribution, we discretize the number of listings under the first range to be [1, 3], [4, 6], [7, infinity) and for other ranges to be: 0, [1, 3], [4, infinity).

Second, we need to discretize the pricing decision. The pricing decision can be represented as between (x%, y%] away from the minimum price. The (x, y] pairs for the pricing discretization is the same as those for the market price discretization discussed above, except that we allow a player to price below the current minimum price on the market. Therefore, we include two additional ranges:(-100%, -5%] and (-5%, 0)[53].

[53] For the range (-5%, 0), it does not include zero because there is a separate range (0, 0], which means pricing exactly at the current minimum price on the market.

Appendix C Counterfactual Algorithm

We follow Goettler *et al.* (2005), which is modified from Pakes and McGuire (2001). We simulate a market session and update the sale probability $q(\cdot)$ until convergence. We use the following procedure to update $q(\cdot)$. We start the algorithm at $t = 0$. Denote the version of $q(\cdot)$ at the beginning of period *t* as $q^t(\cdot)$. During each period:

Step 1. Draw a player for entry. Draw the player's random shocks.

Step 2. Compute the player's optimal strategy (i.e., listing, purchasing or doing nothing) according to $q^t(\cdot)$. If it is not to purchase, go to step 3. If it is to purchase, then we update the sale probability. Suppose the listing being purchased was listed under $\tilde{S}$ and at price p, then $q^{t+1}(\tilde{S}, p) = \frac{n}{n+1} q^t(\tilde{S}, p) + \frac{1}{n+1}$, where n is the number of listings that was listed under $\tilde{S}$ and at price p before t.

Step 3. If there are listings expired after time *t* which were listed under $\tilde{S}$ and at price p, we update the sale probability as $q^{t+1}(\tilde{S}, p) = \frac{n}{n+1} q^t(\tilde{S}, p)$.

Time moves on to next period. We iterate until the sale probability converges.